Lecutre2: Neural Networks

Today:

Today:

- Revisit feature transform (5%)

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- What is a neural net? (10%)

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- Differential computational graph (25%)

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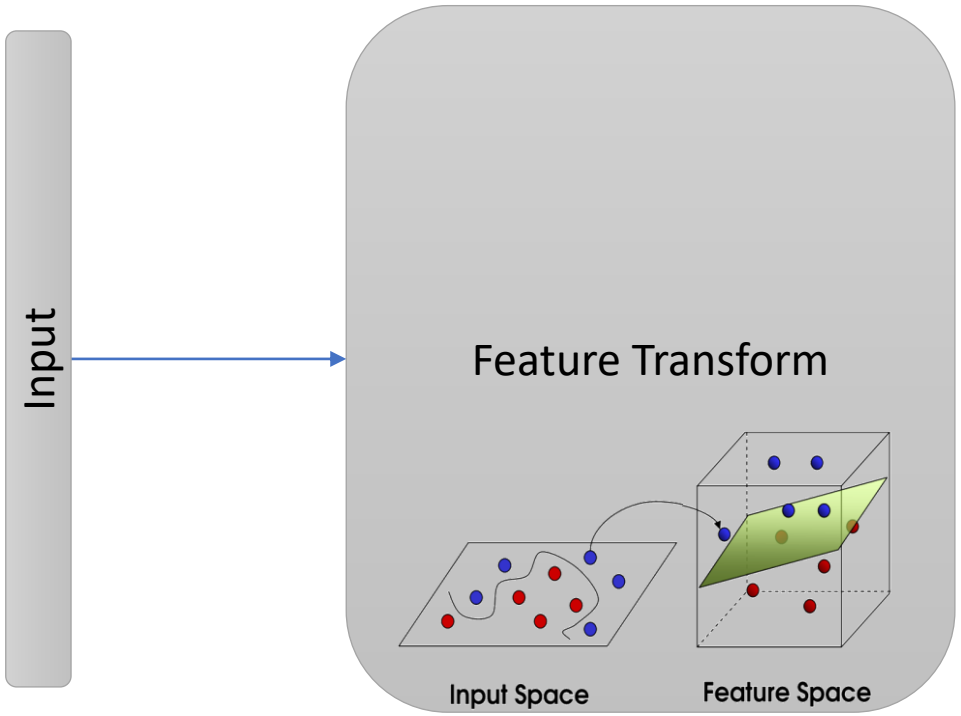
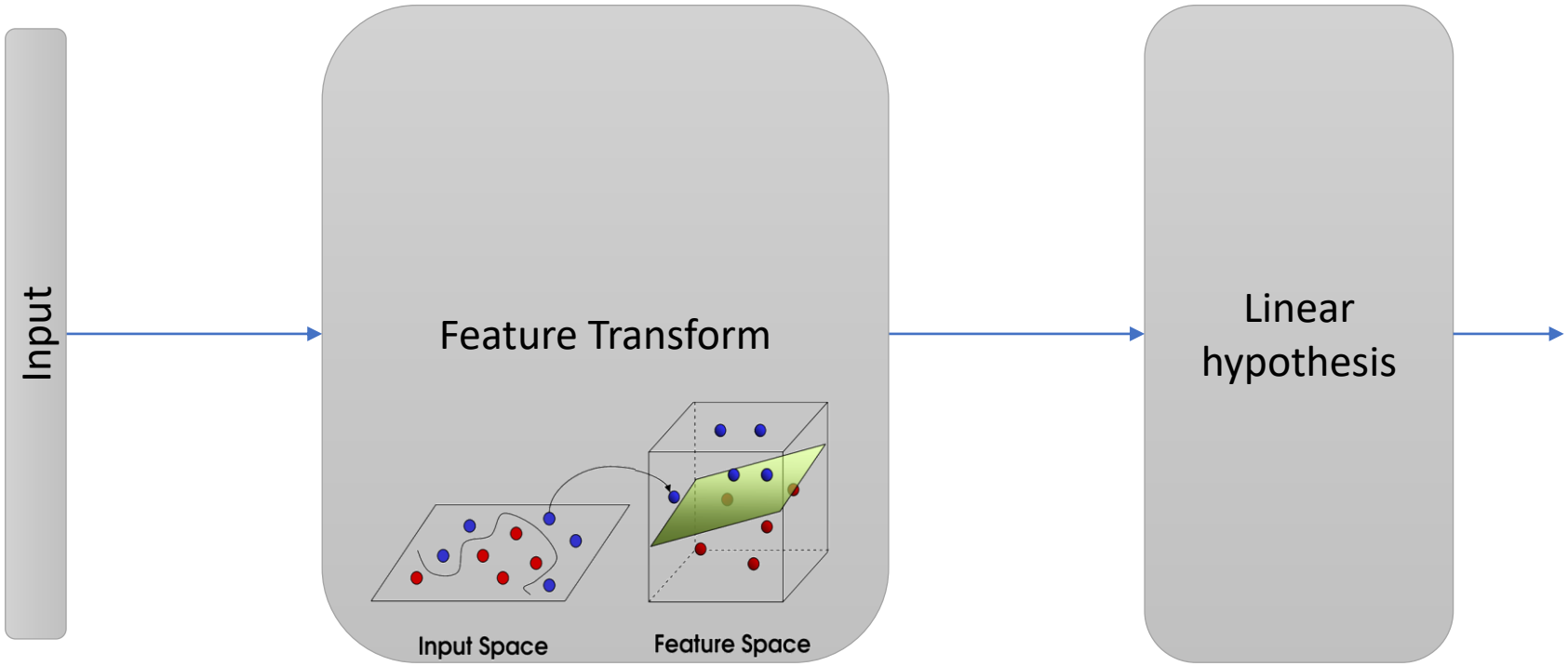
- Revisit feature transform (5%)
- What is a neural net? (10%)
- Derivatives and chain-rule reminder (10%)
- Training a vanilla network (back-prop) (40%)
- Differential computational graph (25%)
- Demo (10%)

Feature transform

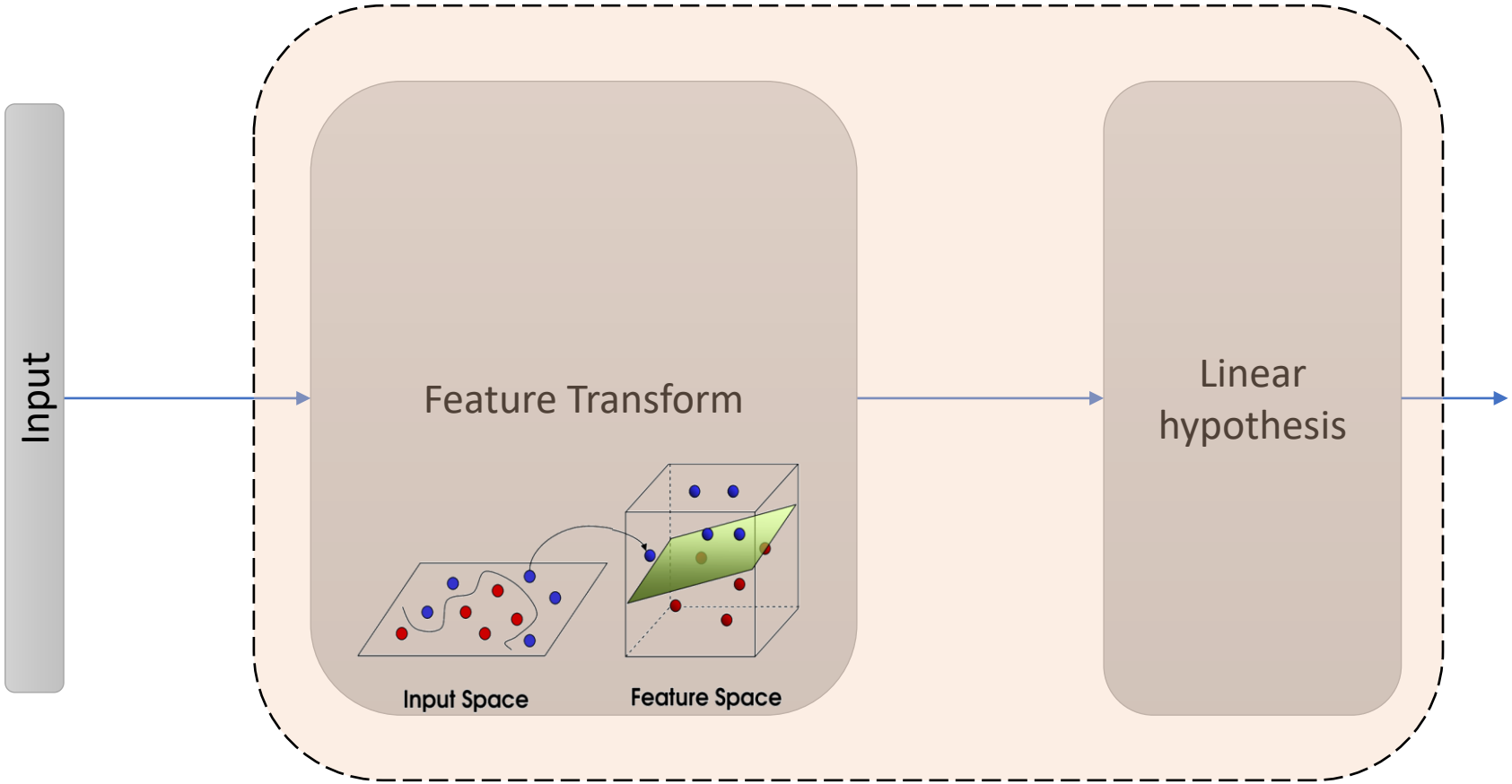
Feature transform



Feature transform

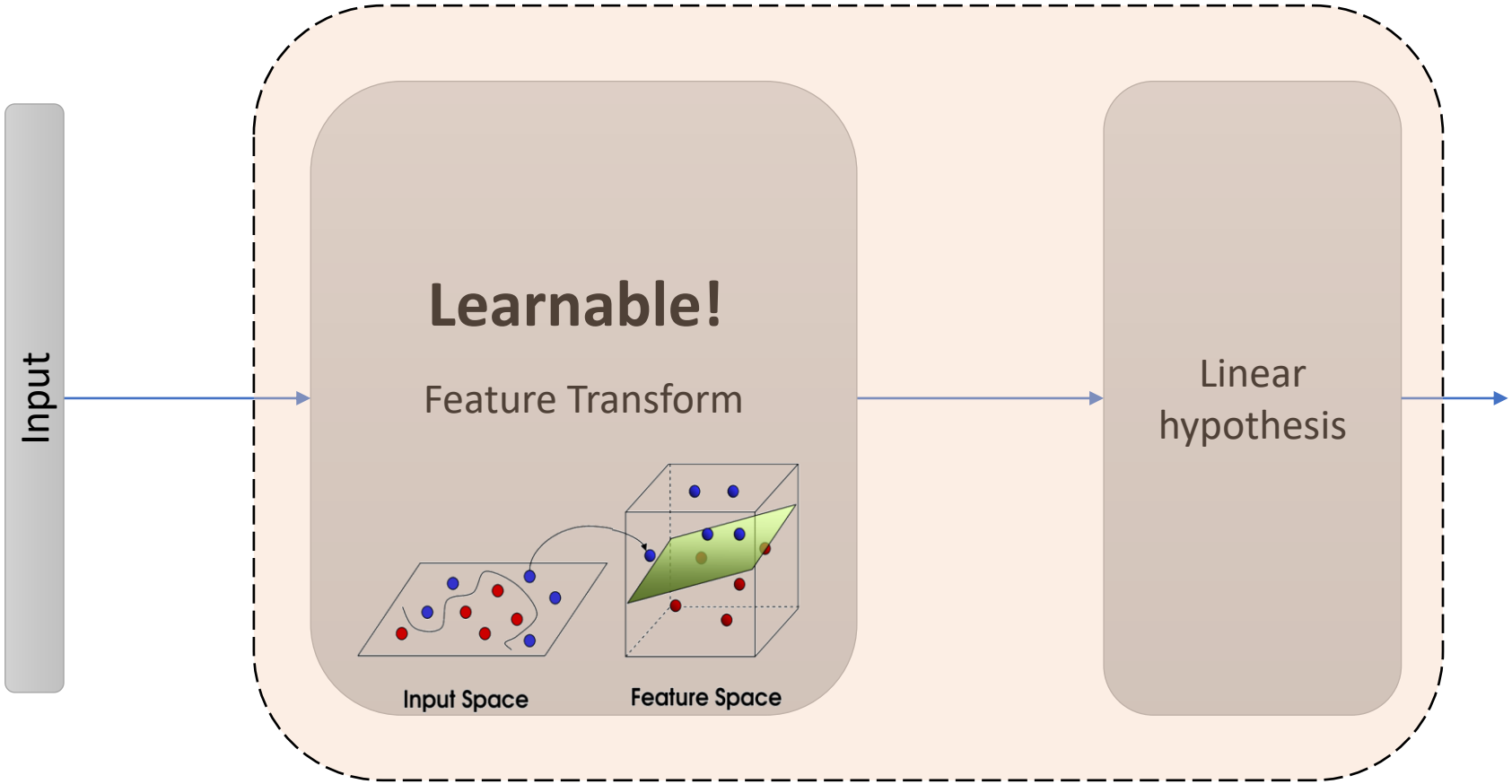


Feature transform



Non-linear hypothesis!

Feature transform



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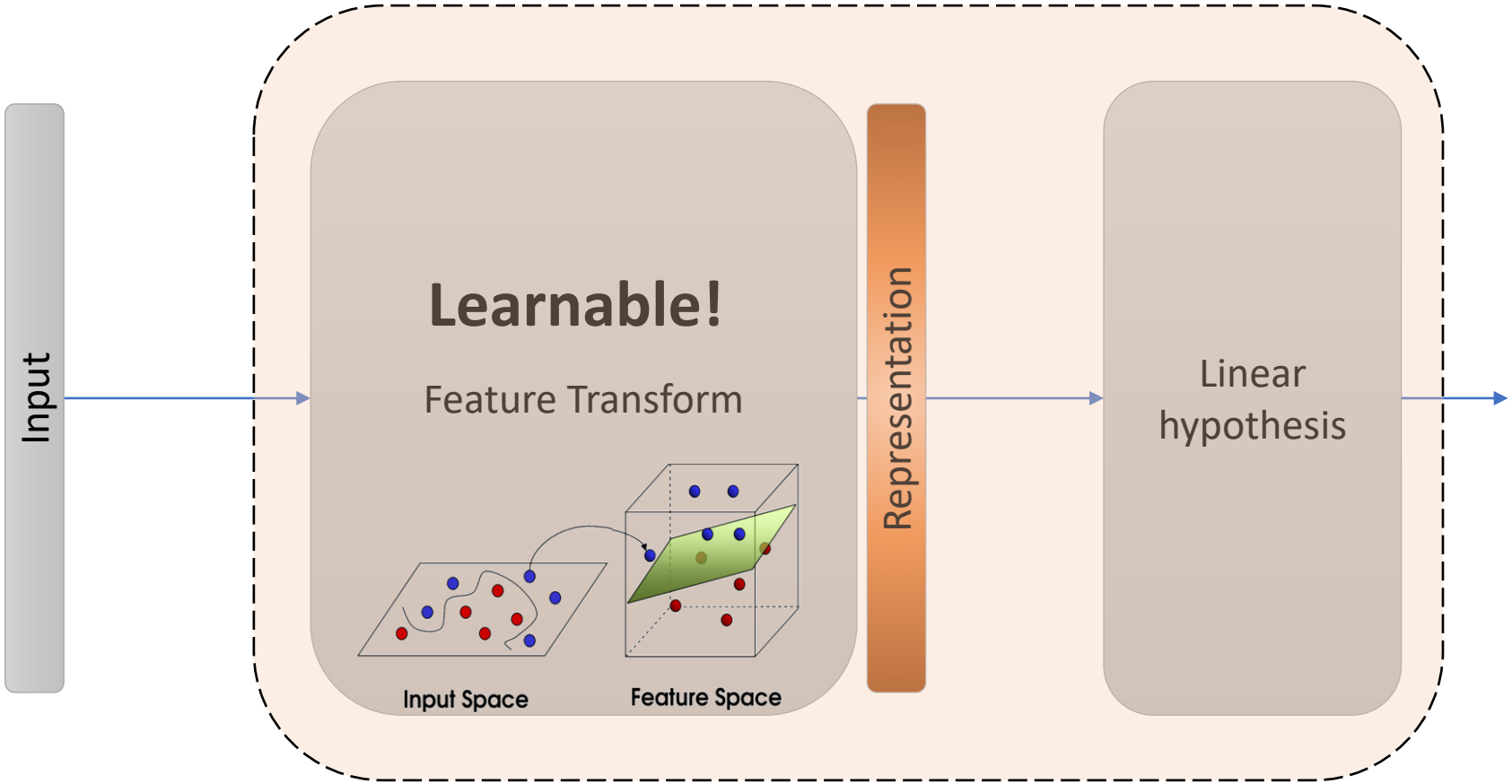

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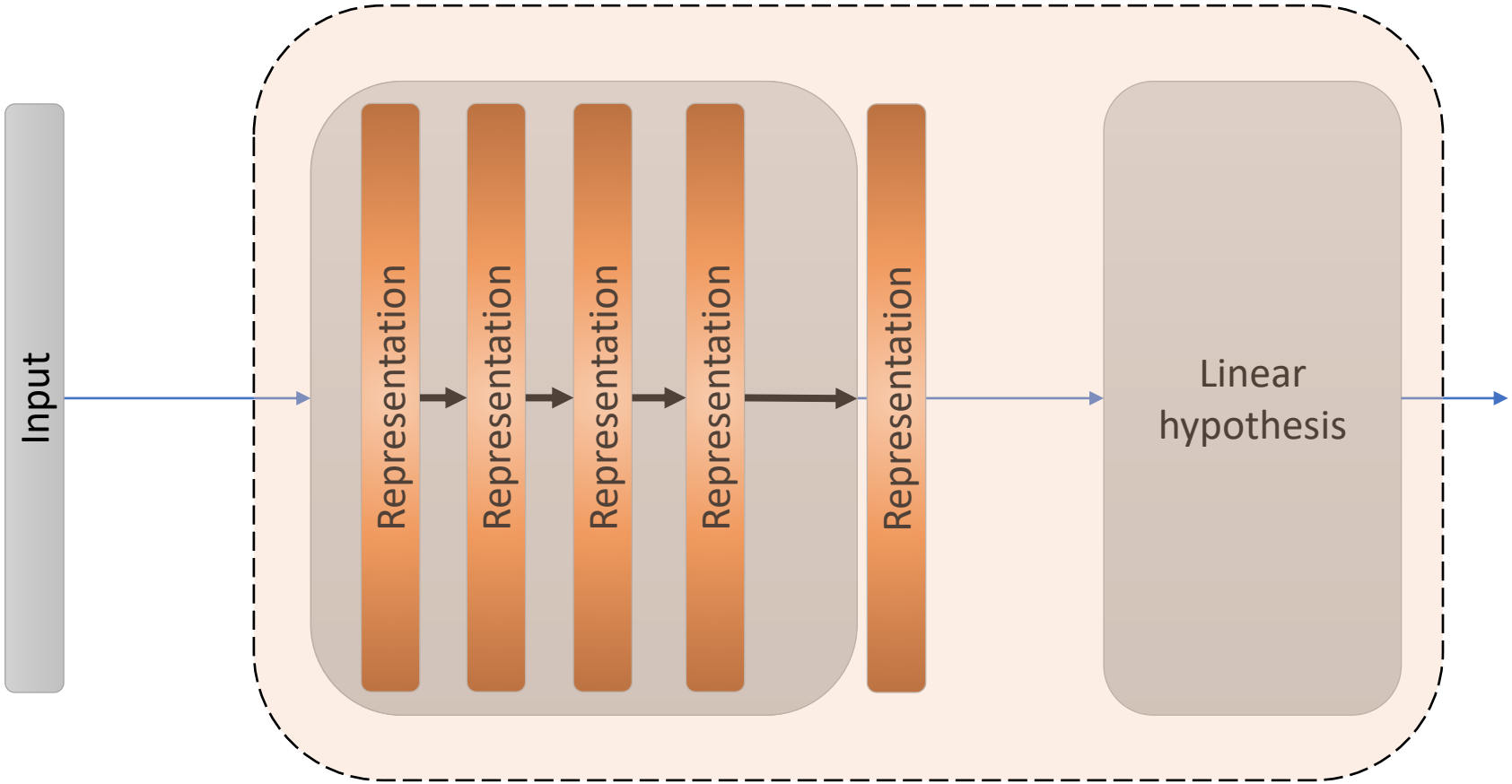
Feature transform



Non-linear hypothesis!

Artificial Neural Networks

Artificial Neural Networks

Vaguely inspired by biological neural networks

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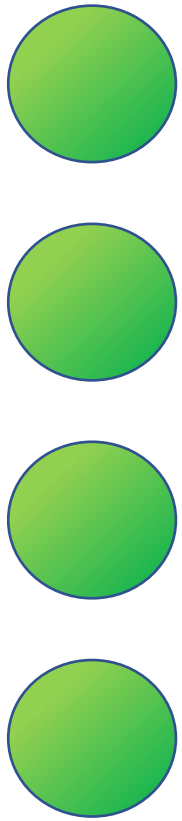
Input vector



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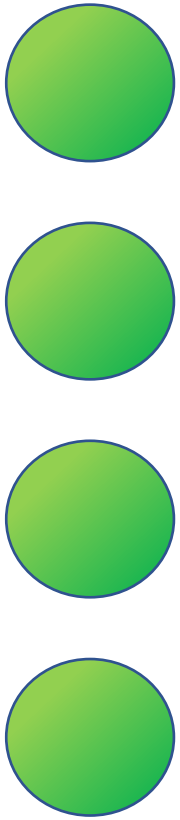
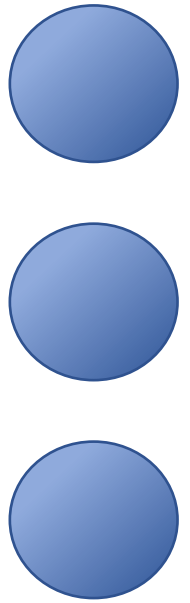
Input vector



Output vector



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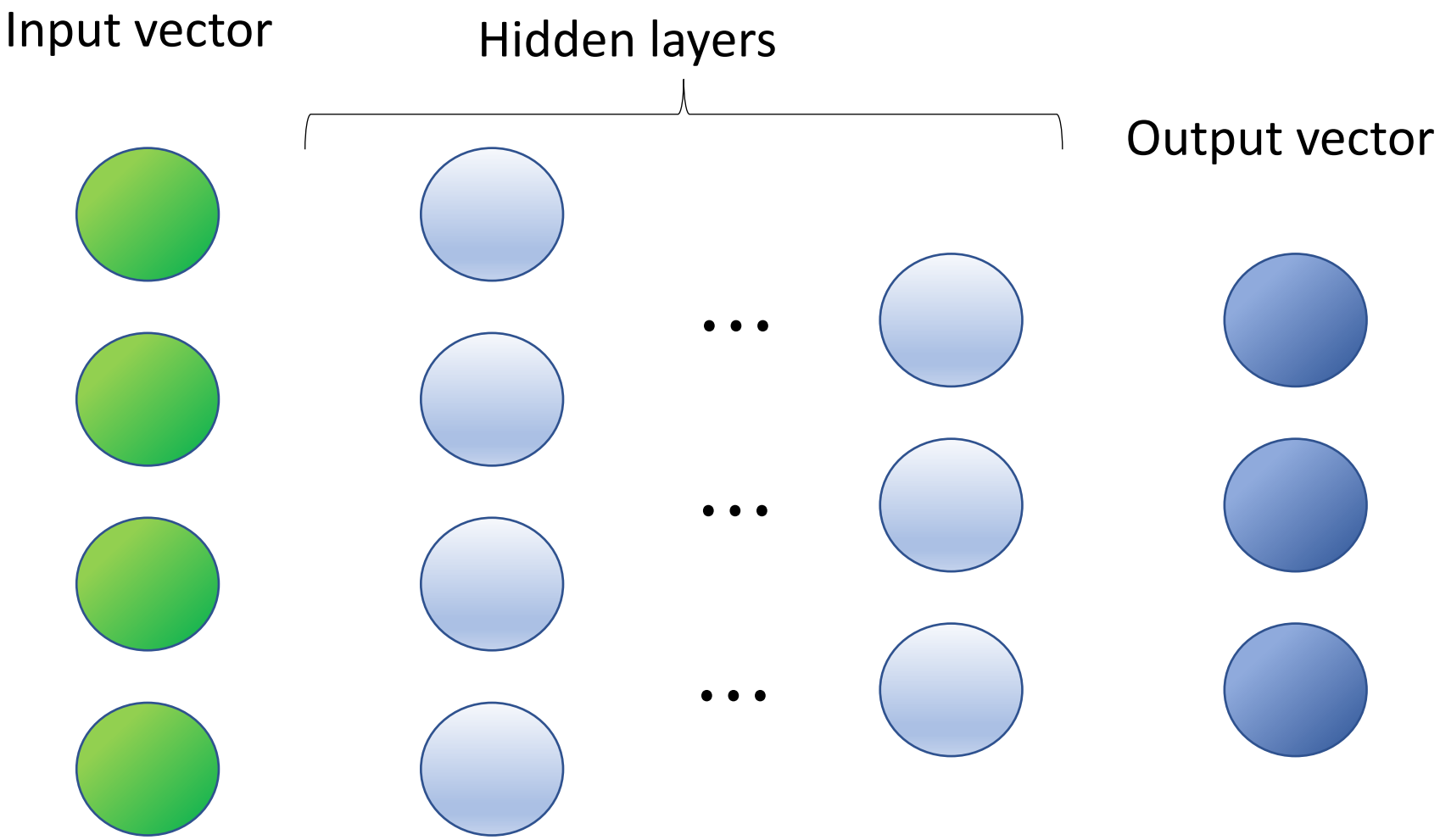

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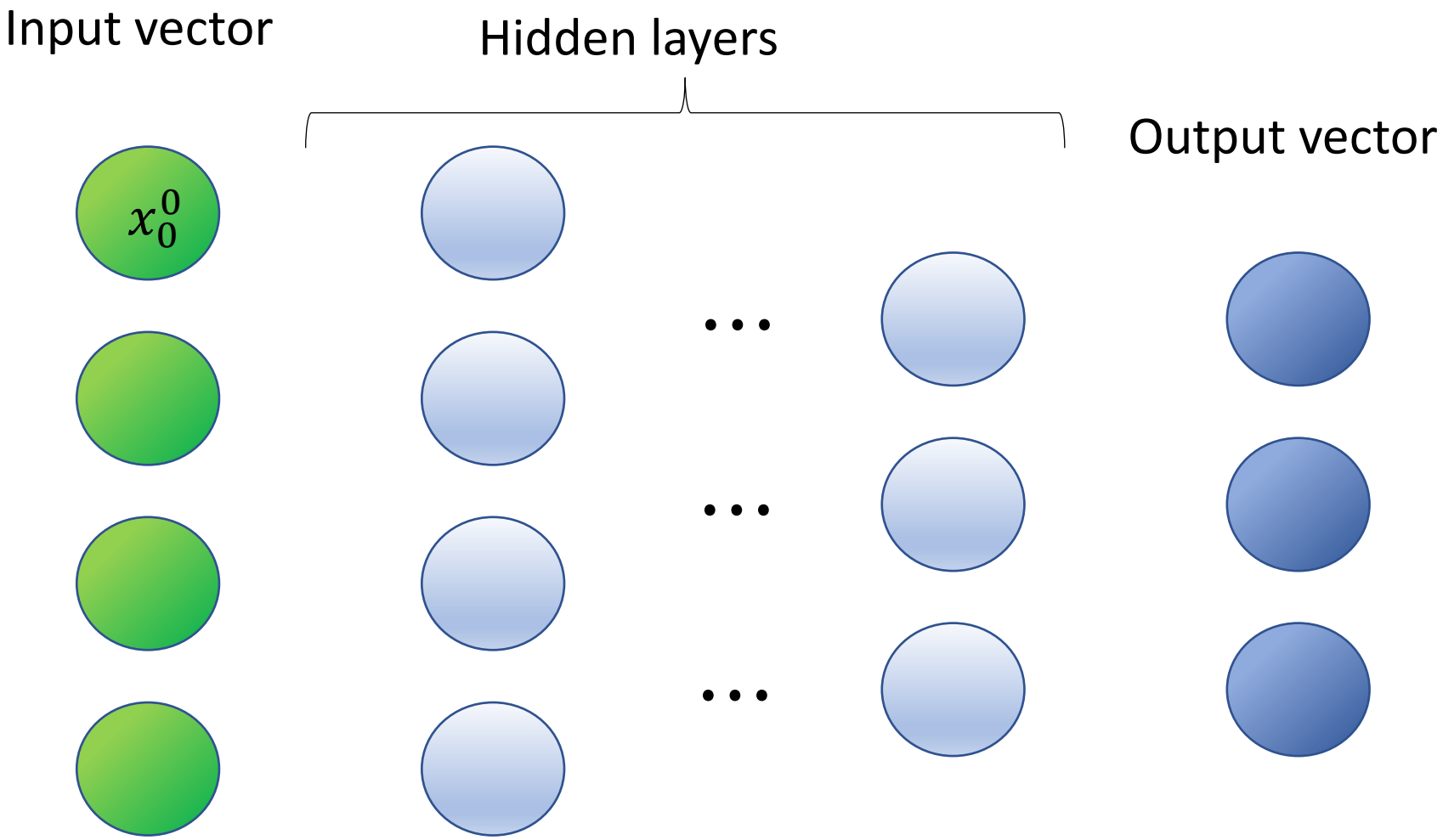

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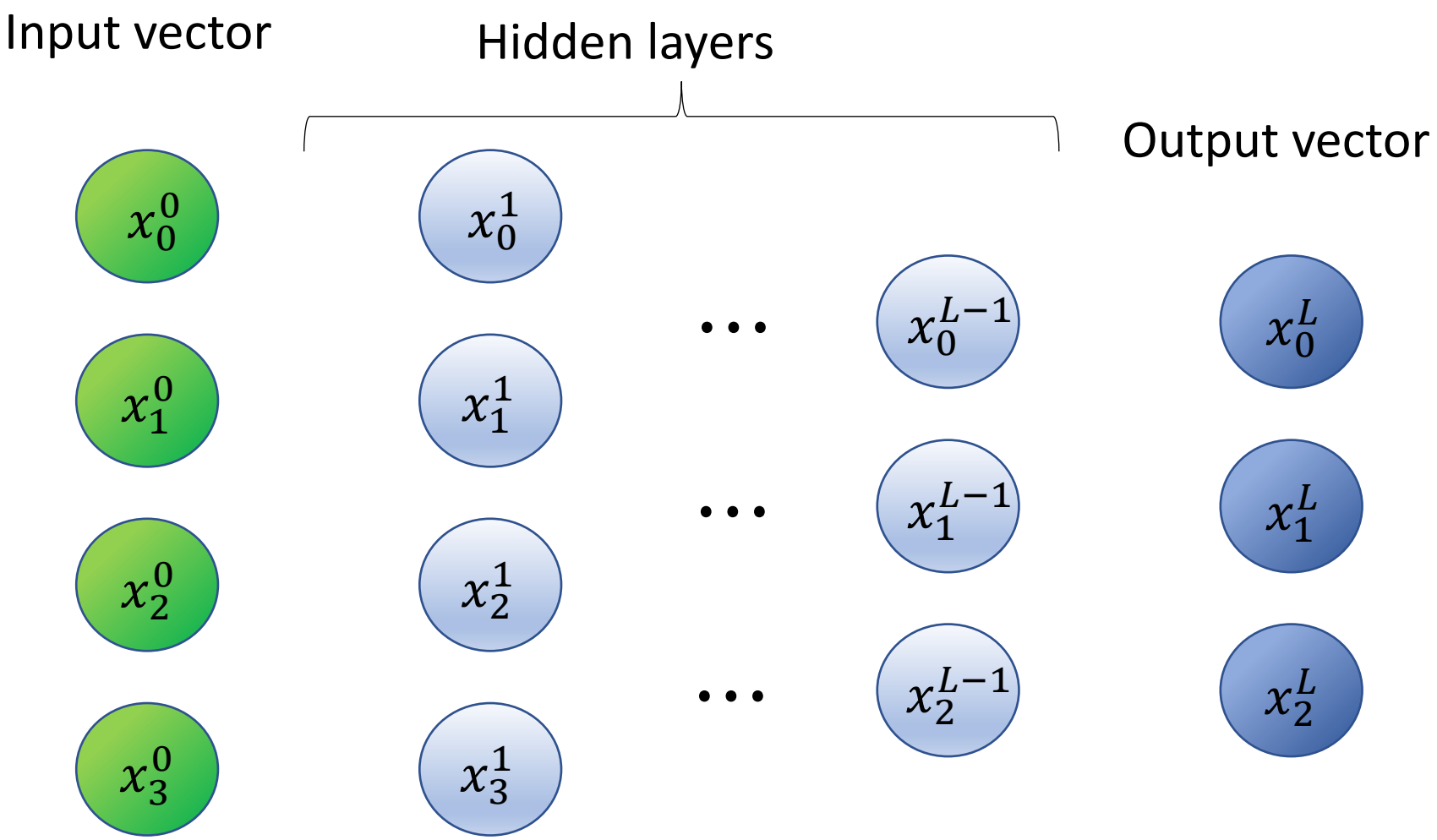

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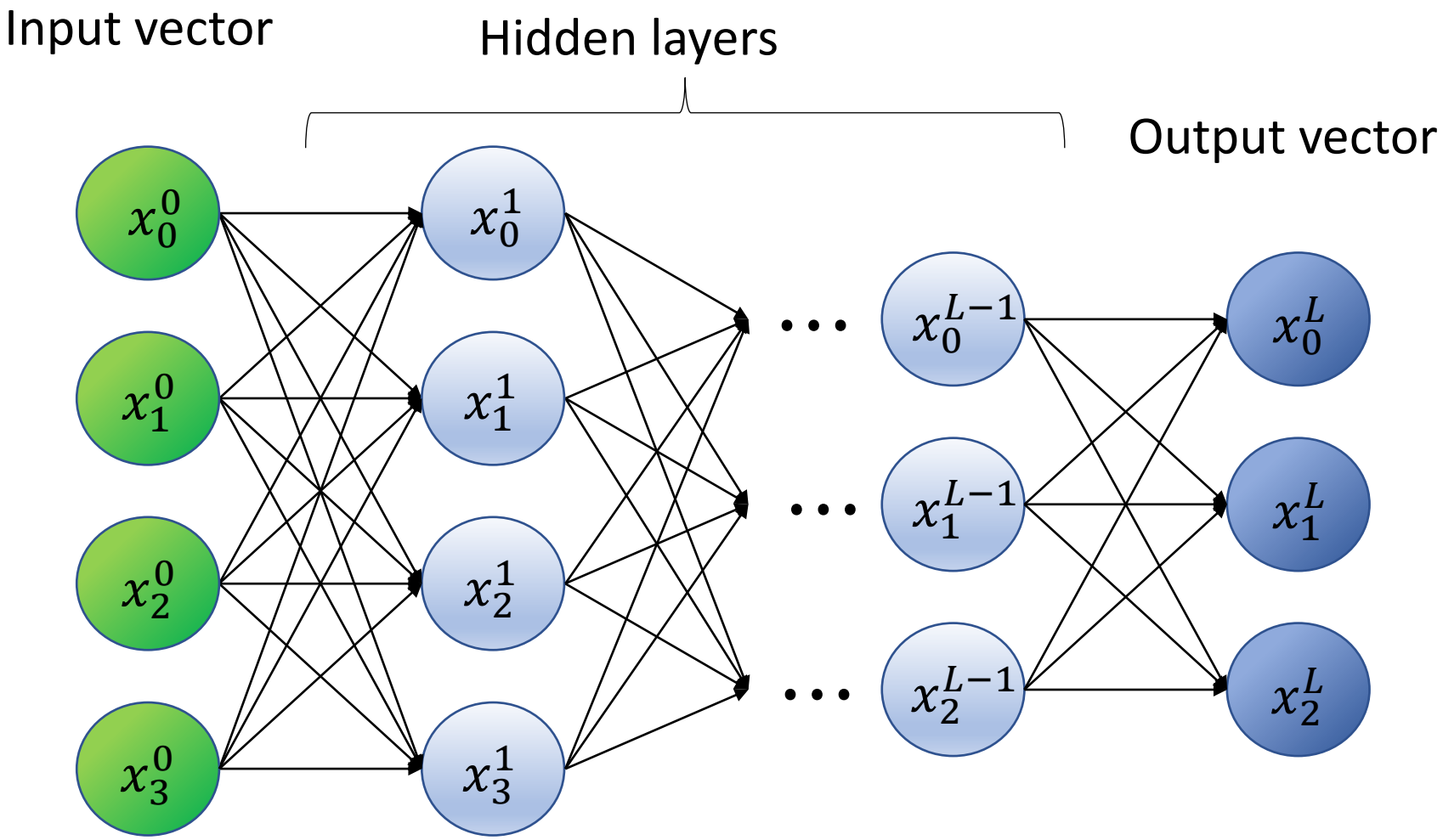

Artificial Neural Networks

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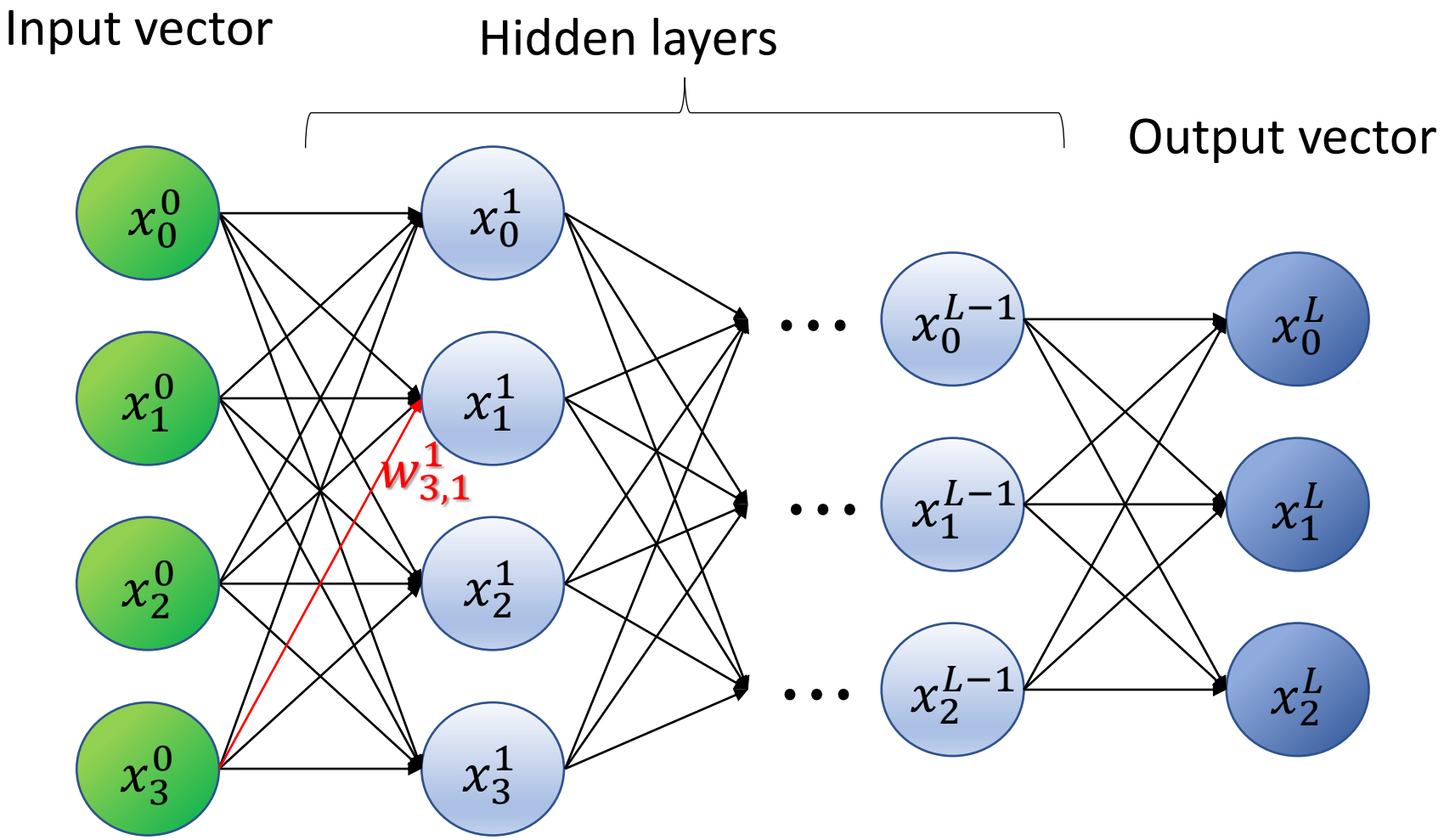
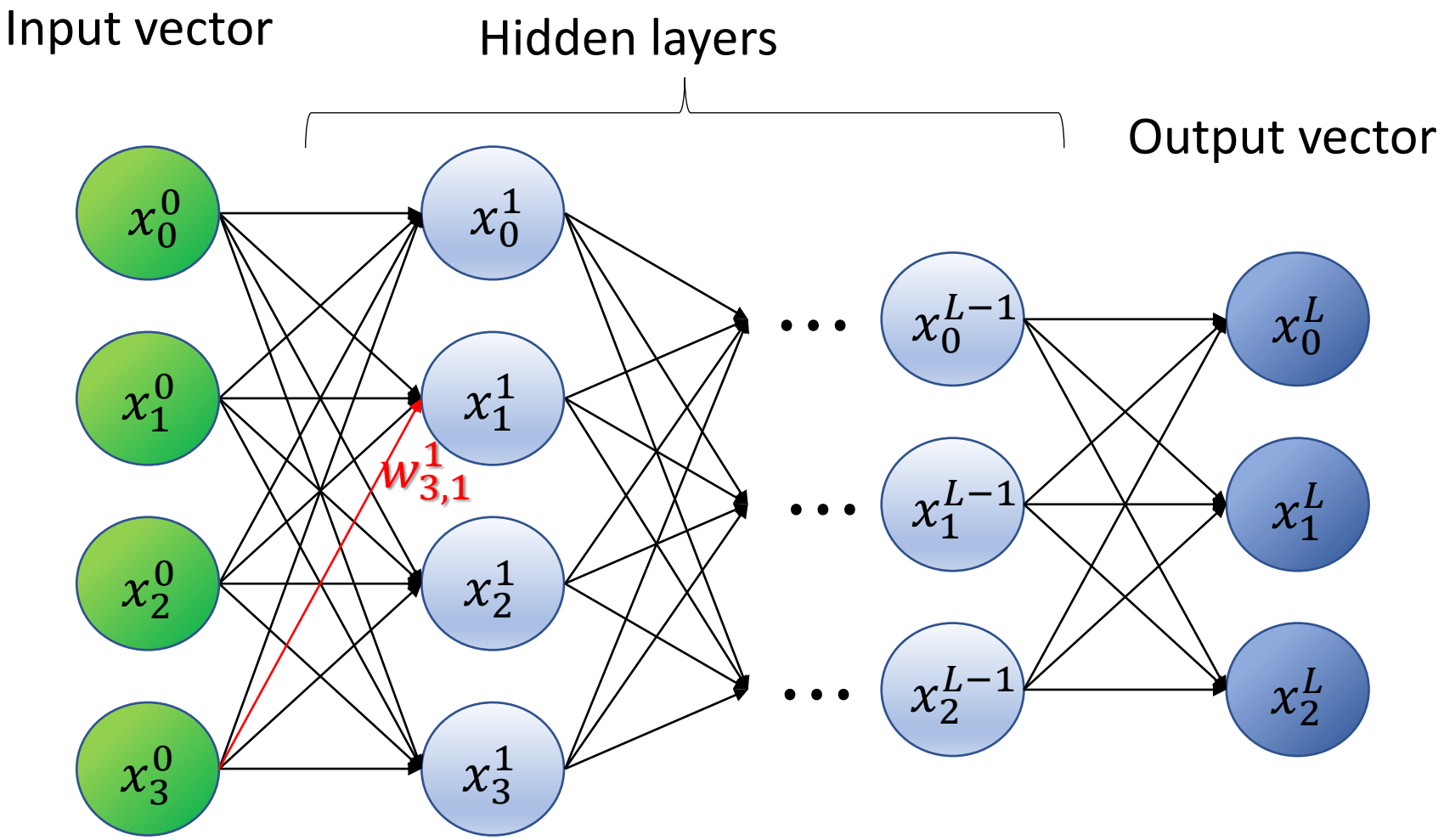
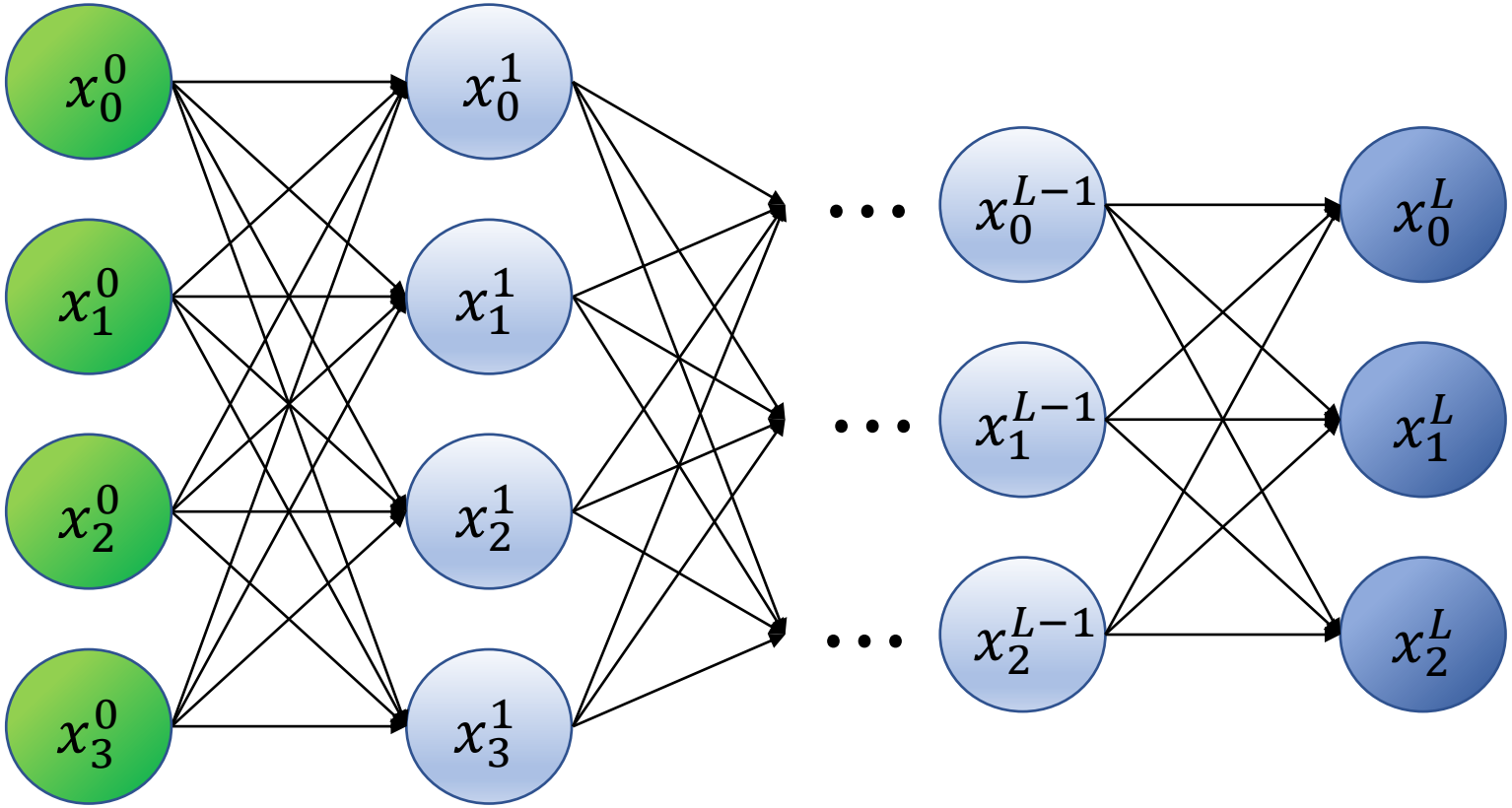



Input vector
Hidden layers
Output vector
x_0^0
x_1^0
x_2^0
x_3^0
x_0^1
x_1^1
x_2^1
x_3^1
$w_{3,1}^1$
x_0^{L-1}
x_1^{L-1}
x_2^{L-1}
x_0^L
x_1^L
x_2^L


x_0^0
x_1^0
x_2^0
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x_0^1
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x_2^1
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x_0^{L-1}
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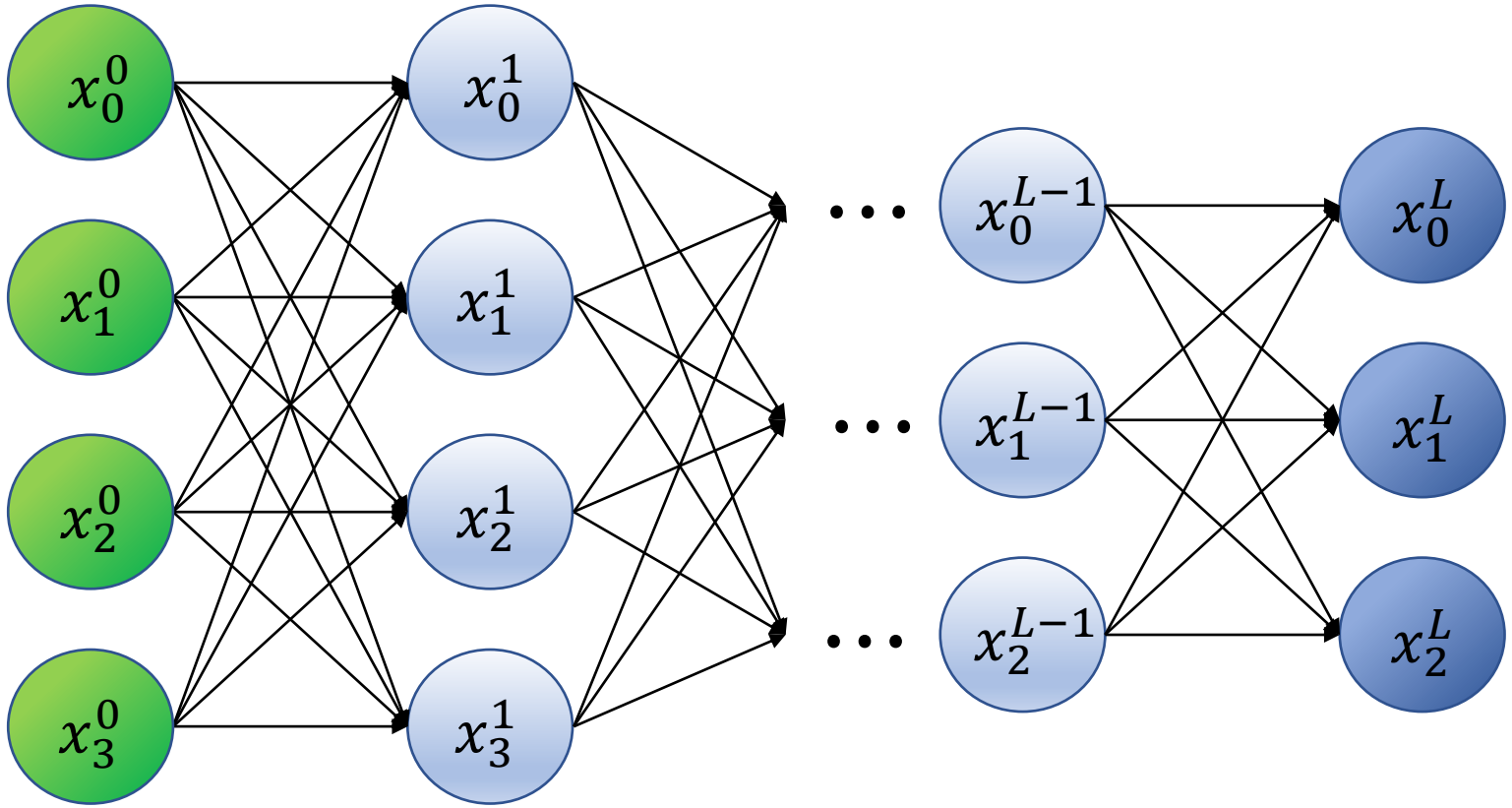
x_0^L
x_1^L
x_2^L

$$x_j^l =$$



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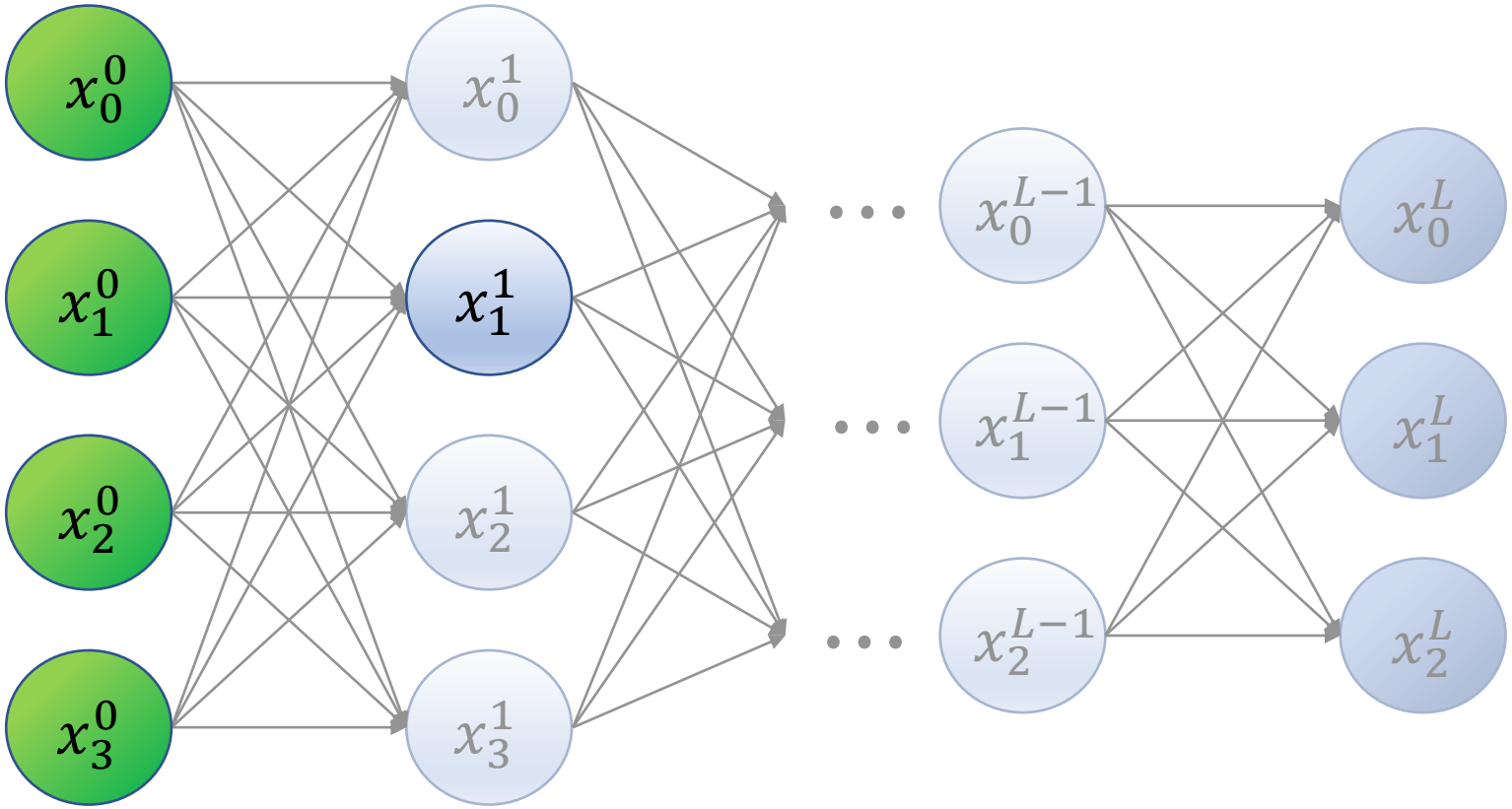

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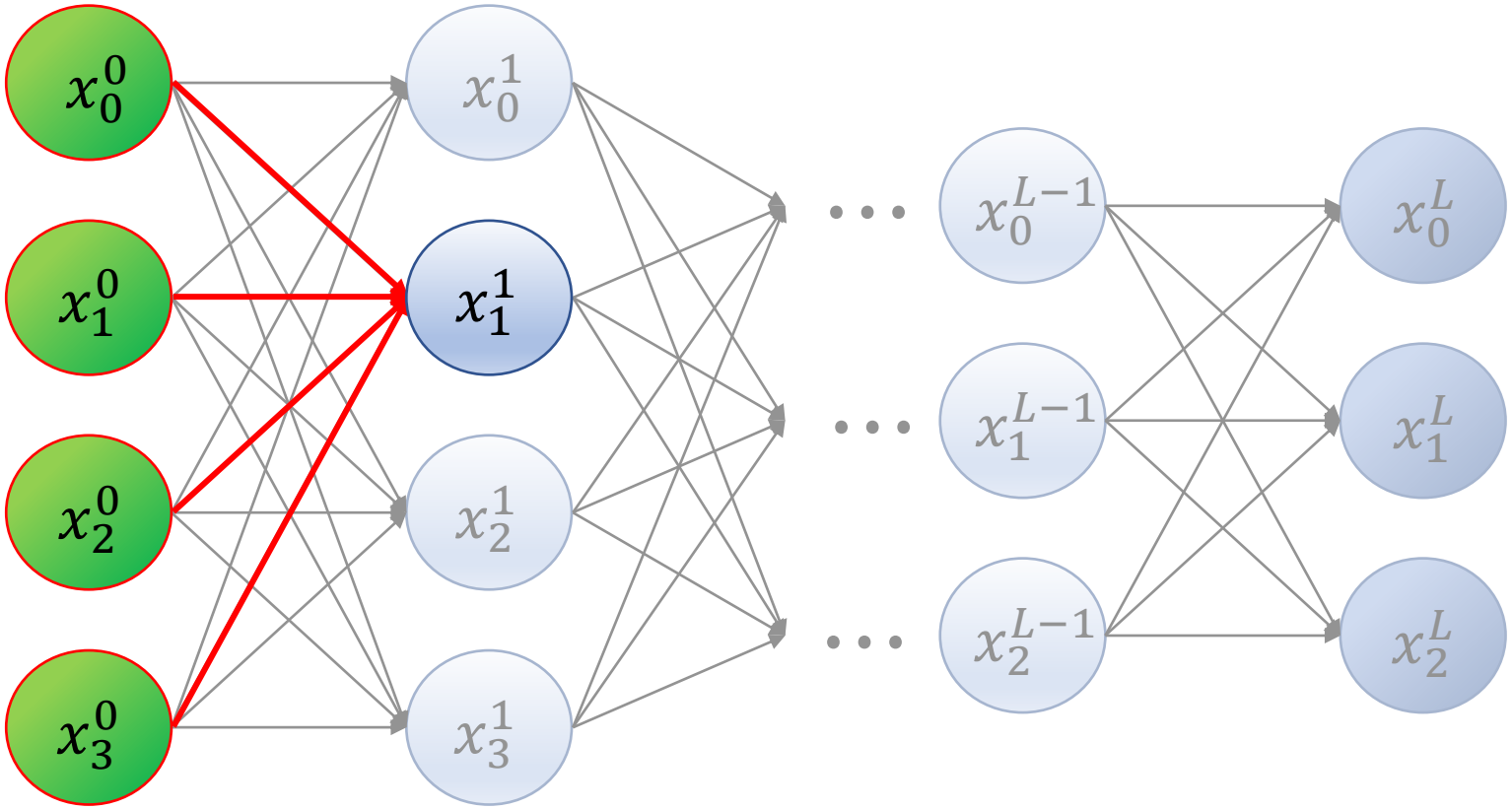
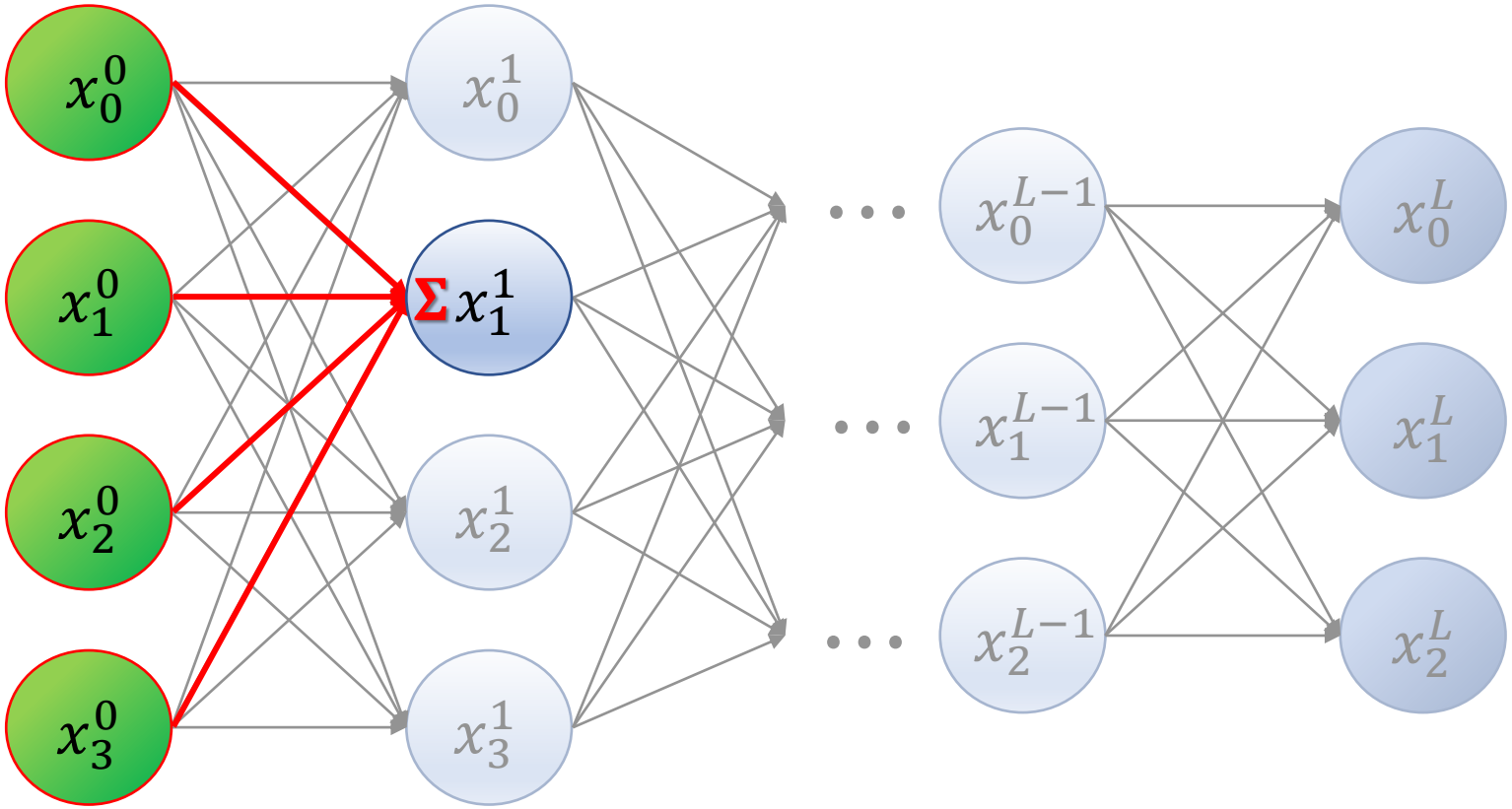
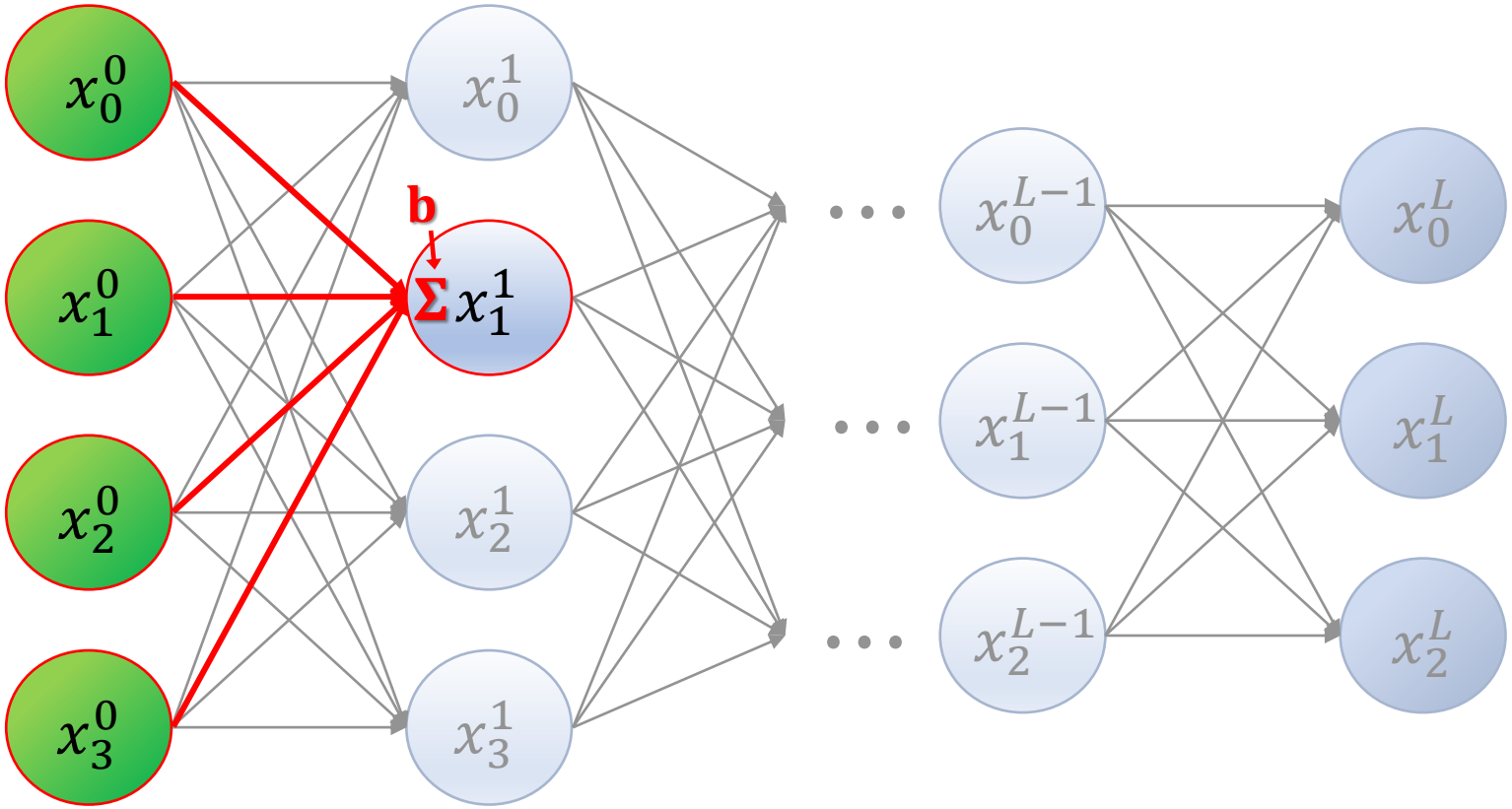


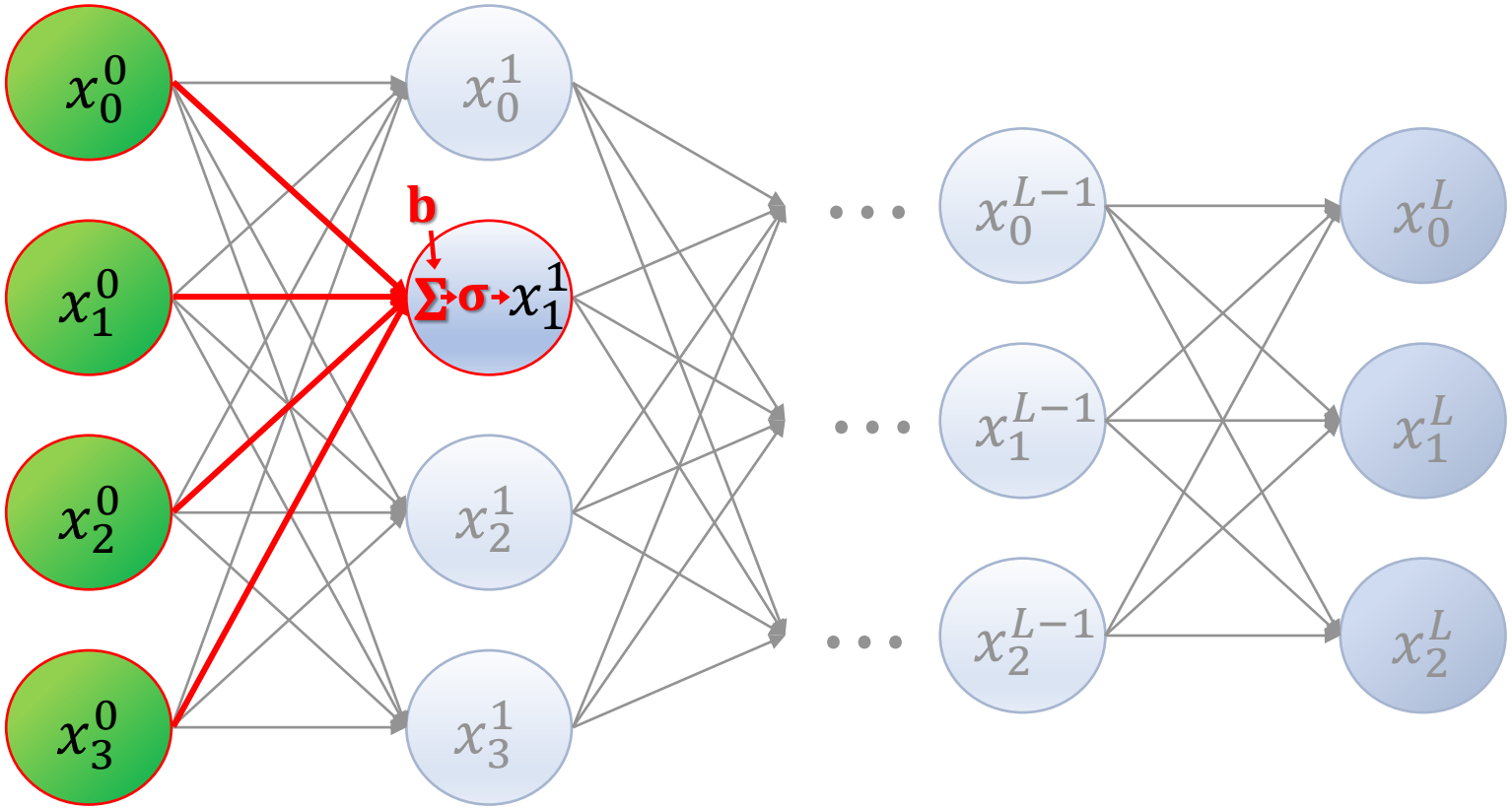
$$x_j^l = \sum_i w_{ij}^l \cdot x_i^{l-1}$$



$$x_j^l = \sum_i w_{ij}^l \cdot x_i^{l-1} + b_j^l$$



$$x_j^l = \sigma\left(\sum_i w_{ij}^l \cdot x_i^{l-1} + b_j^l\right)$$



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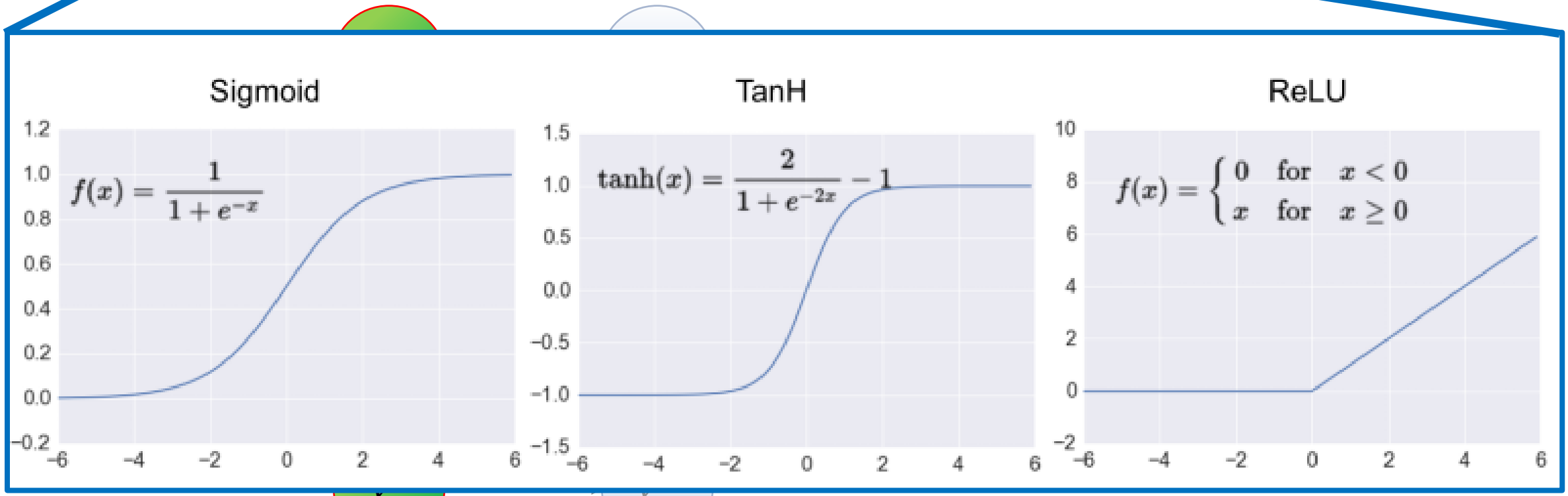

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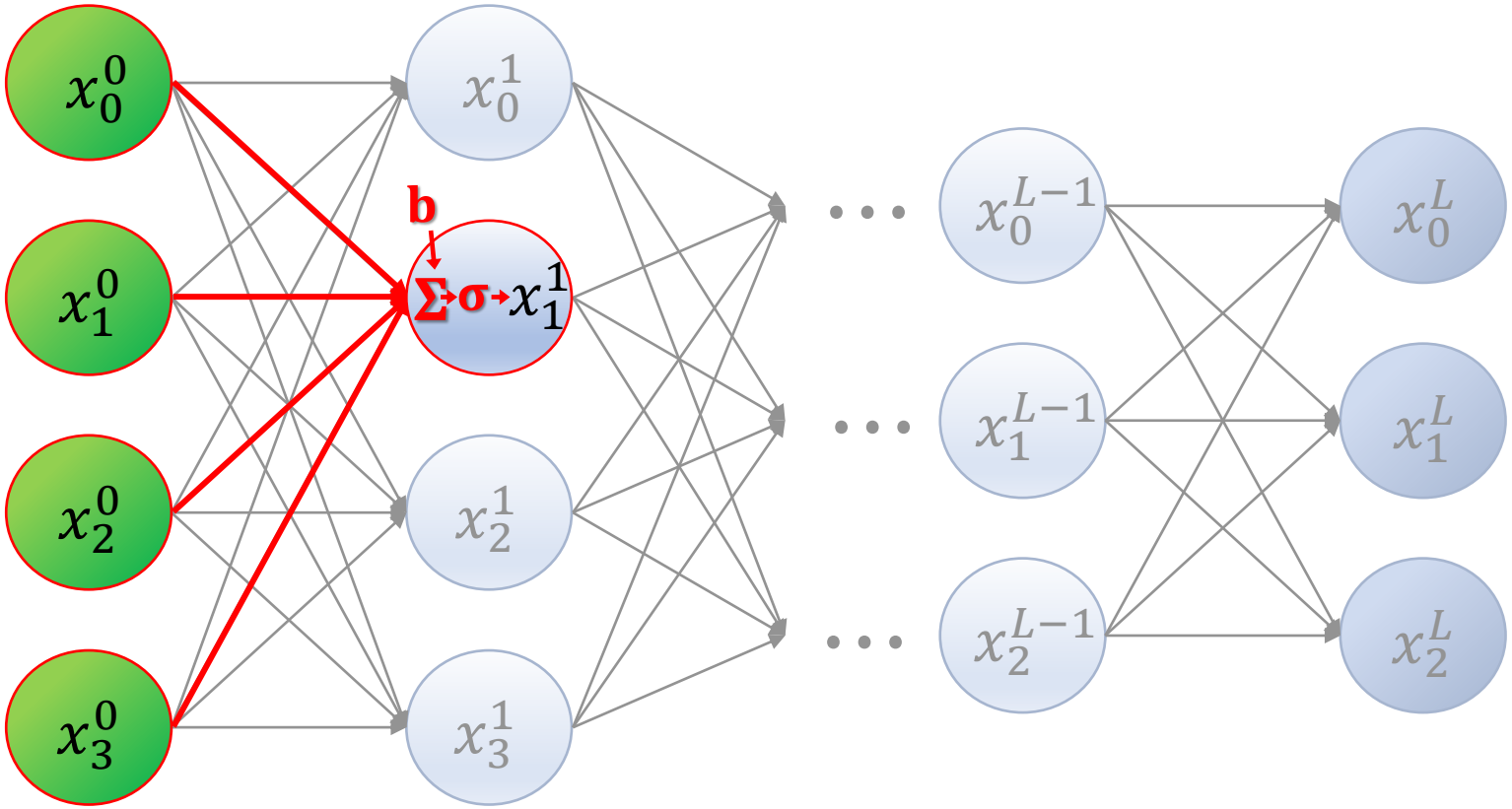

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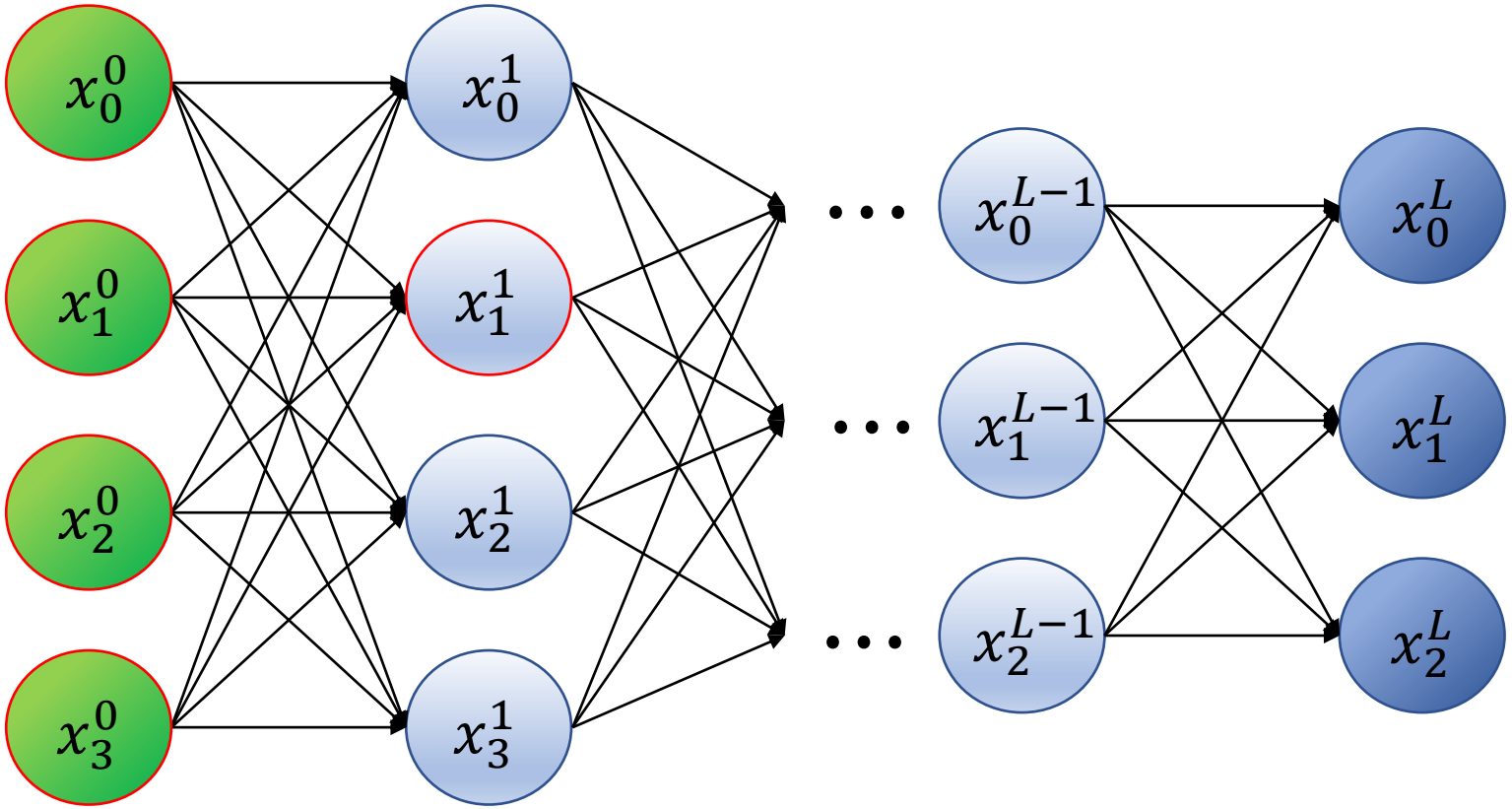

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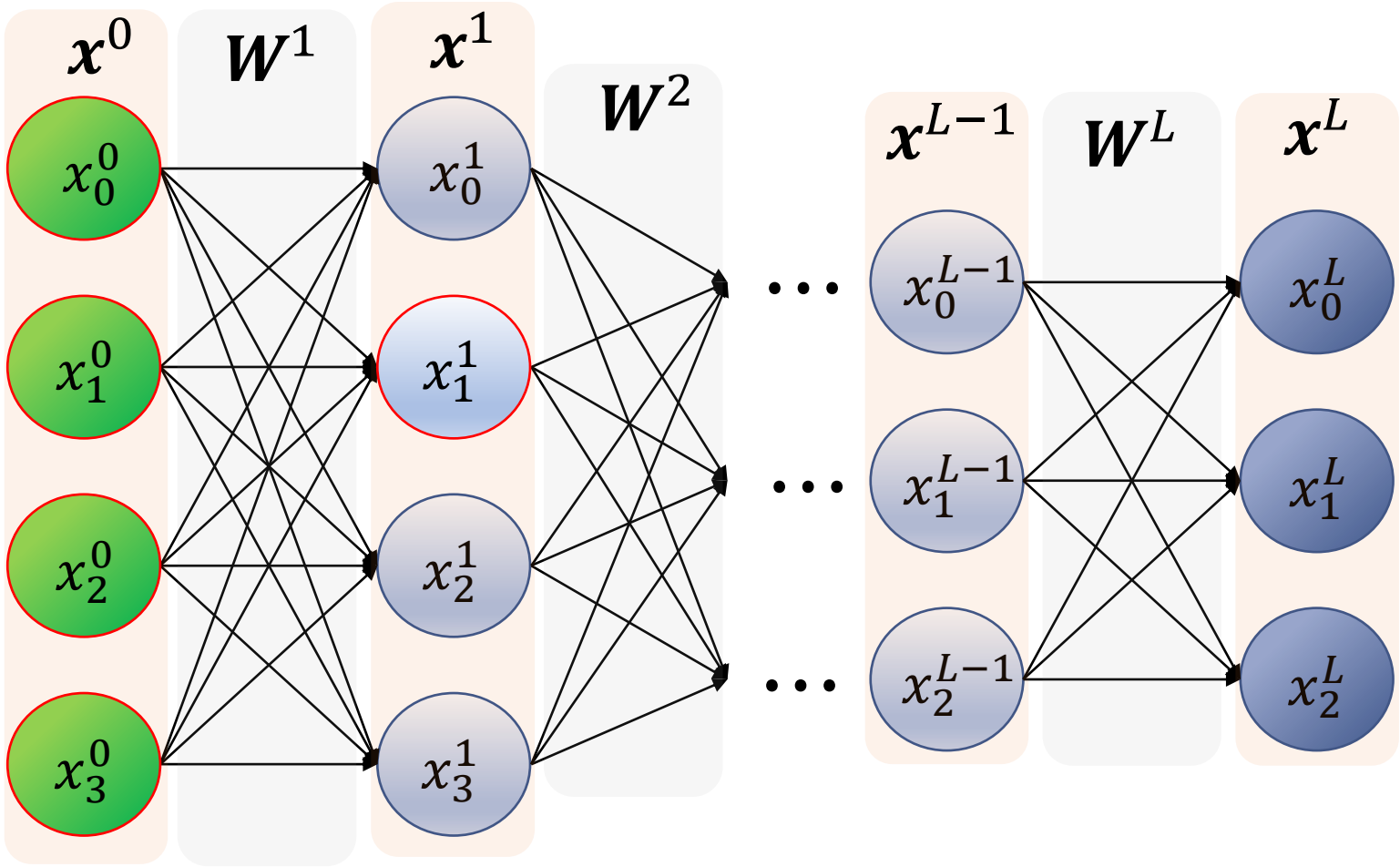
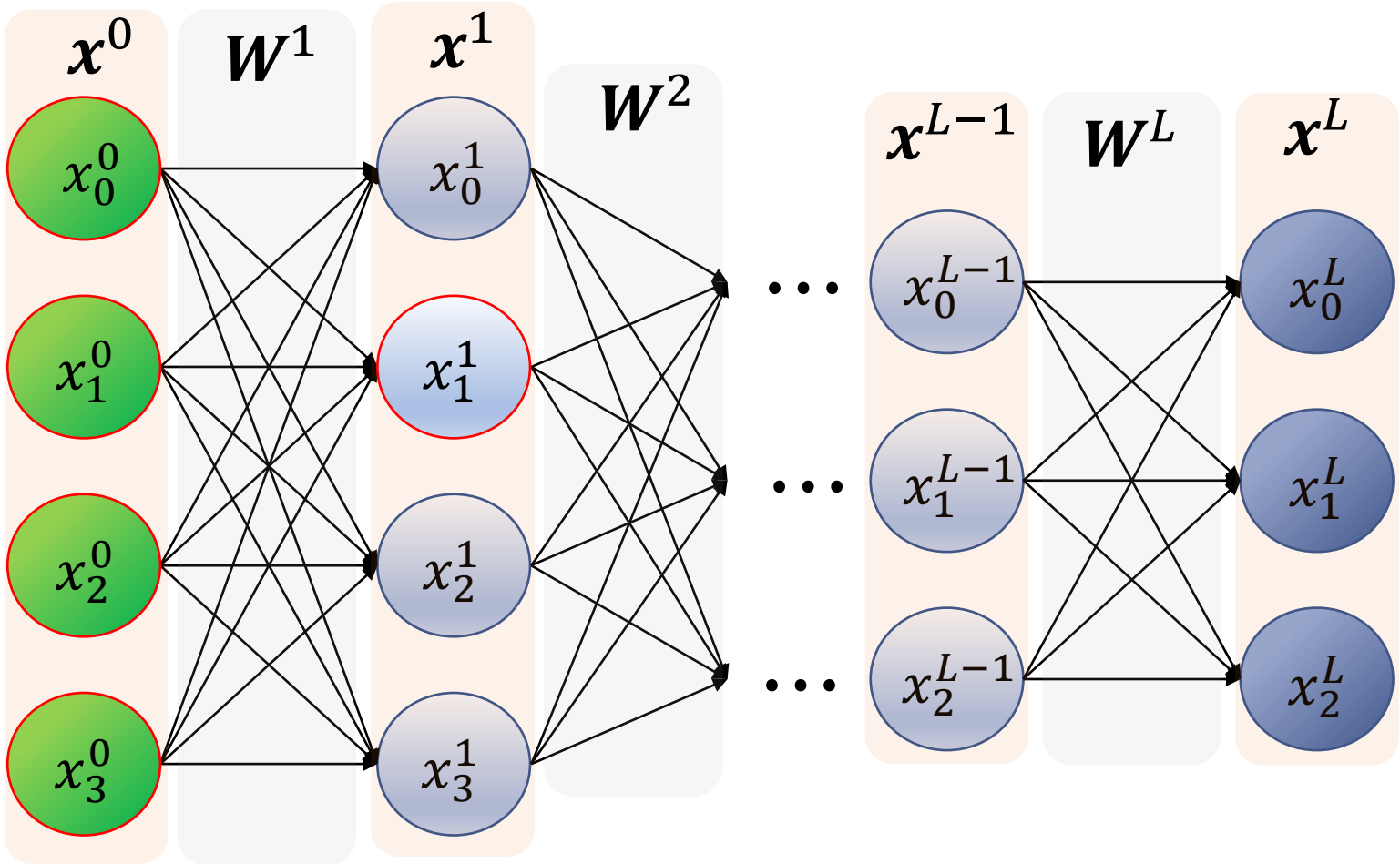


$$x_j^l = \sigma\left(\sum_i w_{ij}^l \cdot x_i^{l-1} + b_j^l\right) \qquad \boldsymbol{x}^l = \boldsymbol{\sigma}\left(\boldsymbol{W}^{l^T}\boldsymbol{x}^{l-1} + \boldsymbol{b}^l\right)$$



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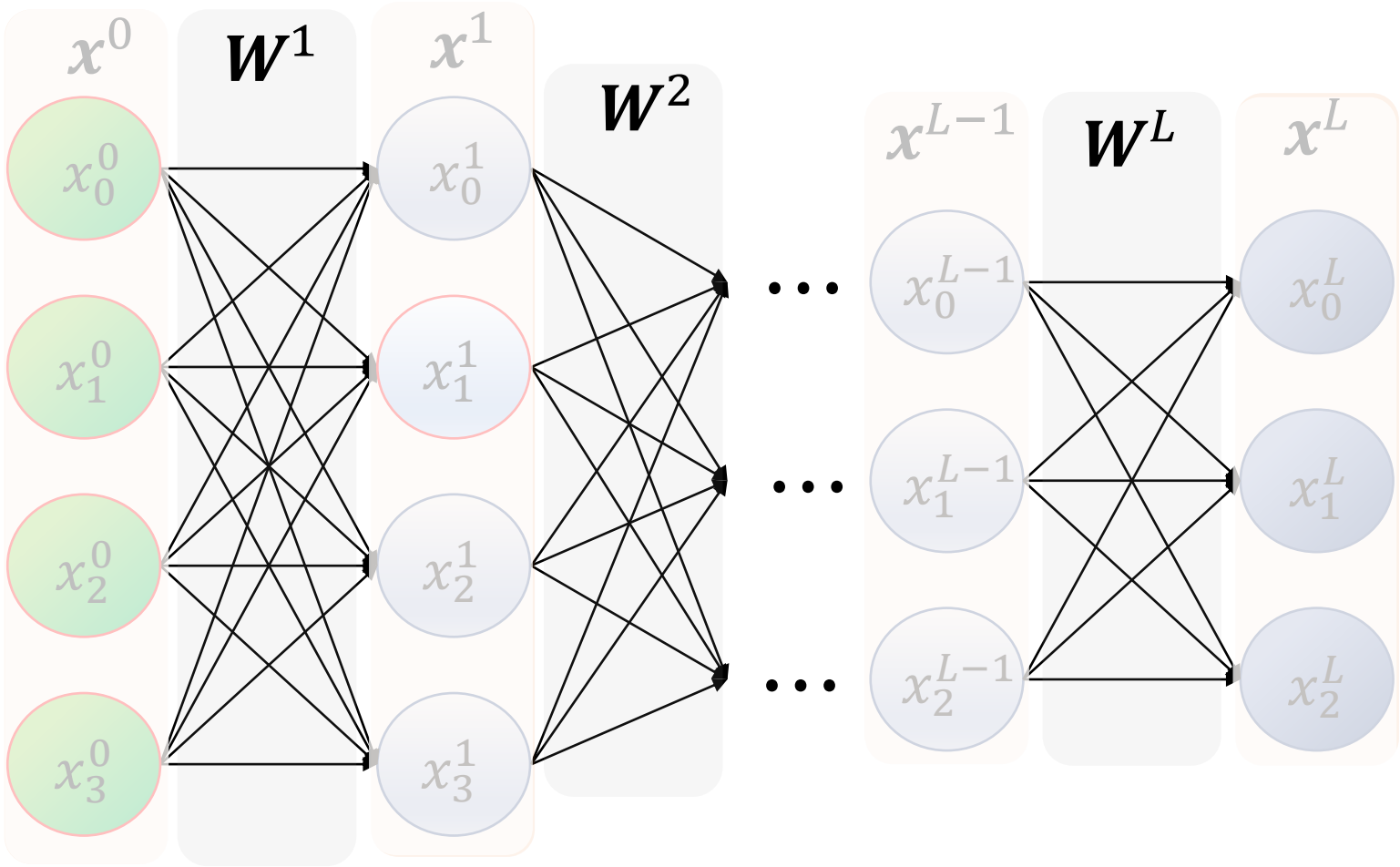

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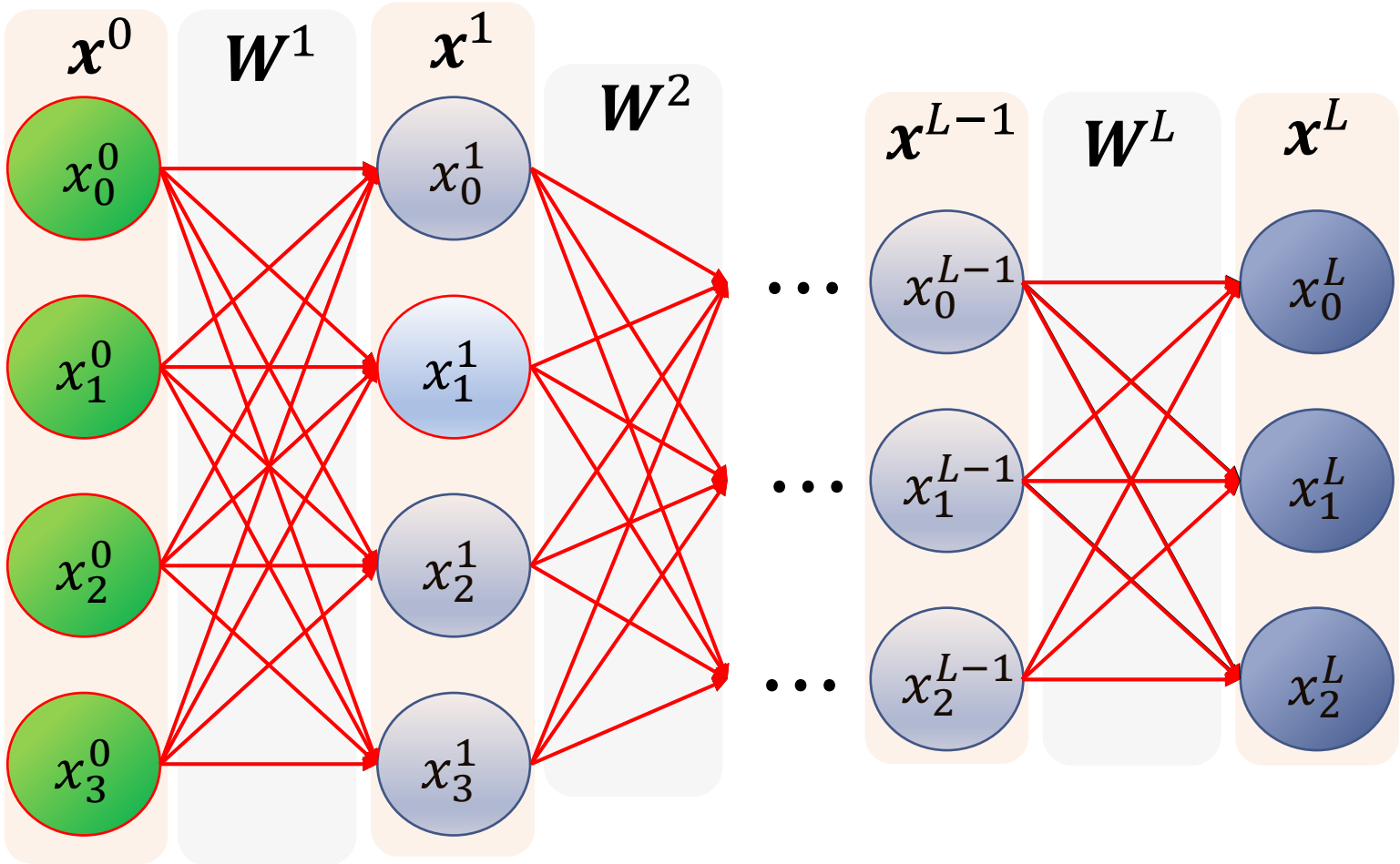

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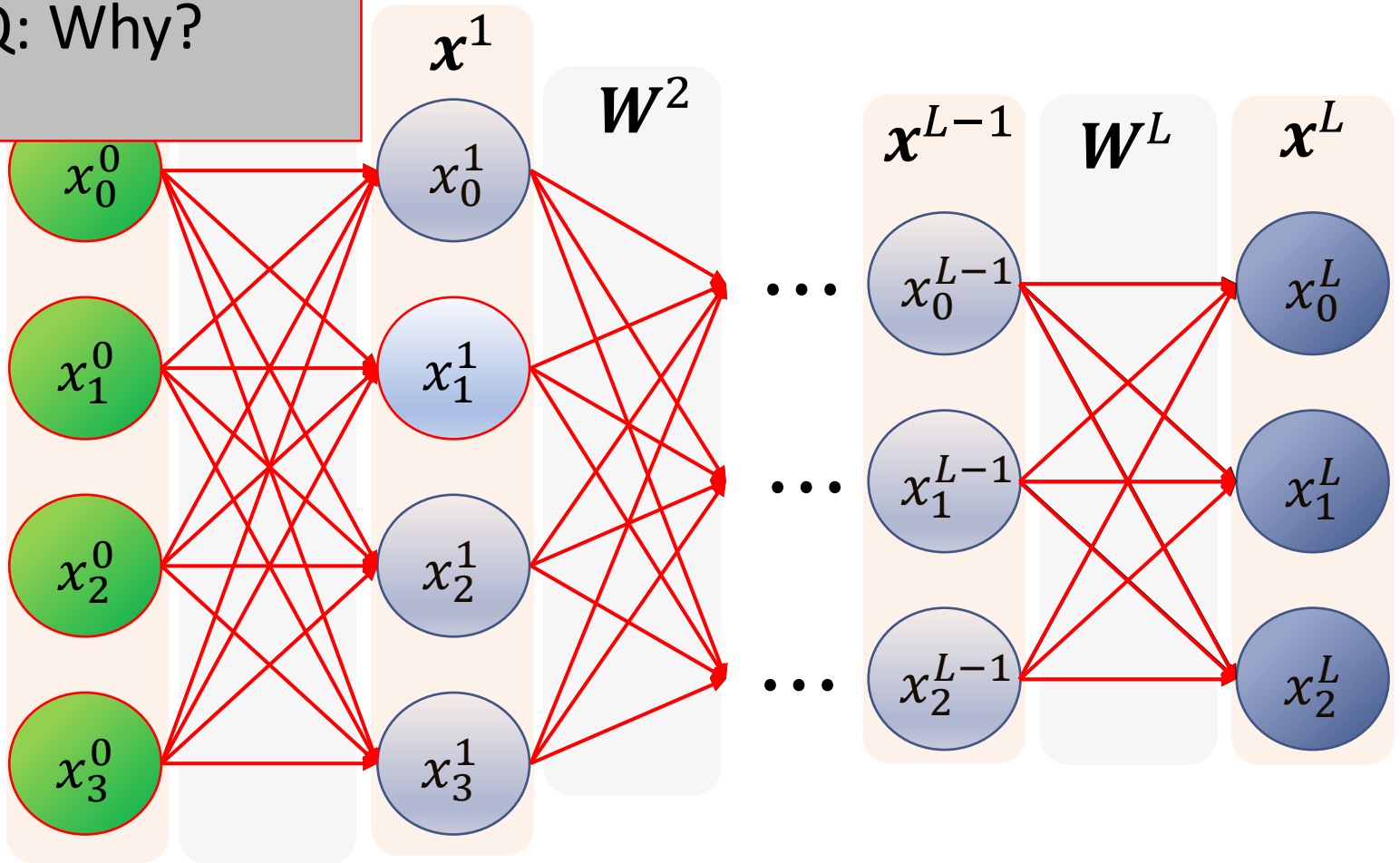
Q: Why?



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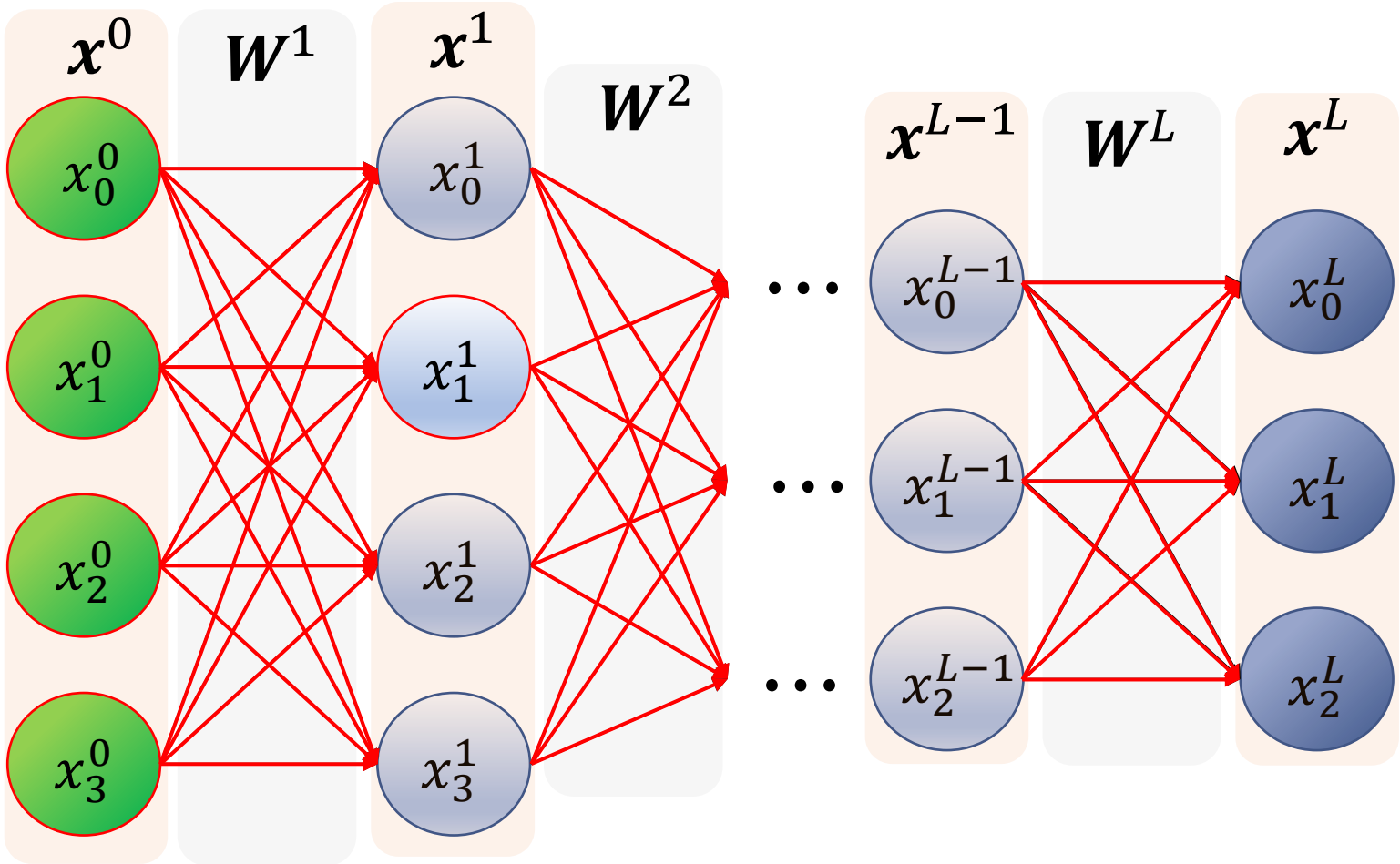

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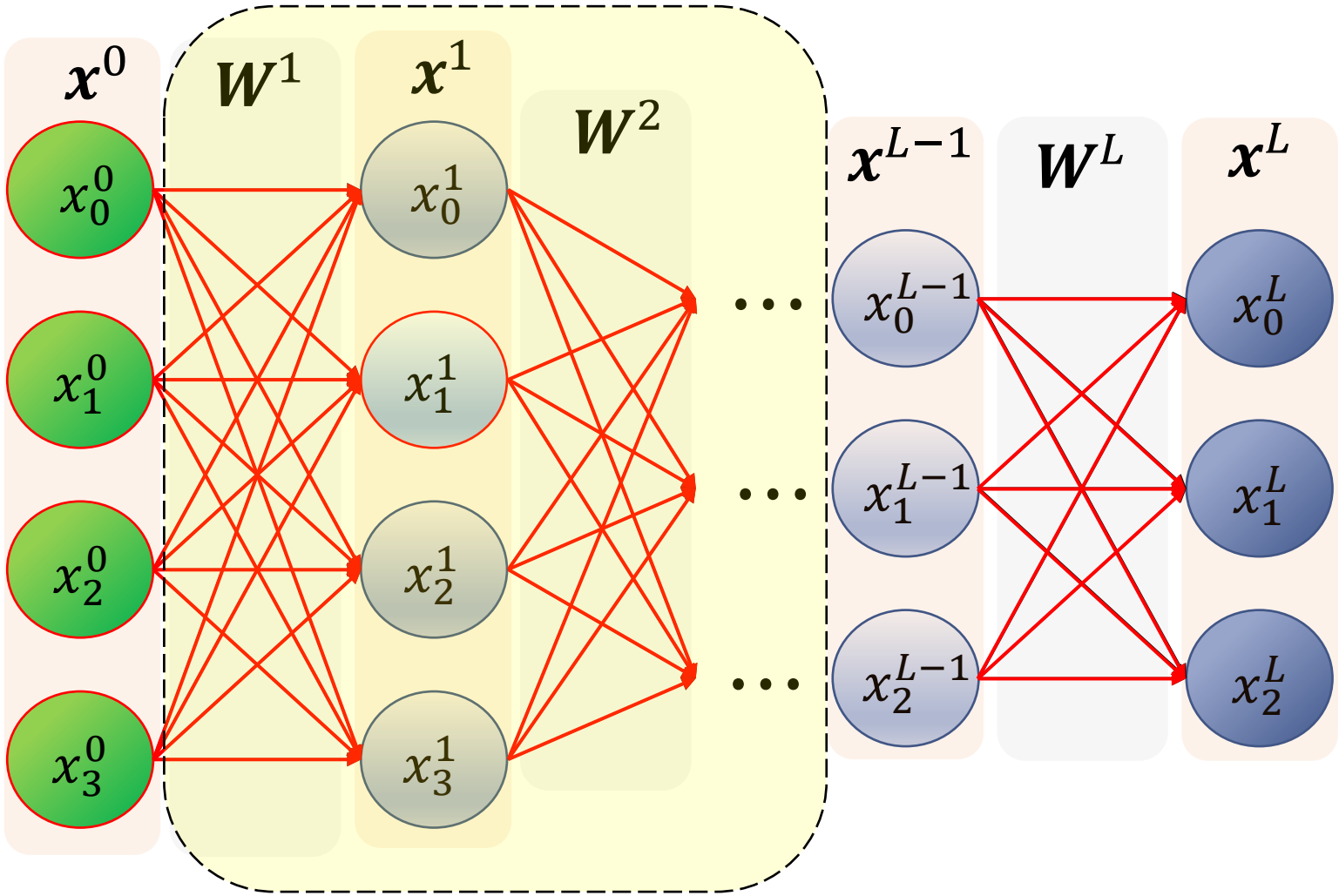
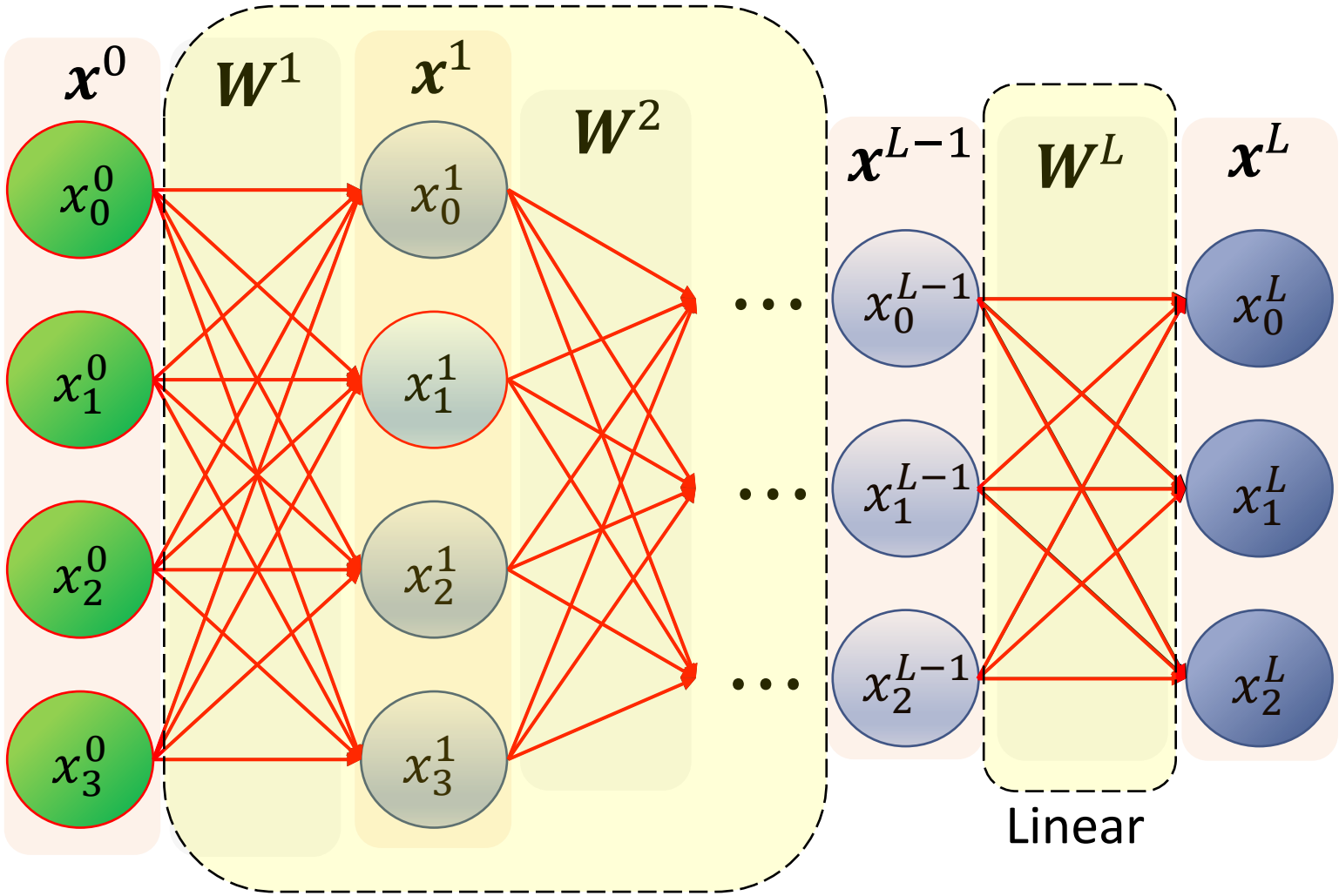


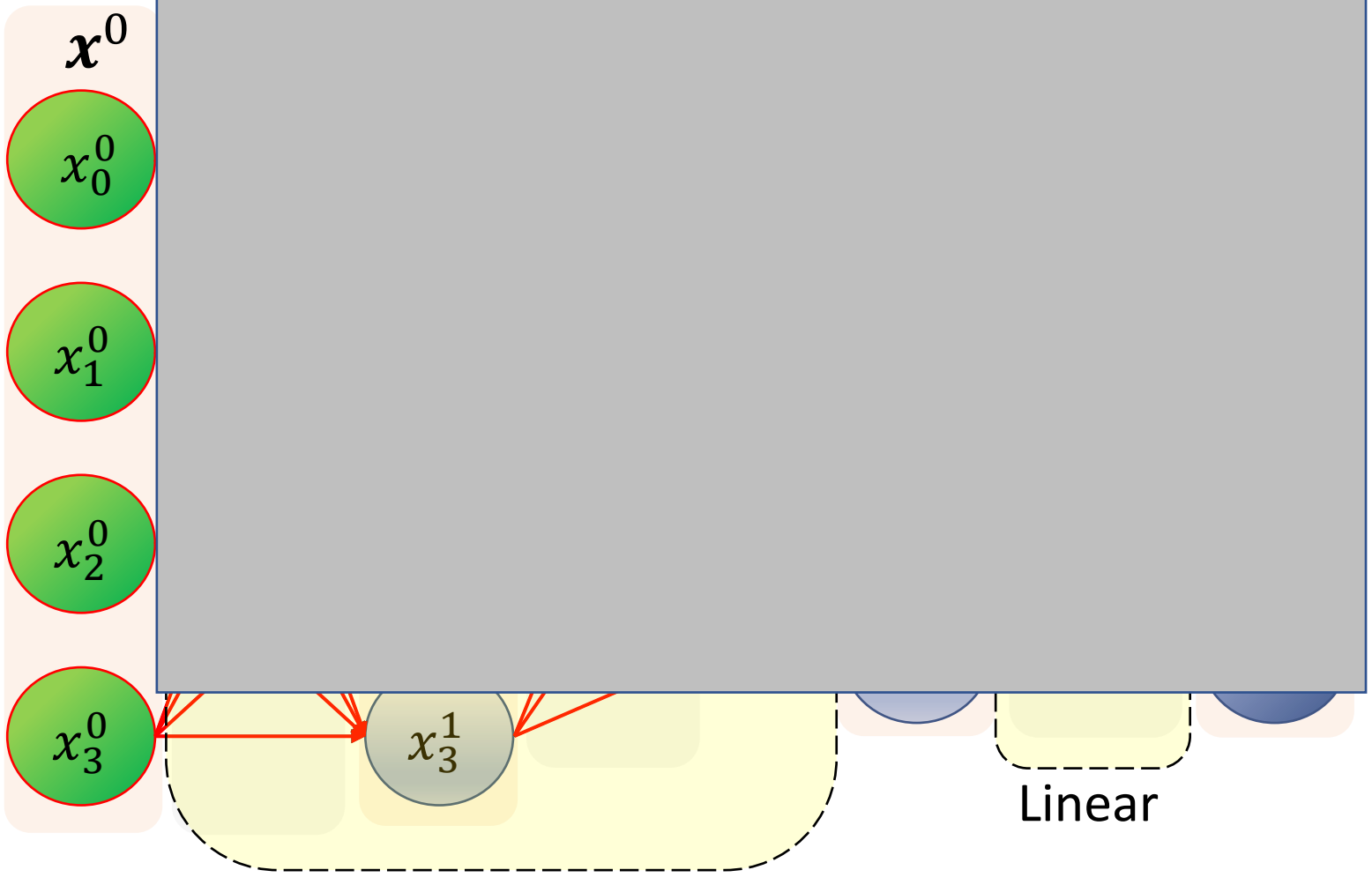
Feature transform

$$x_j^l = \sigma\left(\sum_i w_{ij}^l \cdot x_i^{l-1} + b_j^l\right) \qquad \boldsymbol{x}^l = \boldsymbol{\sigma}\left(\boldsymbol{W}^{l^T}\boldsymbol{x}^{l-1} + \boldsymbol{b}^l\right)$$



$$x_j^l = \sigma\left(\quad \mathbf{b}^l \right)$$

Q: What do you call a single layered net?



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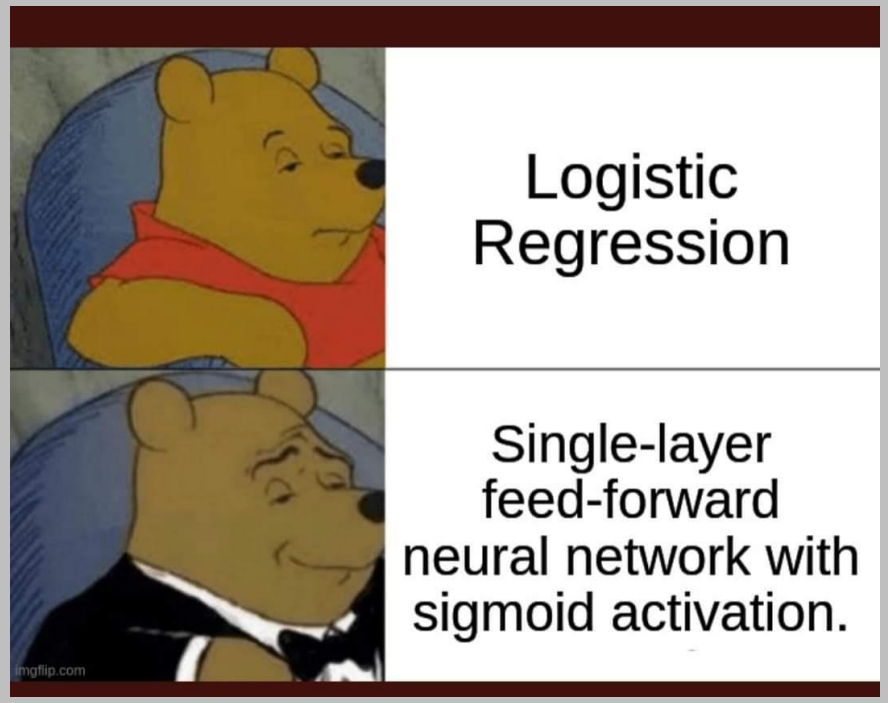
Q: What do you call a single layered net?



$\boldsymbol{x}^0$

x_0^0

x_1^0

x_2^0

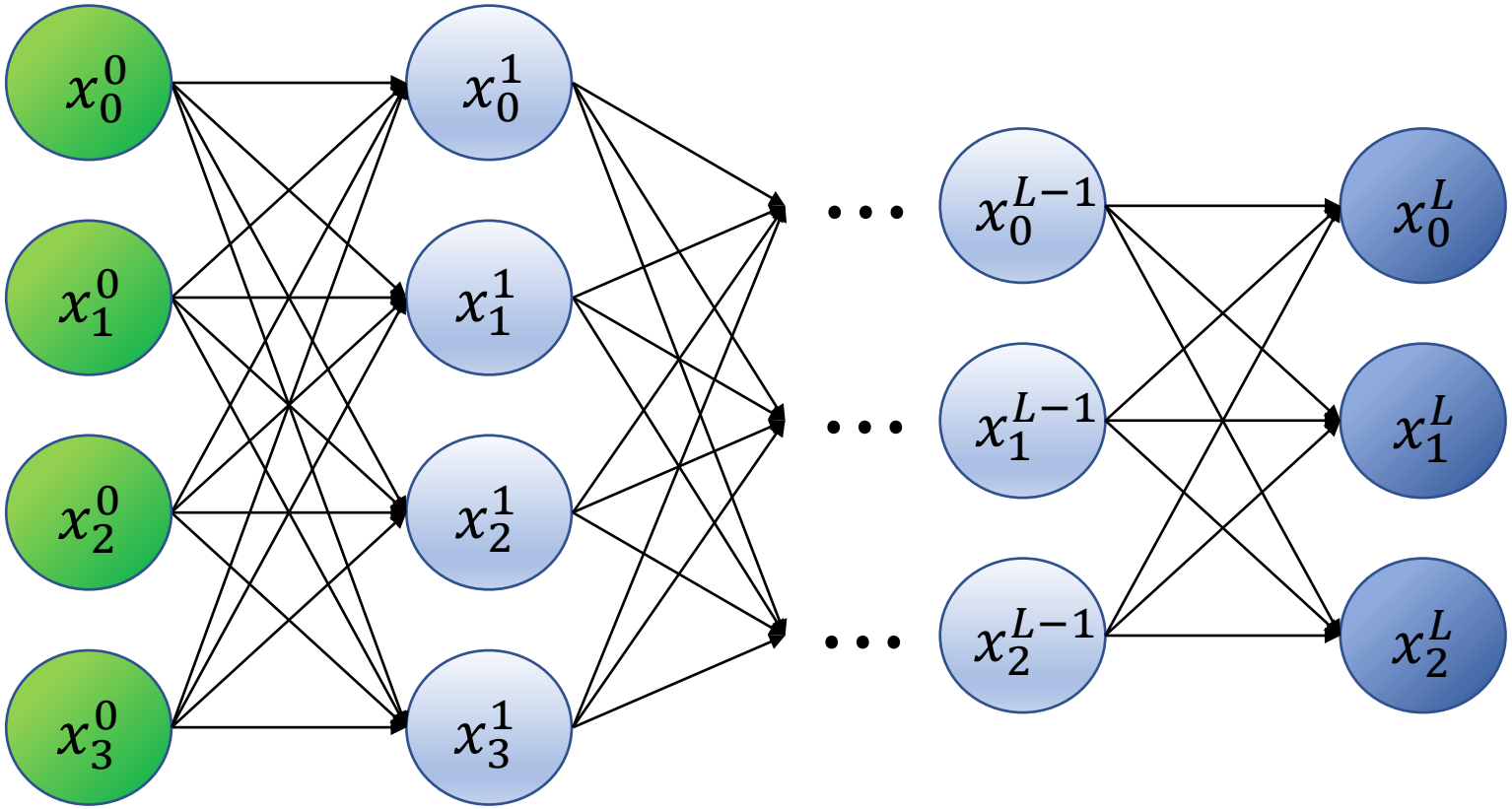
x_3^0

x_3^1

Linear

Feature transform

Learning by SGD



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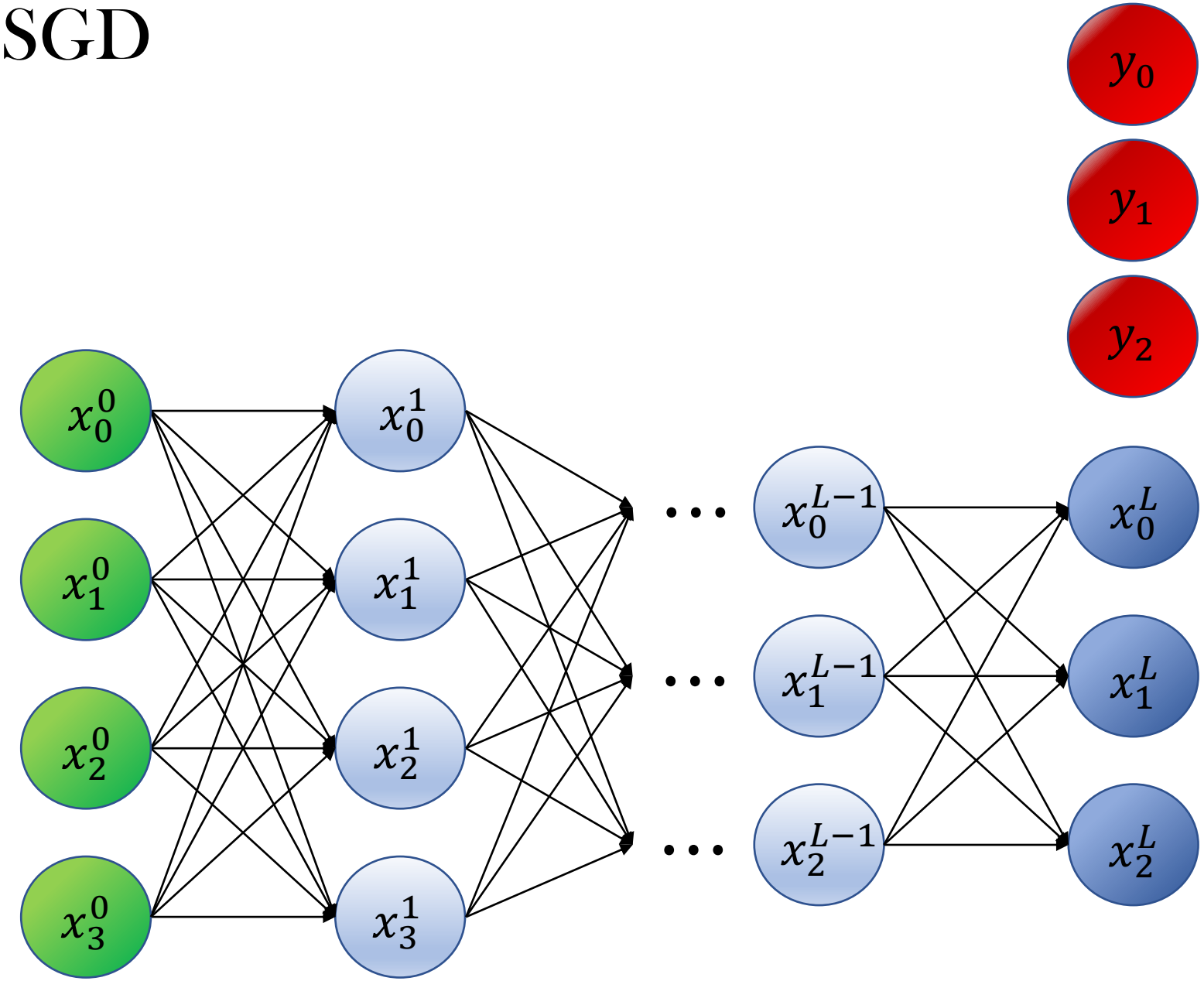
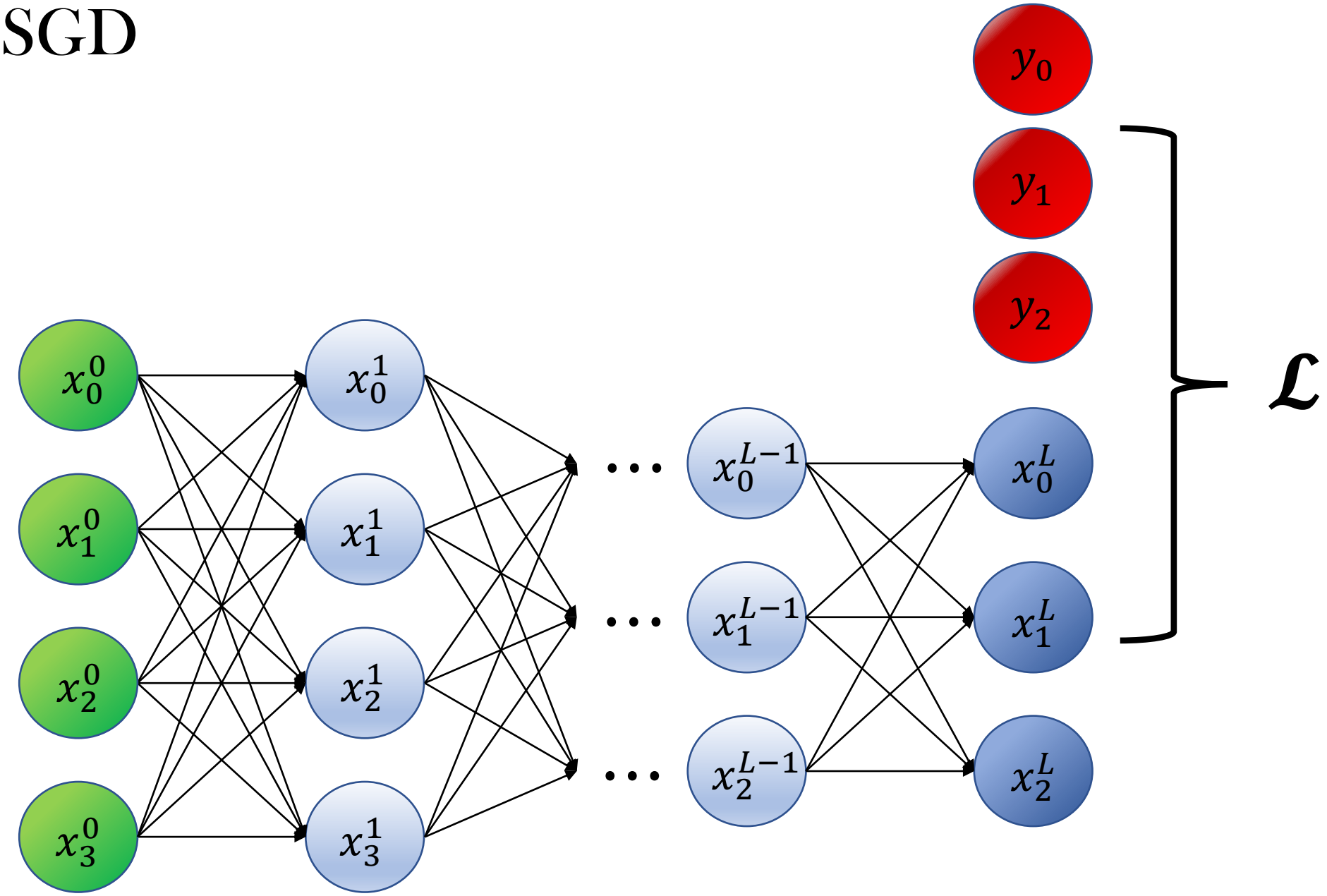
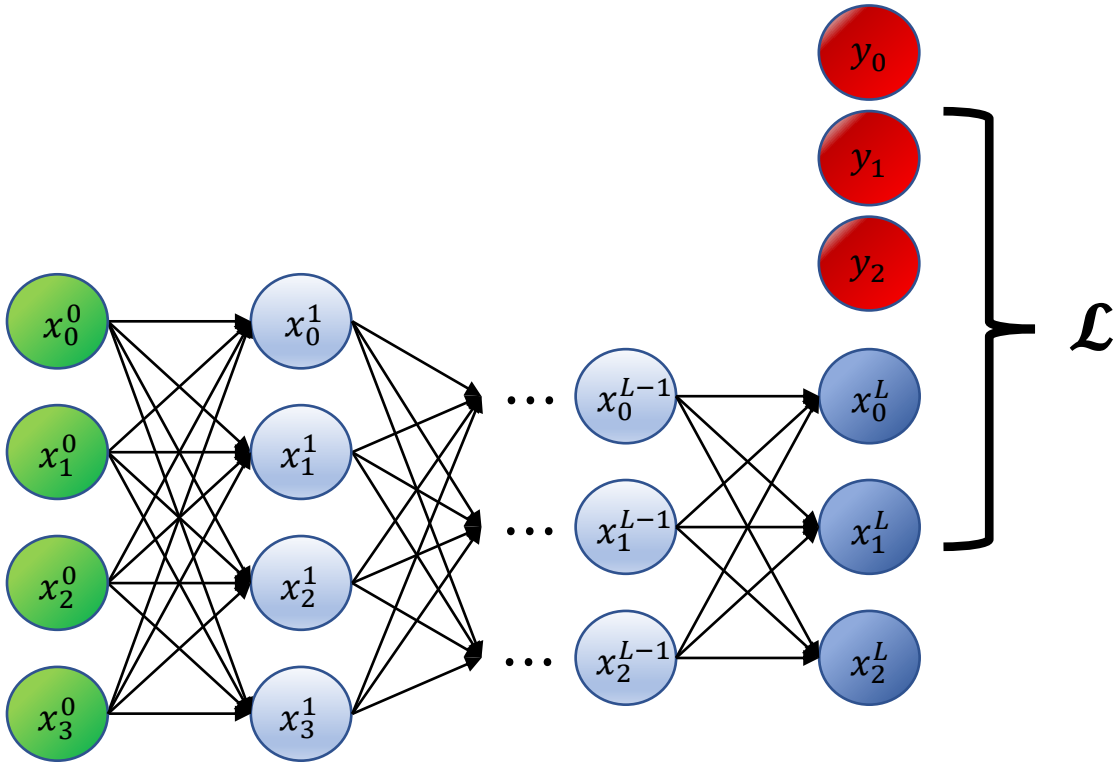


Learning by SGD

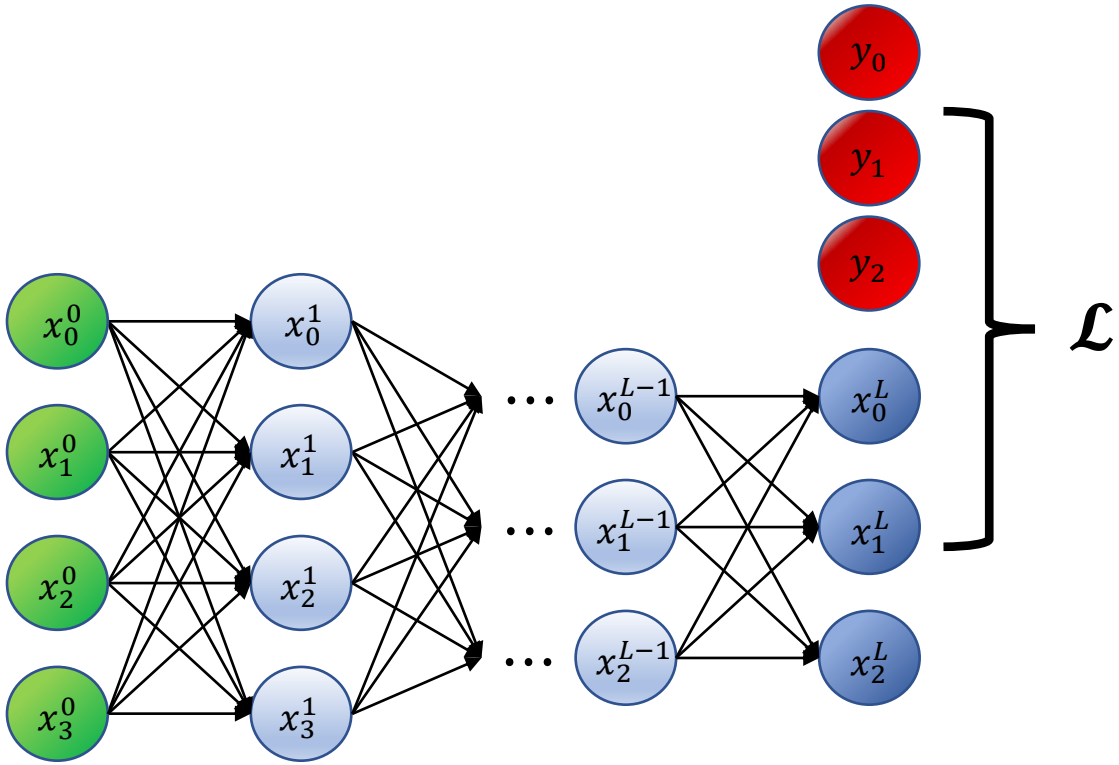


Learning by SGD



Learning by SGD

We need $\frac{\partial \mathcal{L}(\boldsymbol{\theta};\, \boldsymbol{x}, \boldsymbol{y})}{\partial w_{ij}^{l}}$, $\frac{\partial \mathcal{L}(\boldsymbol{\theta};\, \boldsymbol{x}, \boldsymbol{y})}{\partial b_{j}^{l}}$ To all l,I,j



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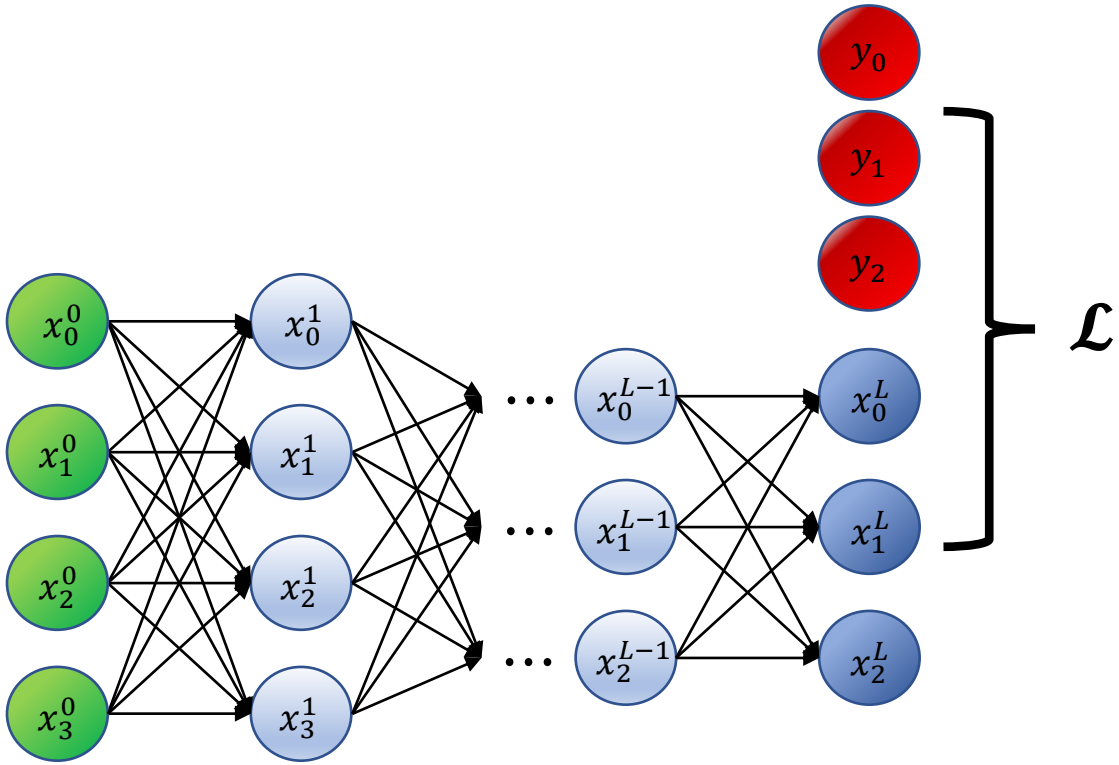
$$x_j^L = \sum_i W_{ij}^L x_i^{L-1} + b_j^L$$



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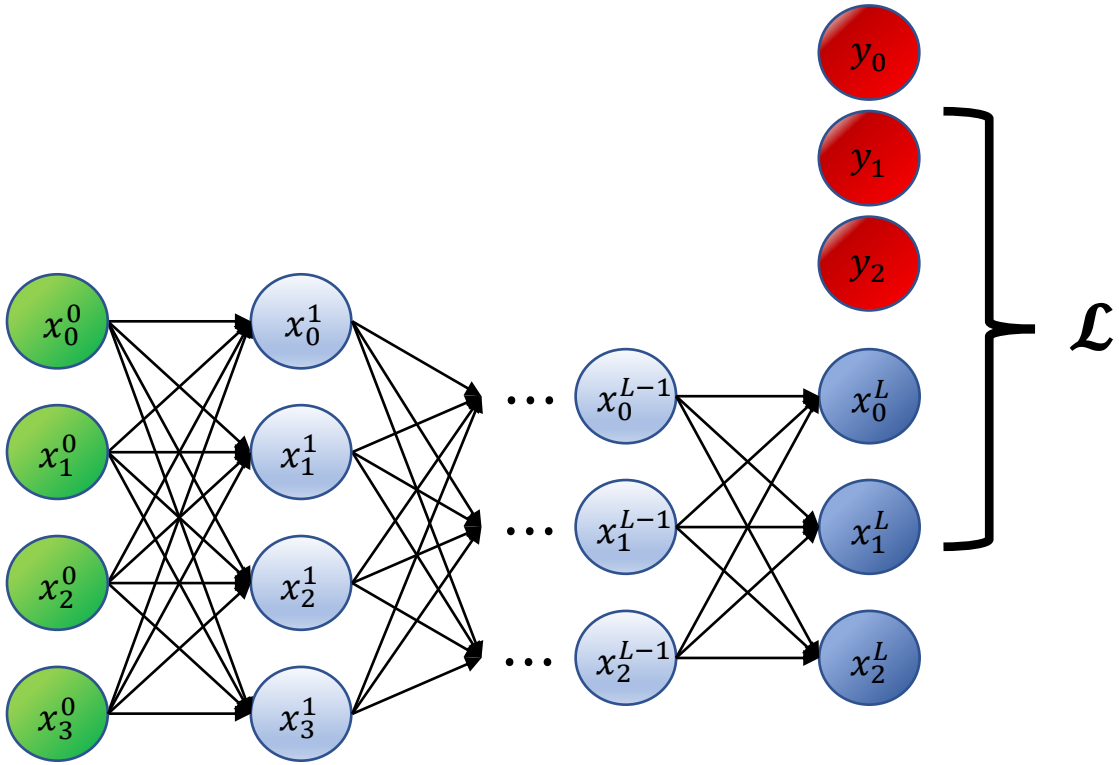

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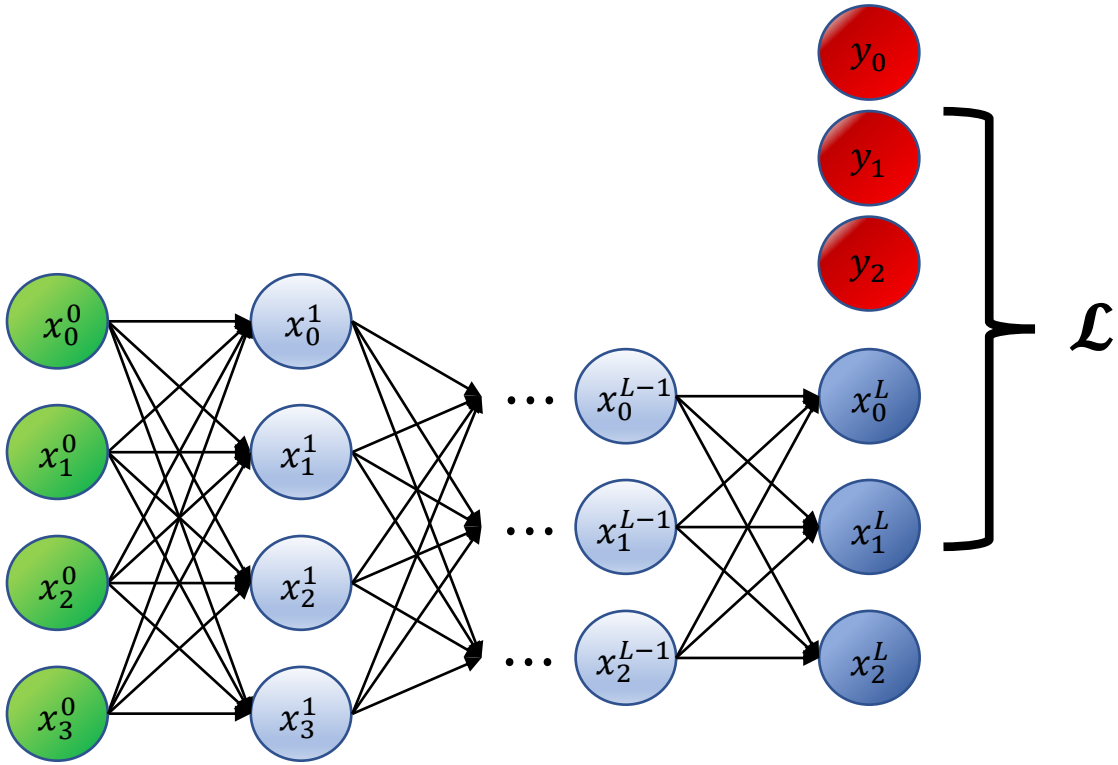
$$x_j^L = \sum_i W_{ij}^L x_i^{L-1} + b_j^L$$

$$= \sum_i W_{ij}^L \sigma\left(\sum_k W_{ki}^{L-1} x_k^{L-2} + b_i^{L-1}\right) + b_j^L$$



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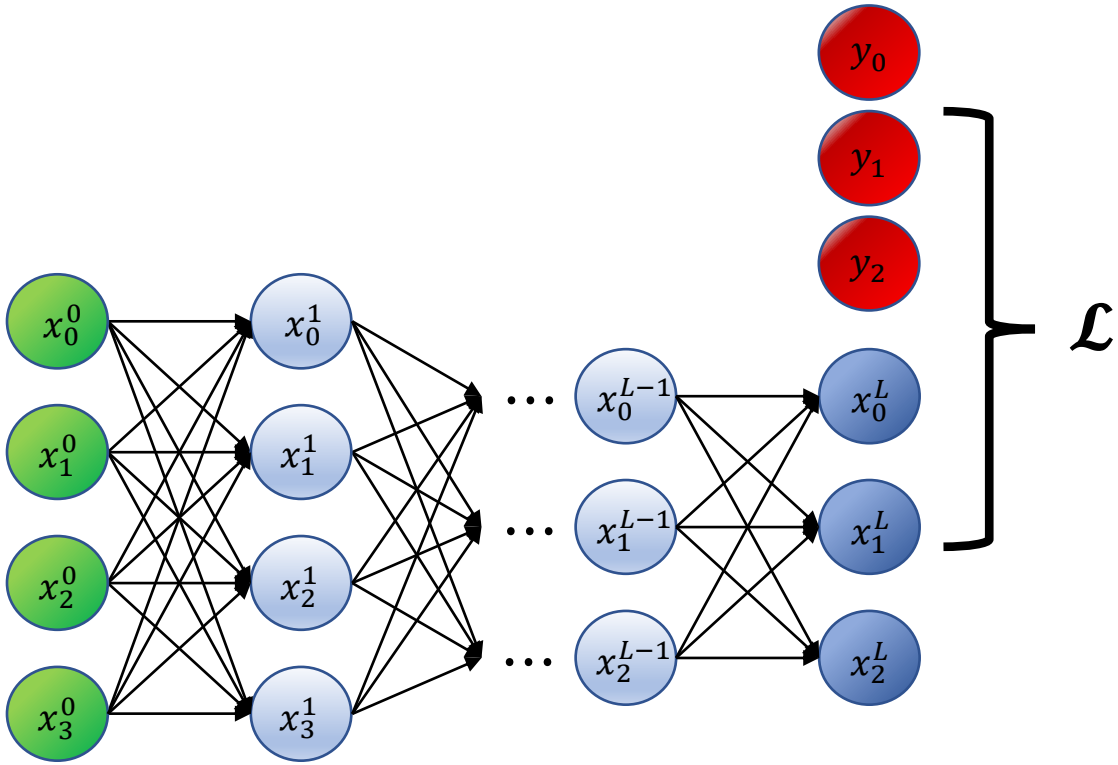

Learning by SGD

We need $\frac{\partial \mathcal{L}(\boldsymbol{\theta};\, \boldsymbol{x}, \boldsymbol{y})}{\partial w_{ij}^{l}}$, $\frac{\partial \mathcal{L}(\boldsymbol{\theta};\, \boldsymbol{x}, \boldsymbol{y})}{\partial b_{j}^{l}}$ To all l,I,j

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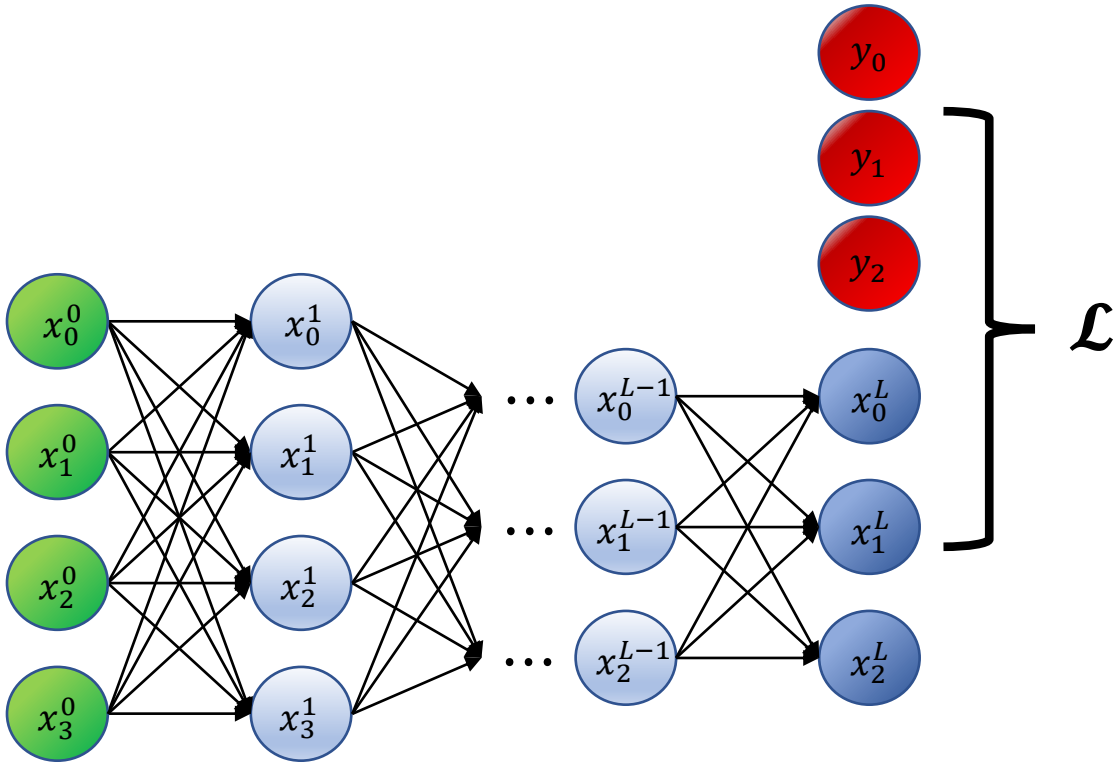
$$= \sum_i W_{ij}^L \sigma\left(\sum_k W_{ki}^{L-1} \sigma\left(\sum_m W_{mk}^{L-2} x_m^{L-3} + b_k^{L-2}\right) + b_i^{L-1}\right) + b_j^L$$

$$\vdots$$



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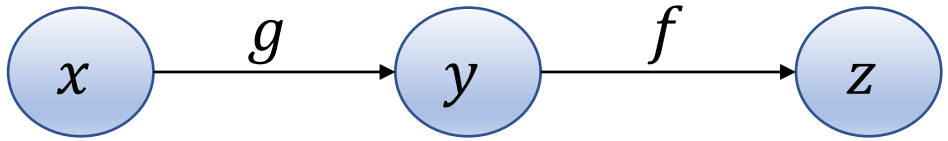

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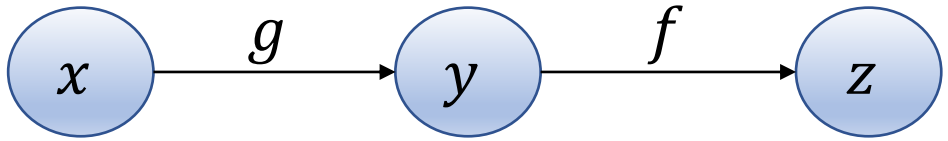

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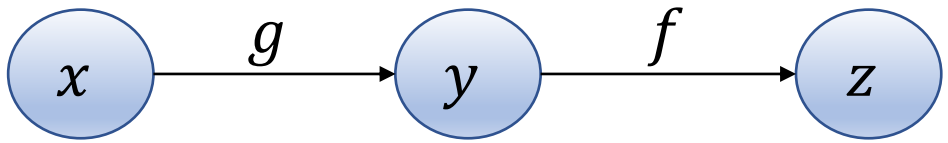

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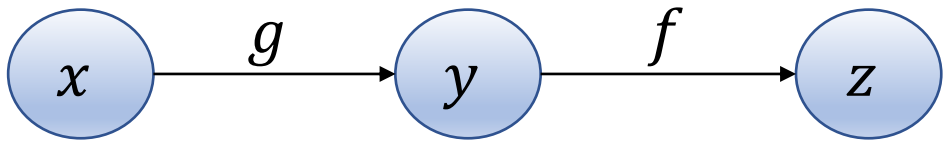

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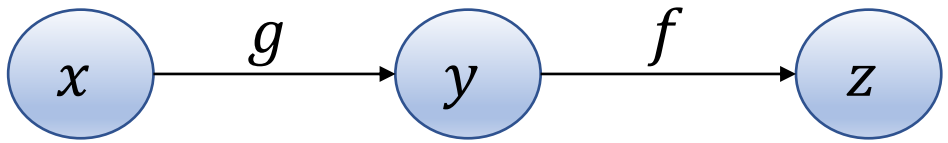

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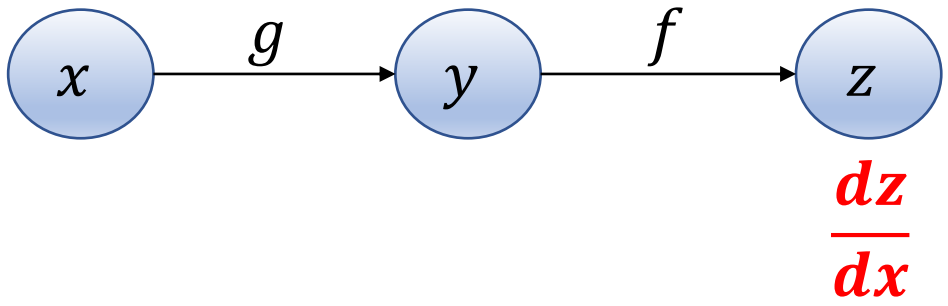

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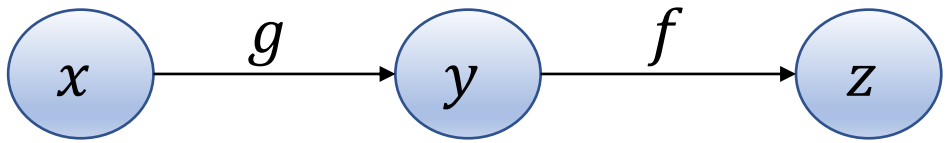

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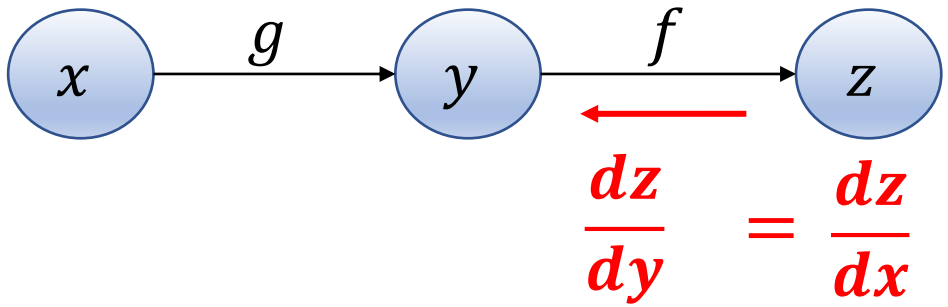

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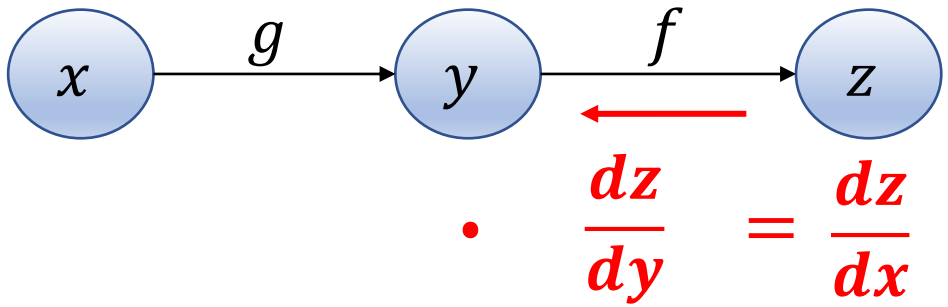

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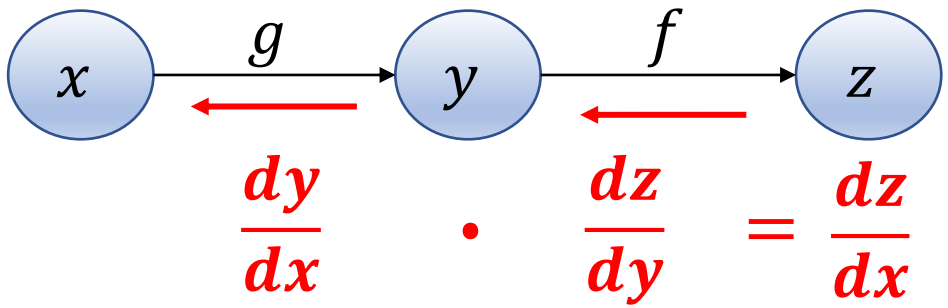

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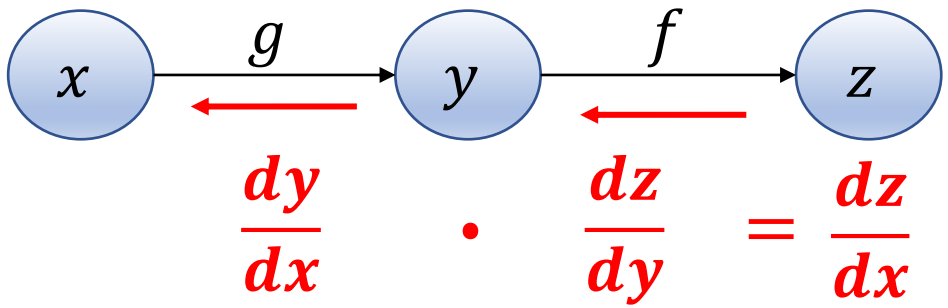

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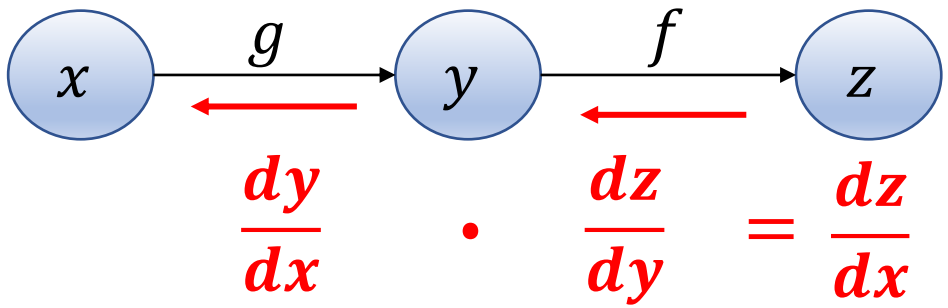

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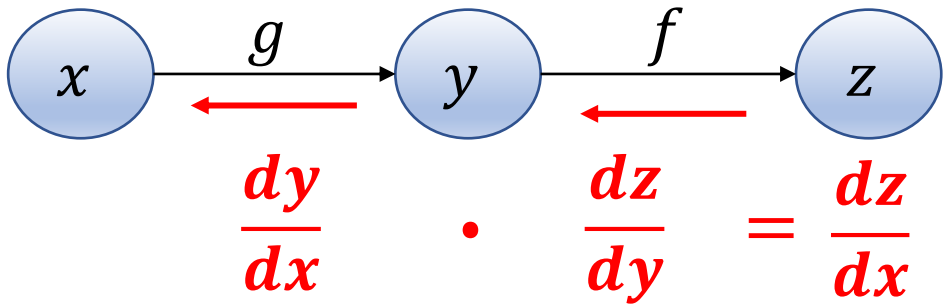

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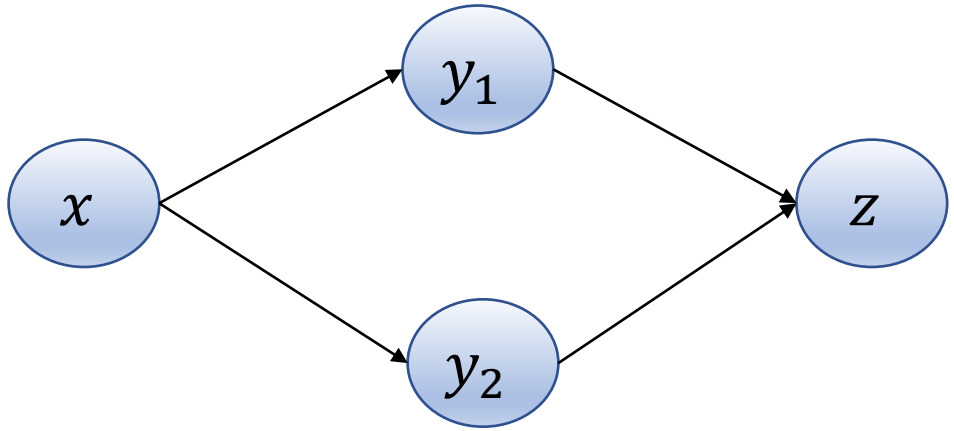

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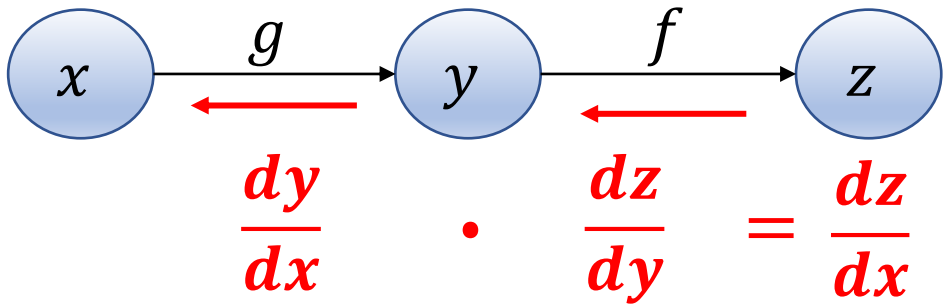

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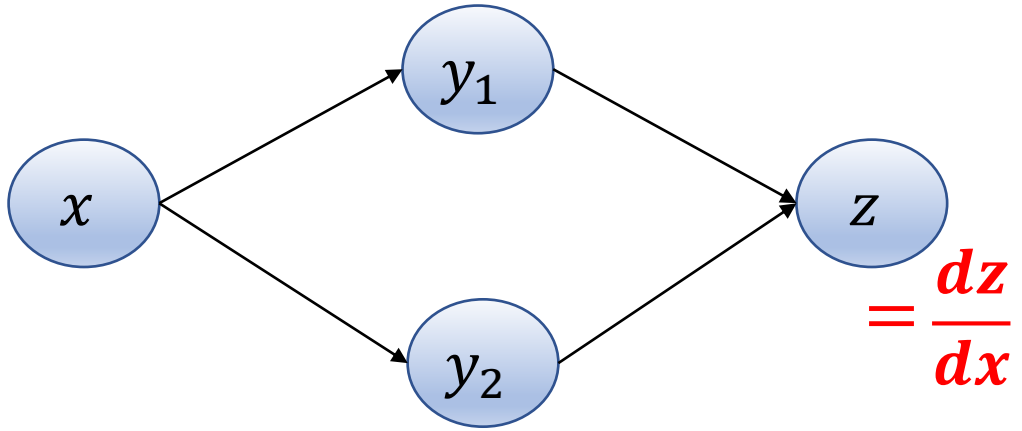

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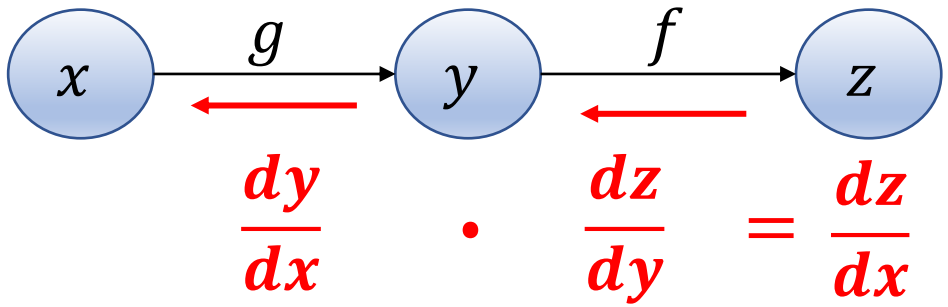

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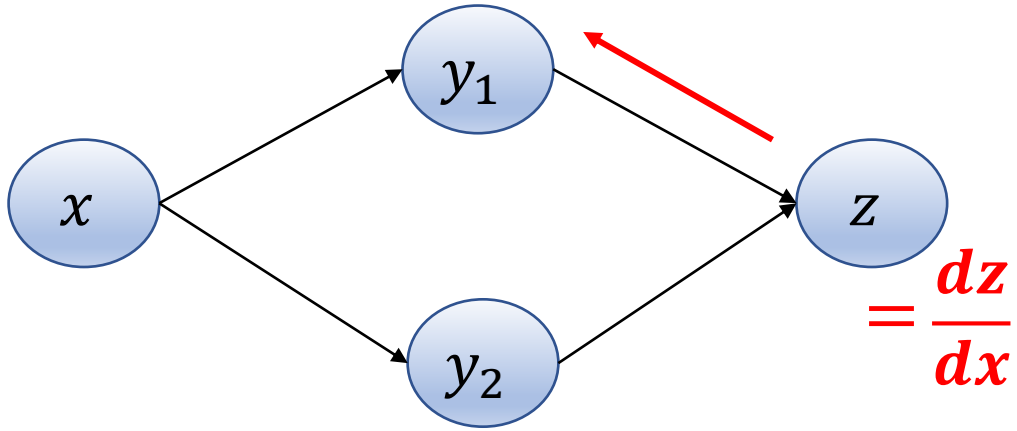

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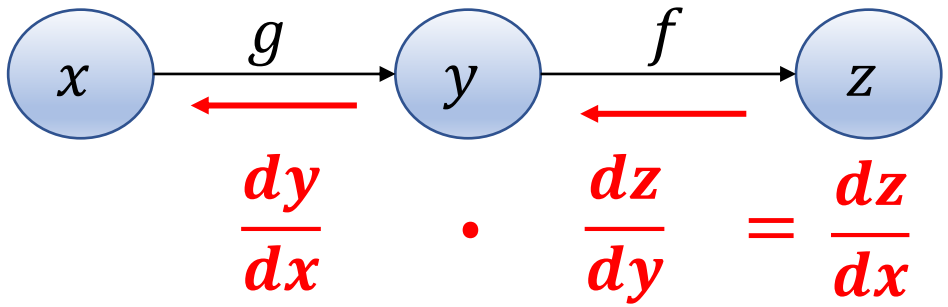

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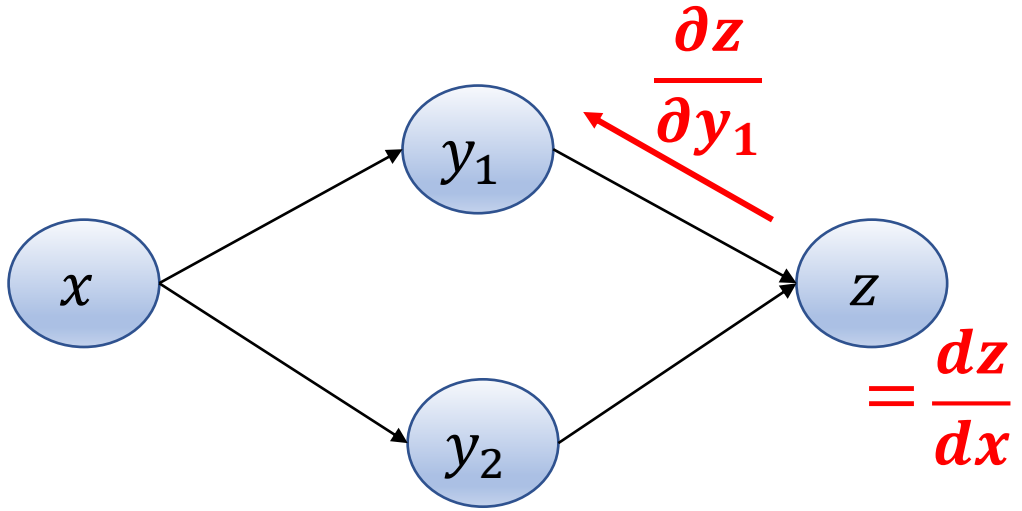

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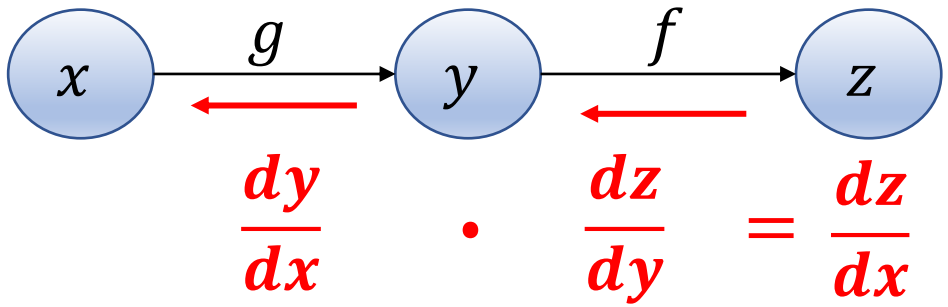

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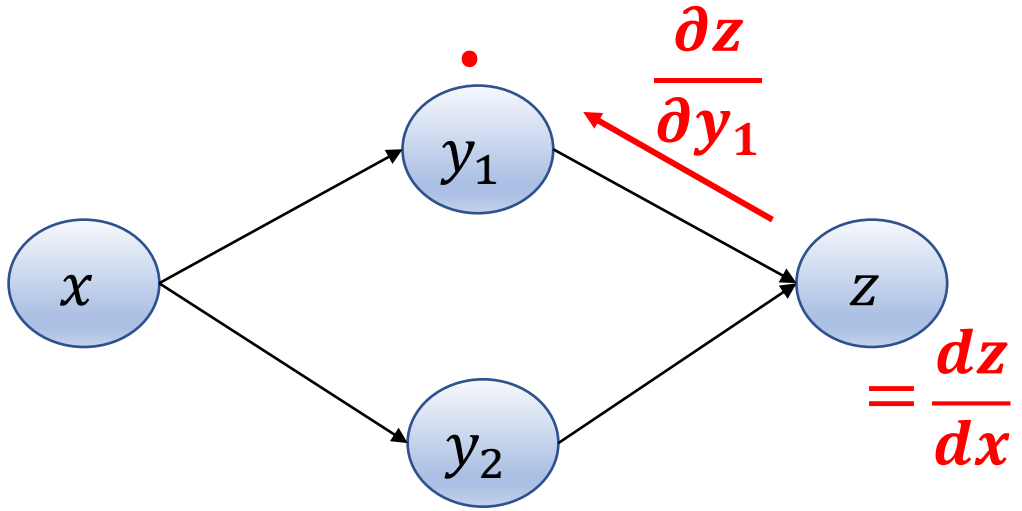

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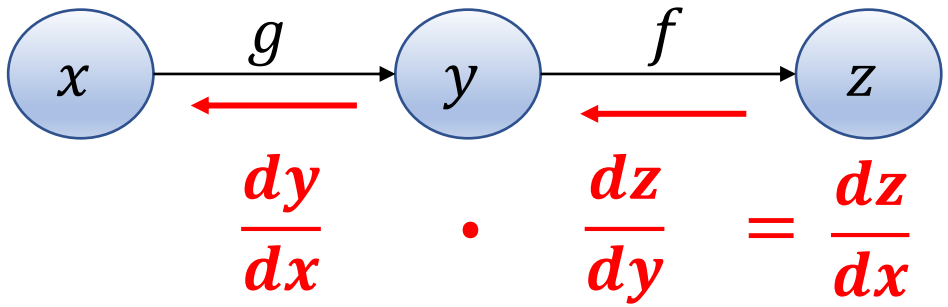

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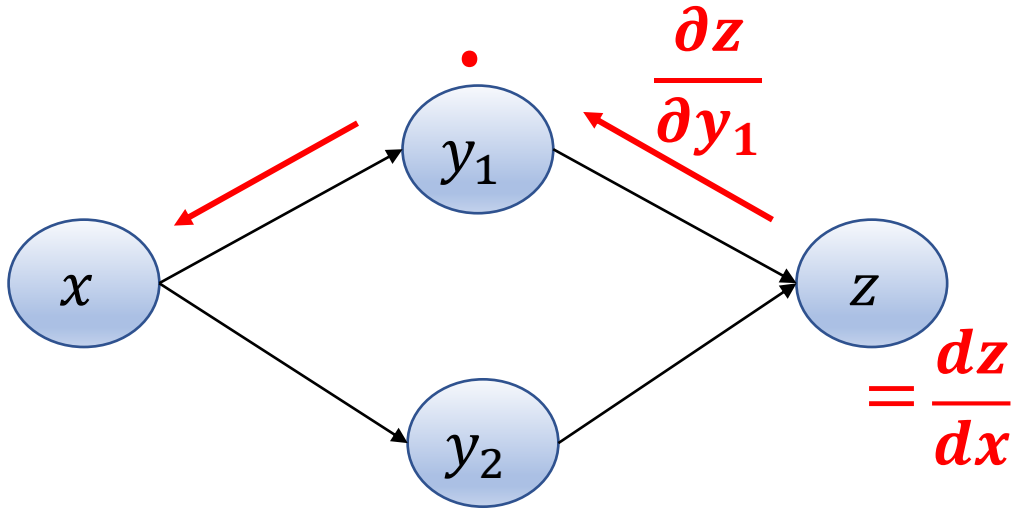

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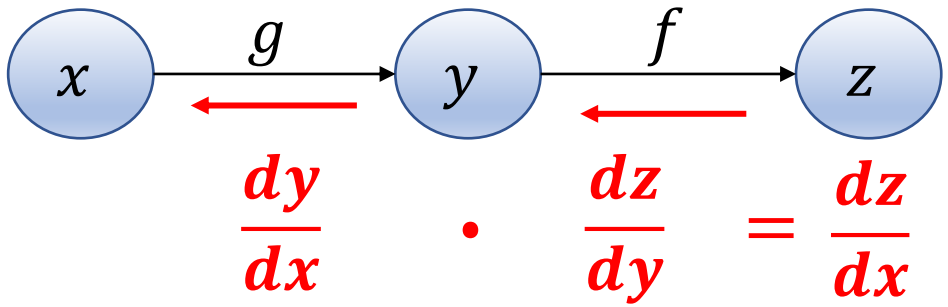

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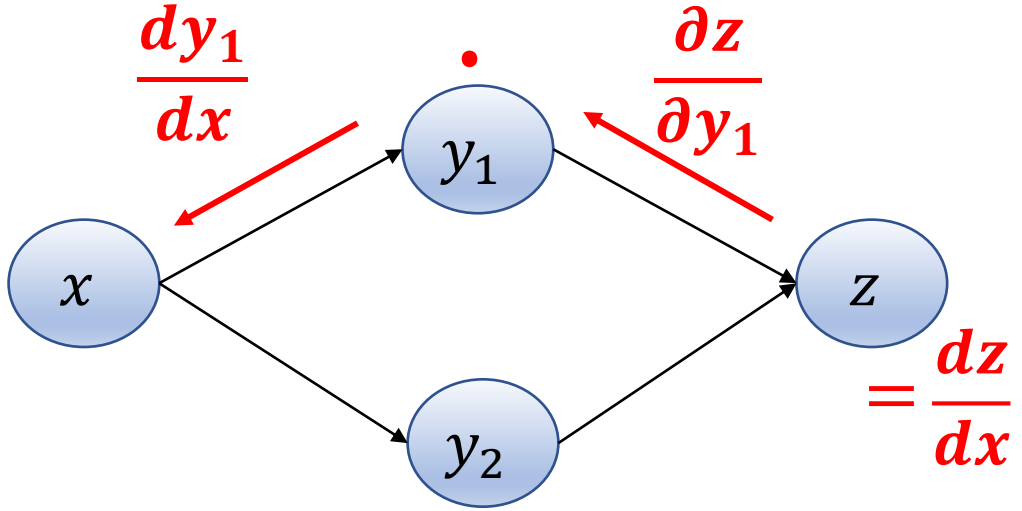

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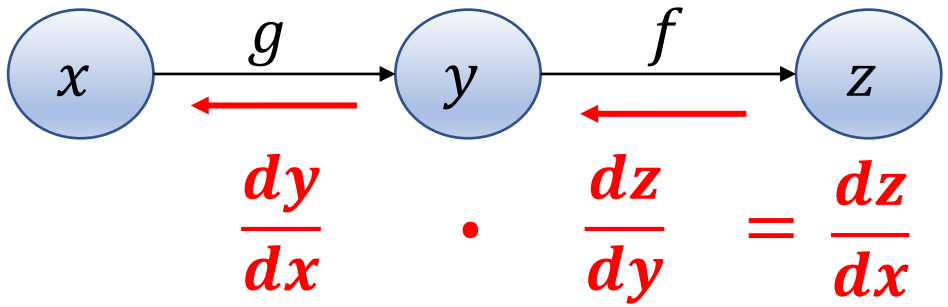

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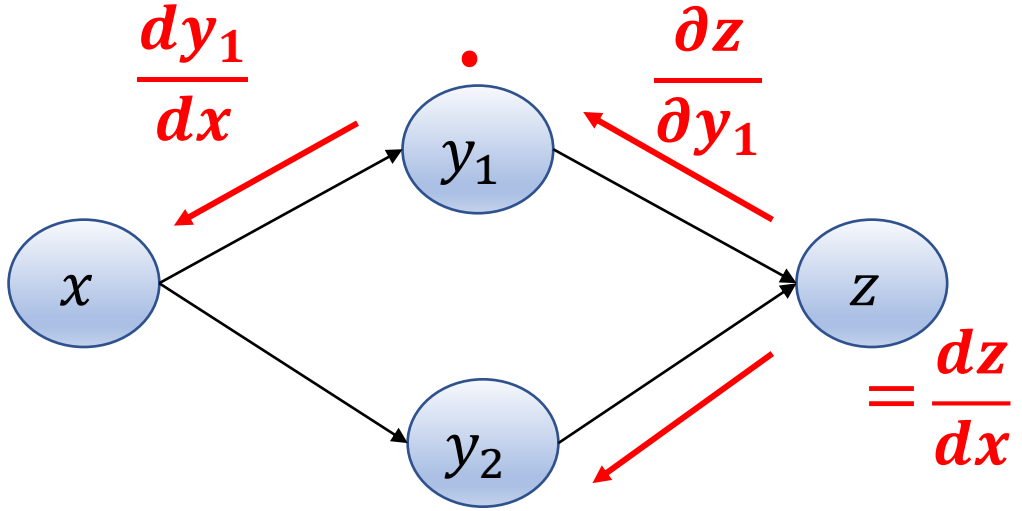

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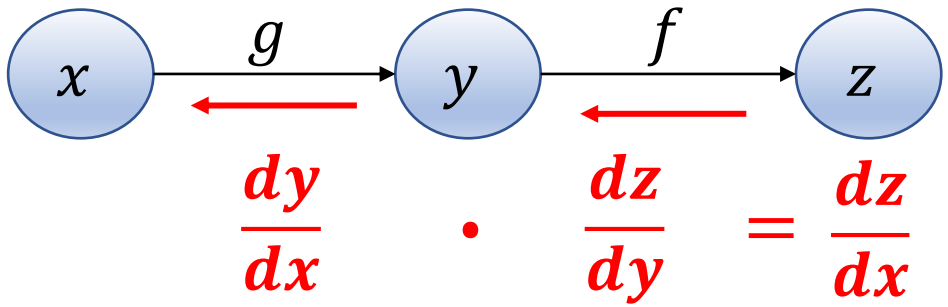

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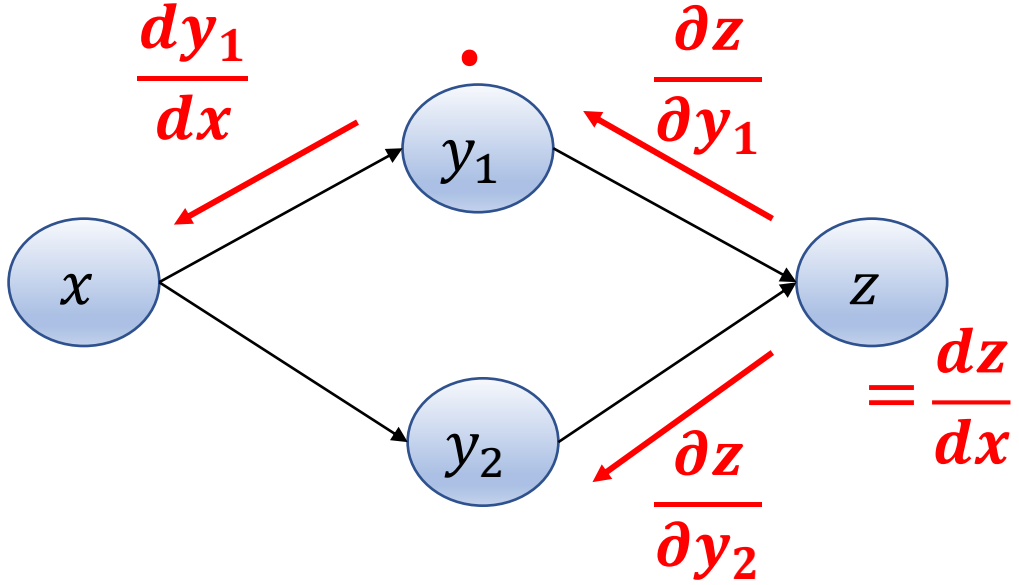

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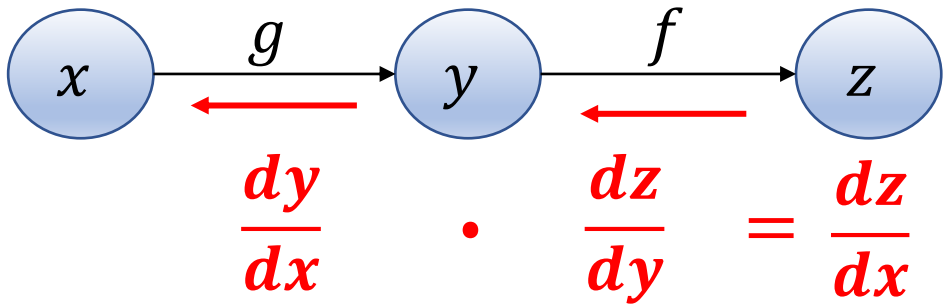

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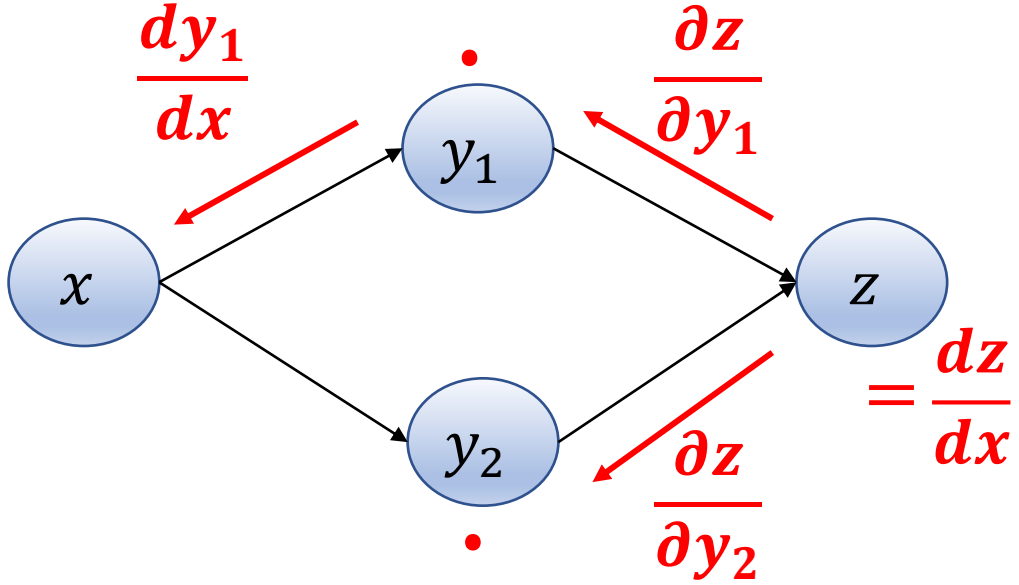

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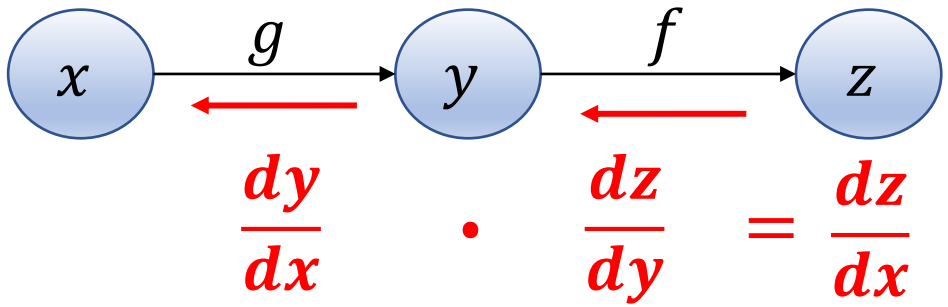

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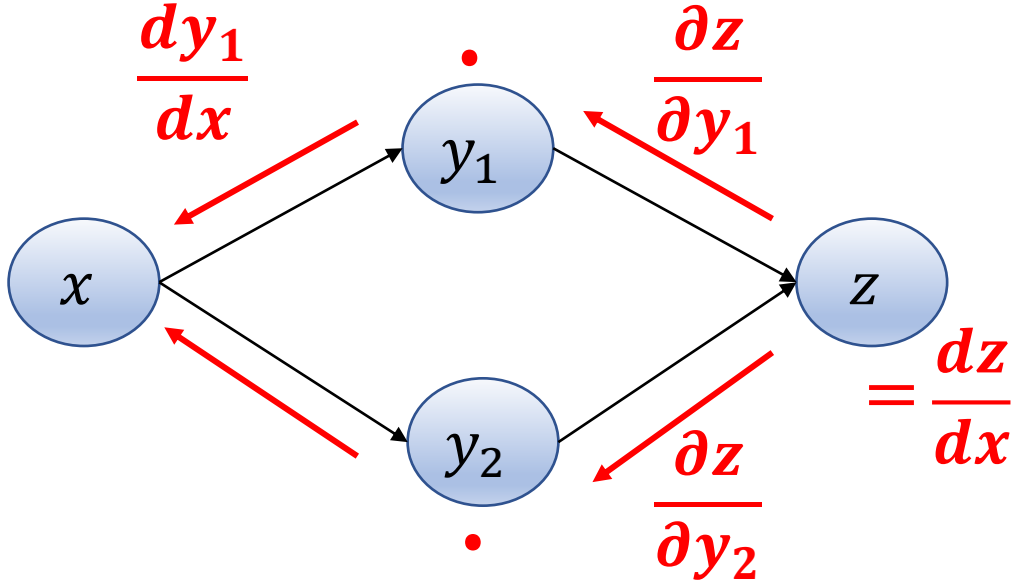

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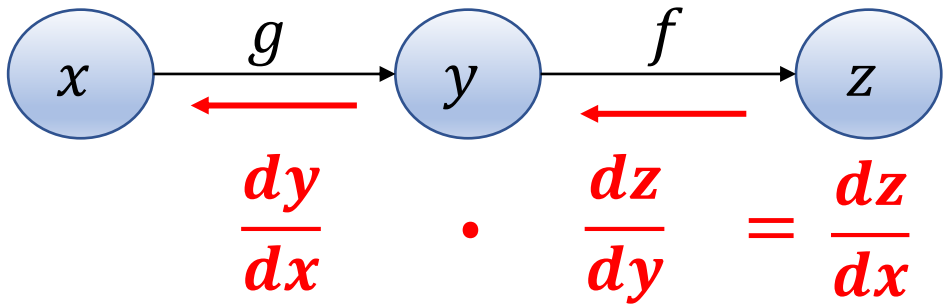

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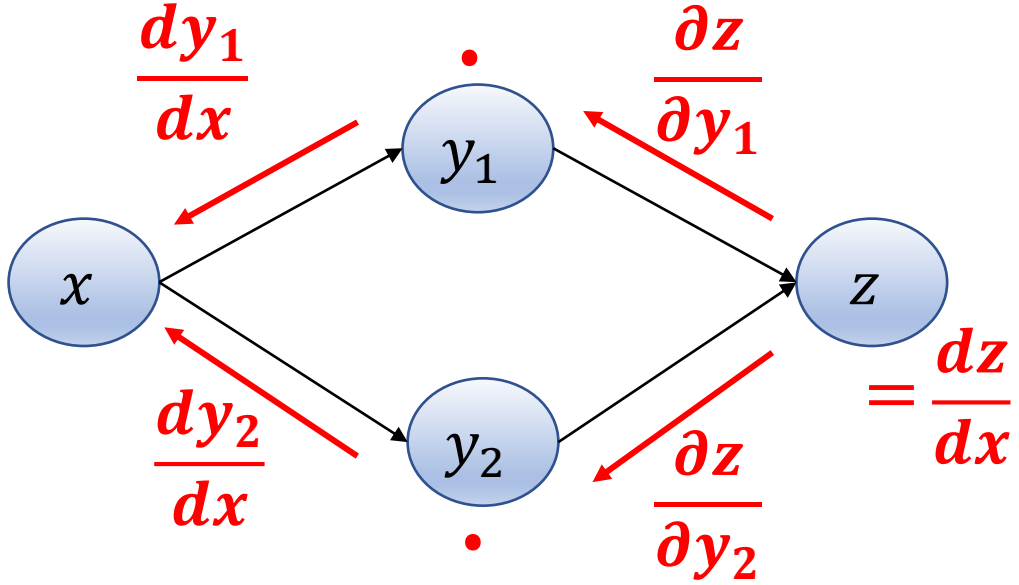

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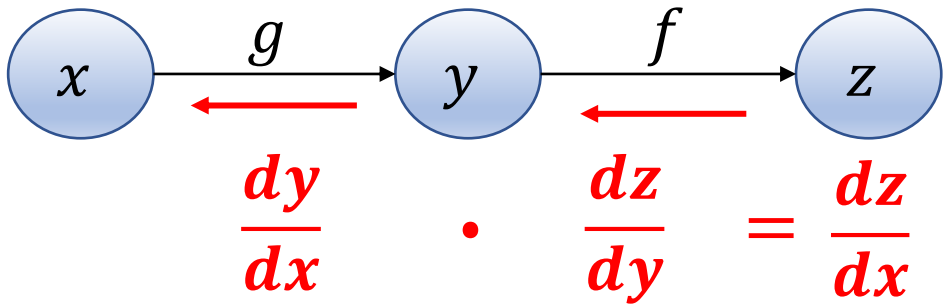

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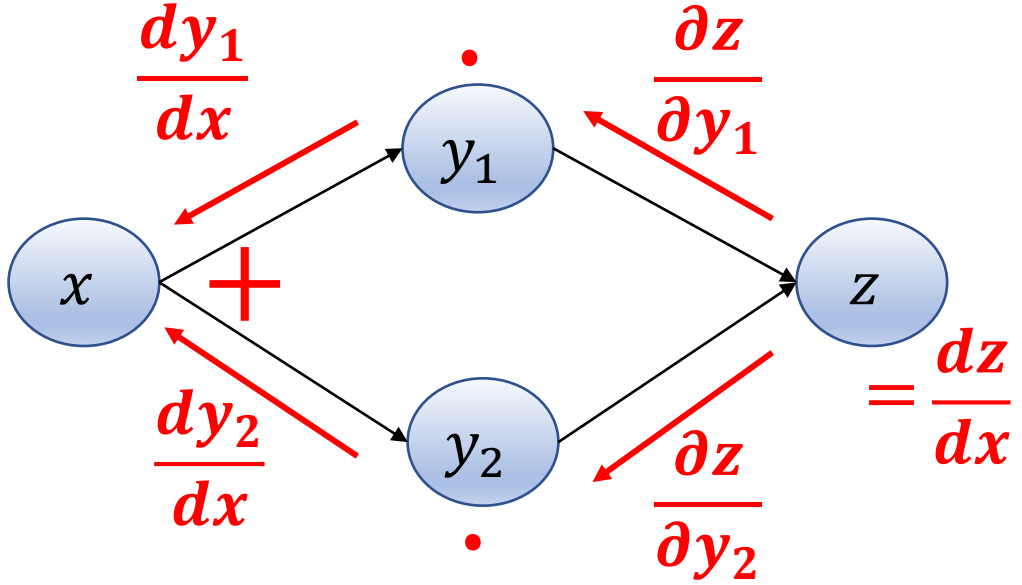
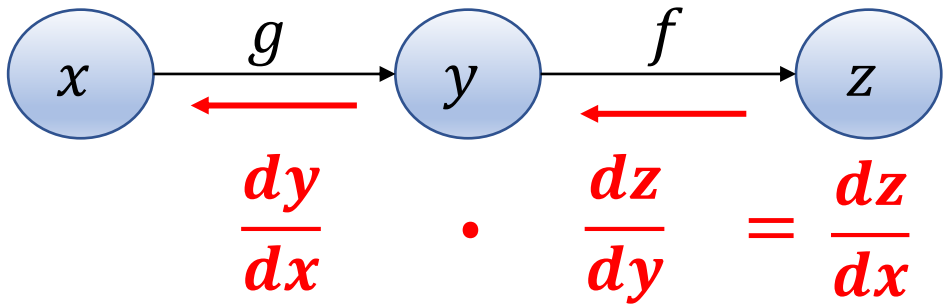


Chain rule reminder

$$f\big(g(x)\big)' = f'\big(g(x)\big) \cdot g'(x)$$

$$y = f(x), \qquad z = g(y)$$

$$\frac{dz}{dx} = \frac{dz}{dy}\,\frac{dy}{dx}$$



$$y_1 = 2x^2 + e^{2x}$$
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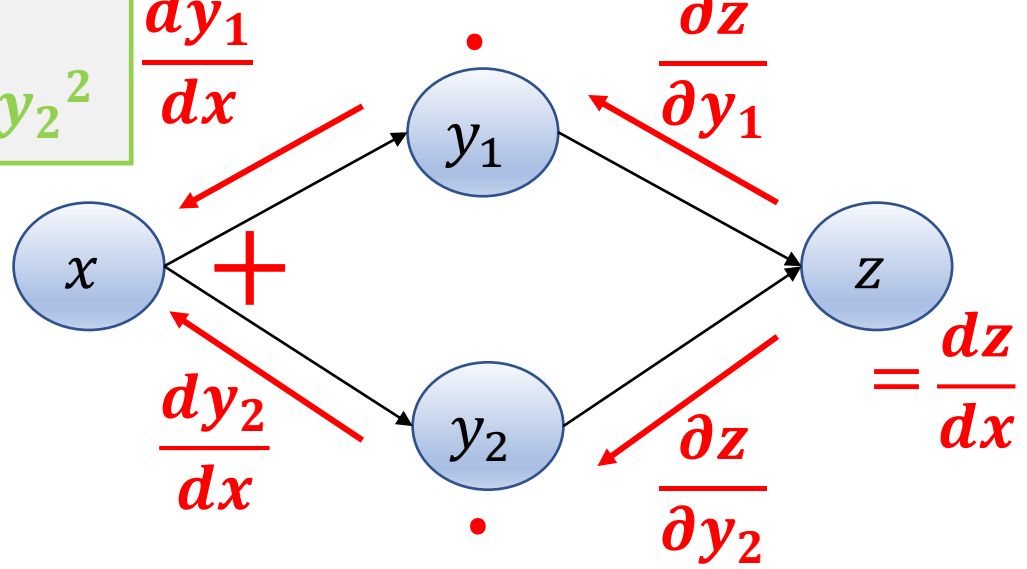

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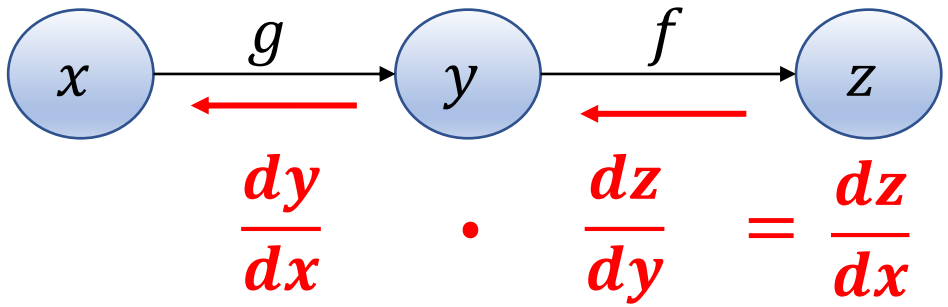

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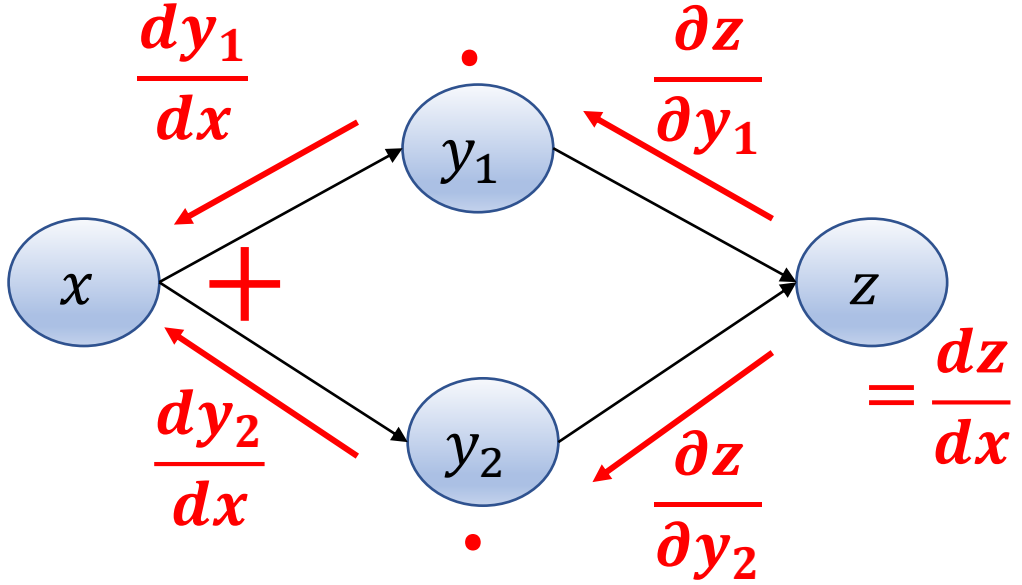

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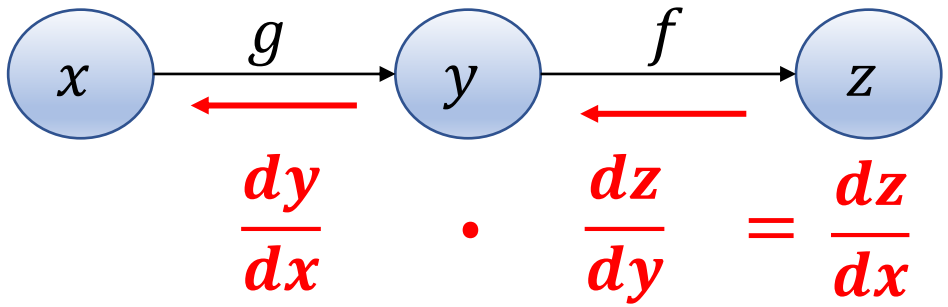

$$\frac{dz(y_1, y_2)}{dx} = \frac{\partial z}{\partial y_1}\,\frac{dy_1}{dx} + \frac{\partial z}{\partial y_2}\,\frac{dy_2}{dx}$$

3

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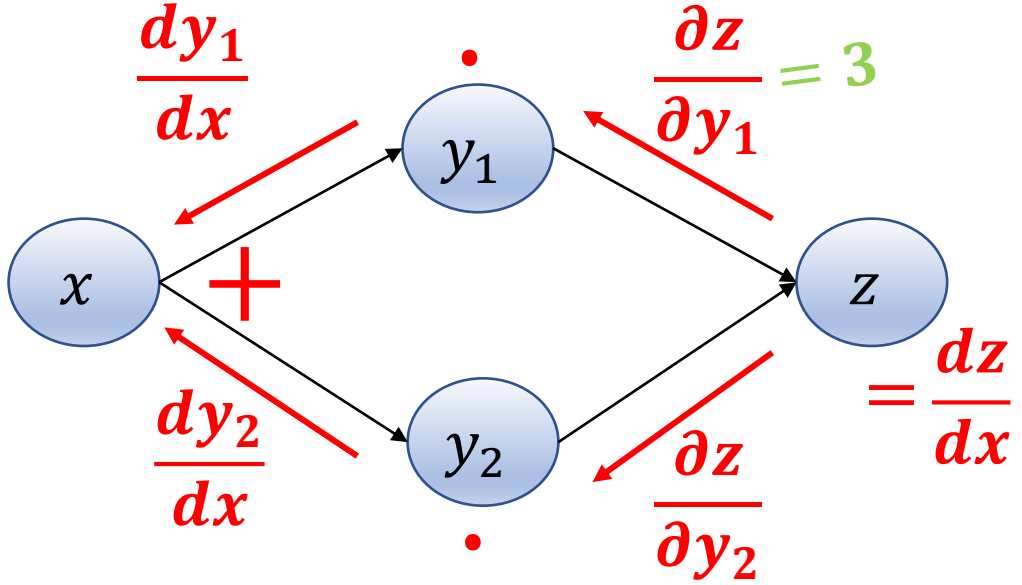

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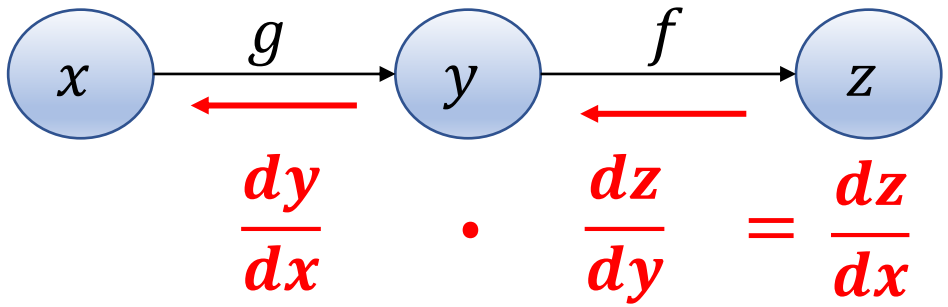

$$\frac{dz(y_1, y_2)}{dx} = \frac{\partial z}{\partial y_1}\,\frac{dy_1}{dx} + \frac{\partial z}{\partial y_2}\,\frac{dy_2}{dx}$$

3 $\qquad$ $4x + 2e^{2x}$

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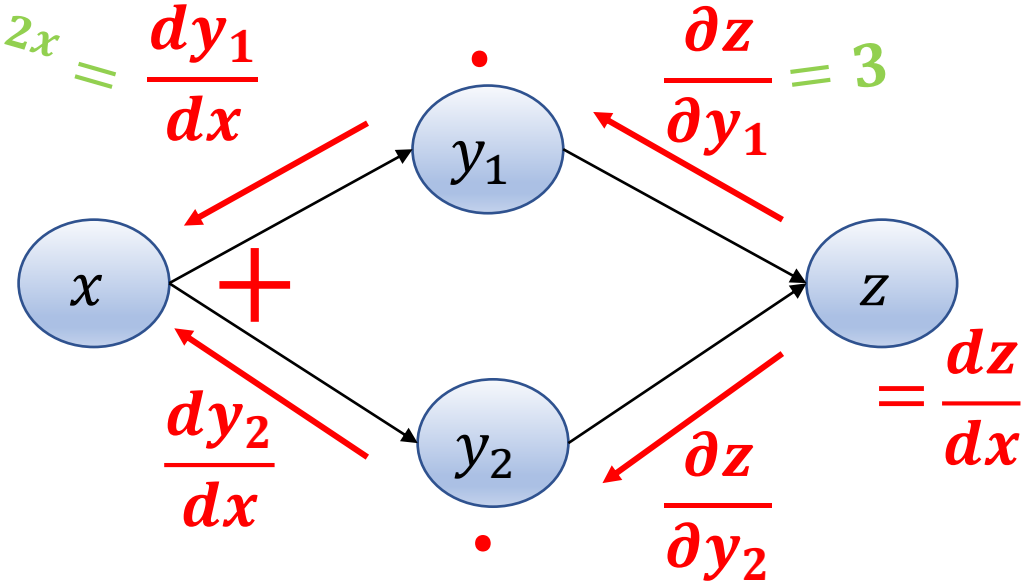

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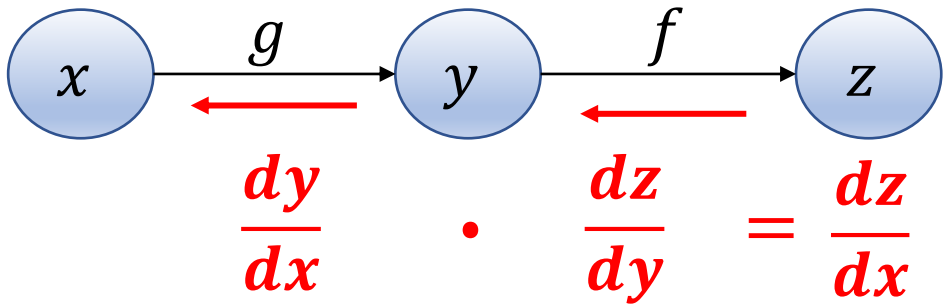

$$\frac{dz(y_1, y_2)}{dx} = \frac{\partial z}{\partial y_1}\,\frac{dy_1}{dx} + \frac{\partial z}{\partial y_2}\,\frac{dy_2}{dx}$$

3 $\qquad$ $4x \times 2e^{2x}$ $\qquad$ $2e^{x}$

$y_1 = 2x^2 + e^{2x}$
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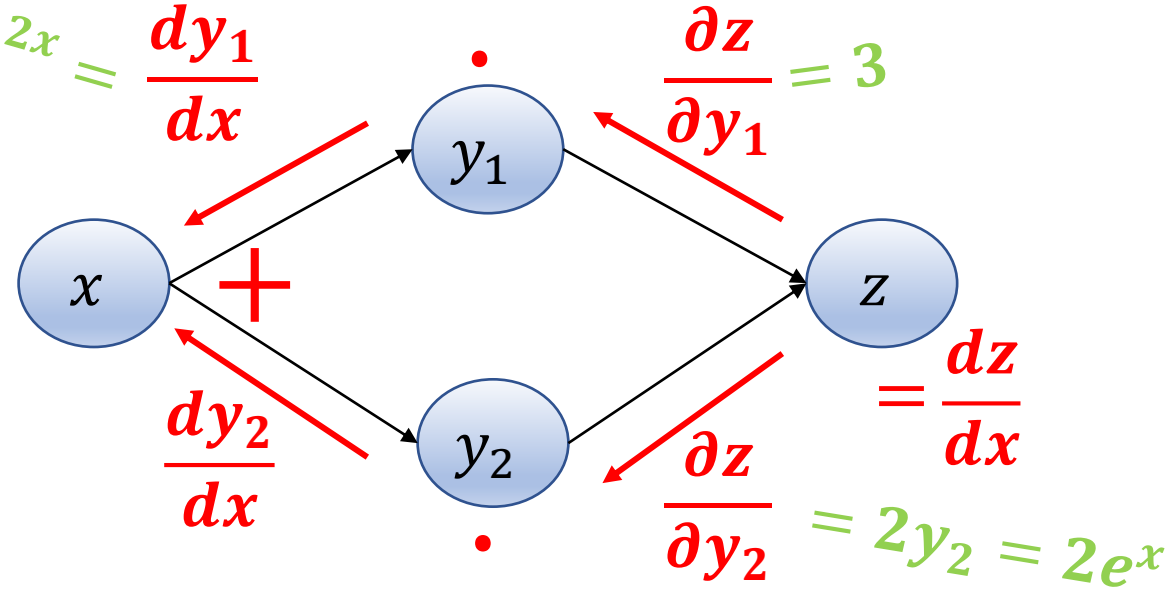

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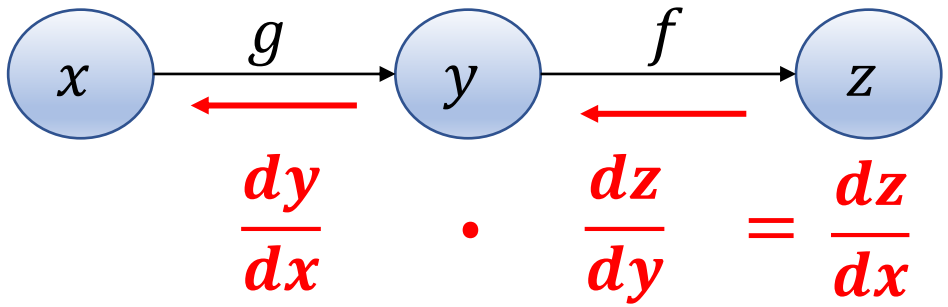

$$\frac{dz(y_1, y_2)}{dx} = \frac{\partial z}{\partial y_1}\,\frac{dy_1}{dx} + \frac{\partial z}{\partial y_2}\,\frac{dy_2}{dx}$$

3 $\quad 4x + 2e^{2x} \quad 2e^x \quad e^x$

$$y_1 = 2x^2 + e^{2x}$$
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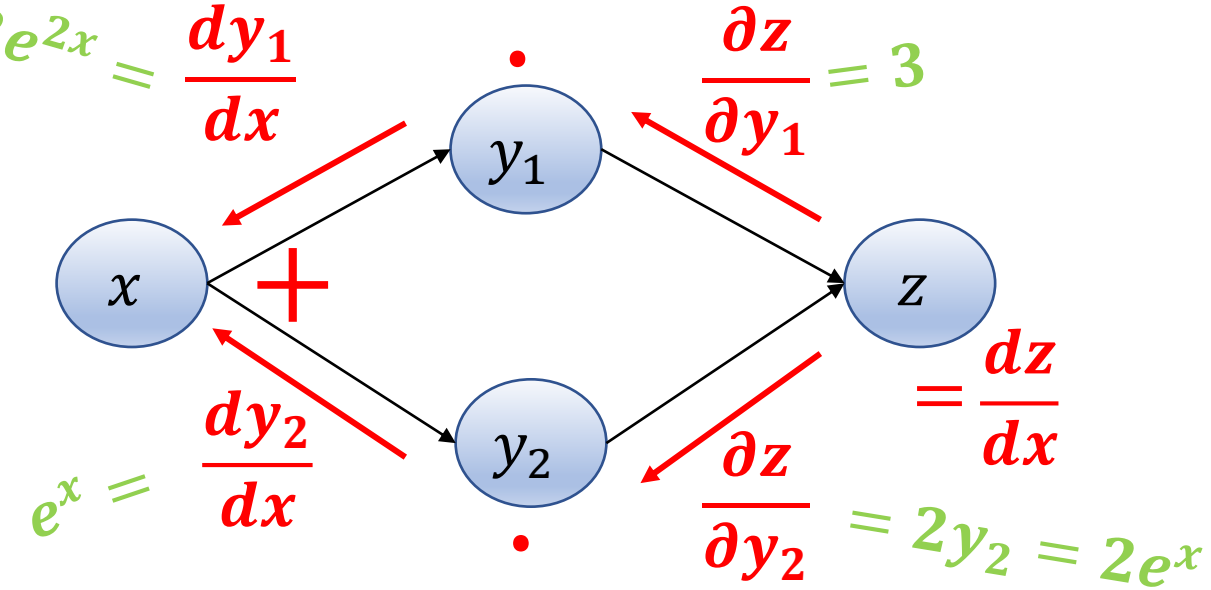

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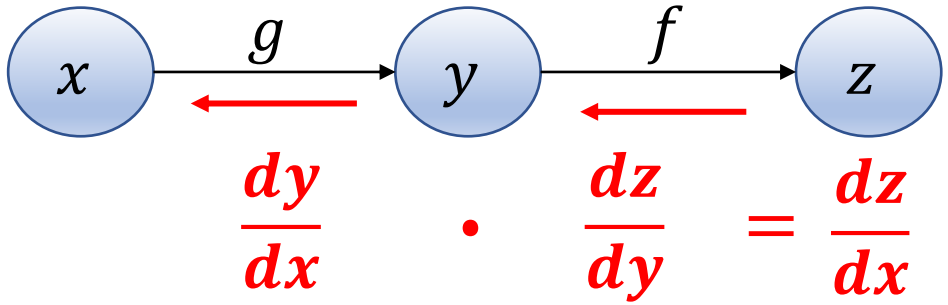

$$\frac{dz(y_1, y_2)}{dx} = \frac{\partial z}{\partial y_1}\,\frac{dy_1}{dx} + \frac{\partial z}{\partial y_2}\,\frac{dy_2}{dx}$$

3 $\quad 4x + 2e^{2x} \quad 2e^{x} \quad e^{x}$

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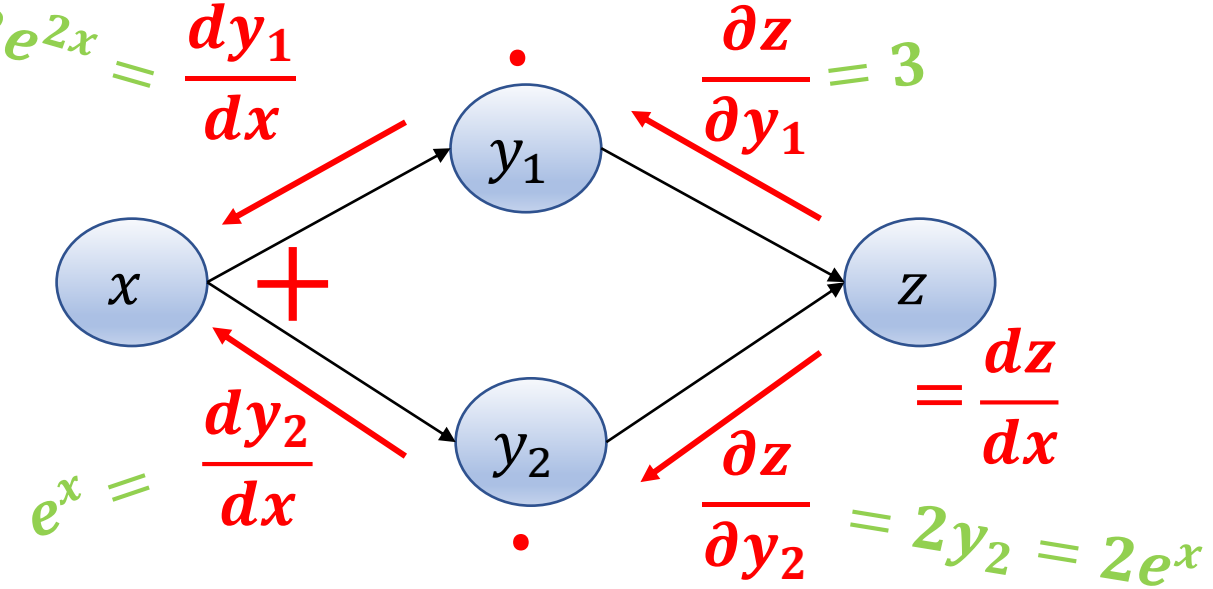

$$\frac{dz(y_1, y_2)}{dx} = 12x + 8e^{2x}$$

Chain rule reminder

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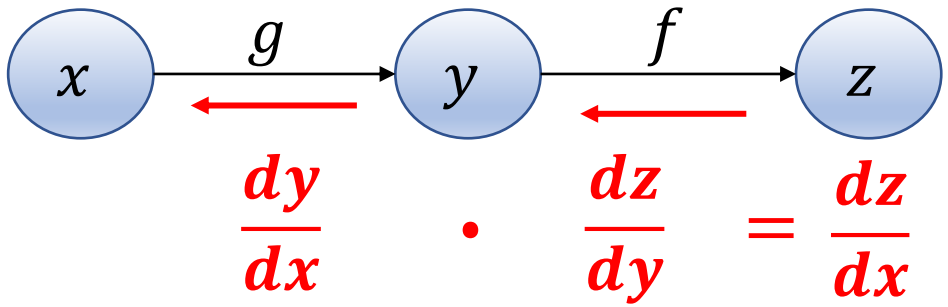

$$\frac{dz(y_1, y_2)}{dx} = \frac{\partial z}{\partial y_1}\,\frac{dy_1}{dx} + \frac{\partial z}{\partial y_2}\,\frac{dy_2}{dx}$$

3 $\quad 4x \times 2e^{2x}$ $\quad 2e^{x}$ $\quad e^{x}$

$y_1 = 2x^2 + e^{2x}$
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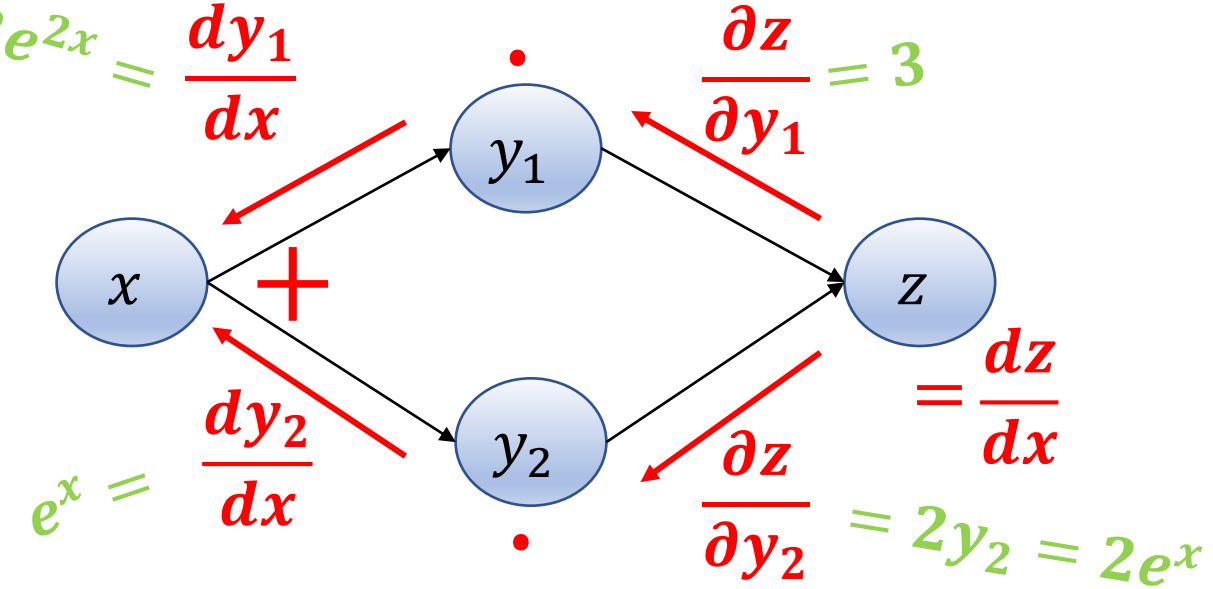
$z = 3y_1 + y_2{}^2$

$= 6x^2 + 4e^{2x}$



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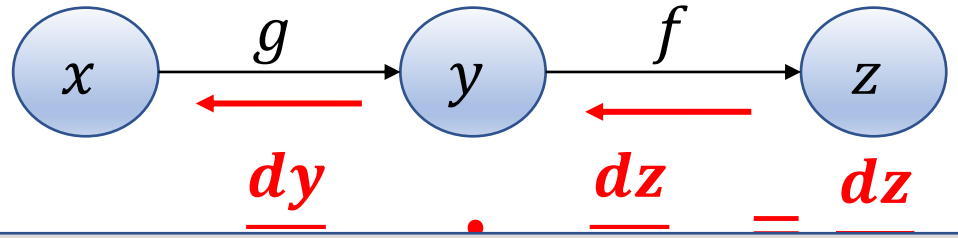
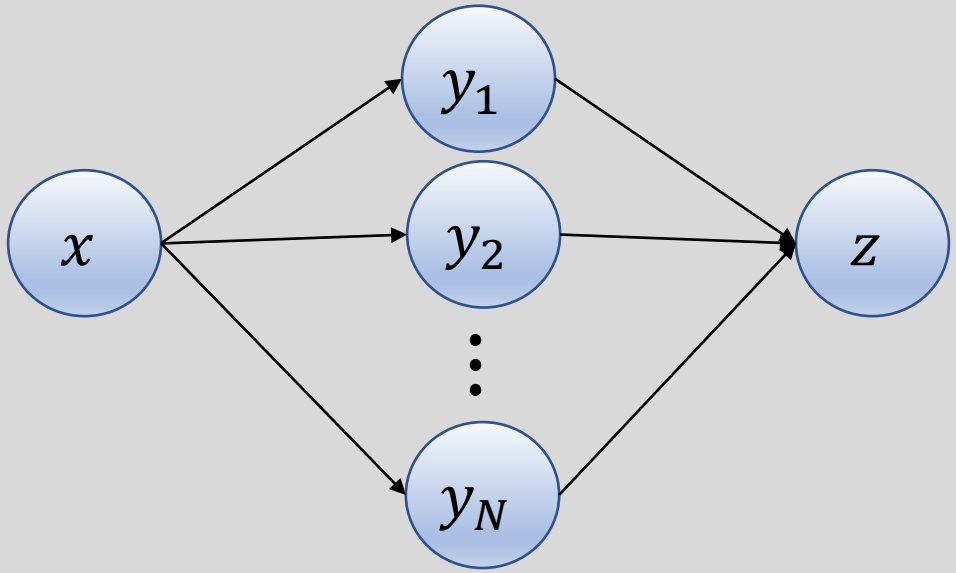


Conclusion:

$$\frac{df(y_1, y_2 \dots y_N)}{dx} = \sum_n \frac{\partial f}{\partial y_n} \frac{dy_n}{dx}$$



$z = 3y_1 + y_2^2$ $= 6x^2 + 4e^{2x}$

$\frac{dz(y_1, y_2)}{dx} = 12x + 8e^{2x}$

Back Propagation - preliminaries

$$x_j^l = \sigma\left(\sum_i w_{ij}^l \cdot x_i^{l-1} + b_j\right)$$

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$$\frac{\partial \mathcal{L}}{\partial w_{ij}^l}$$

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Easy!

$$x_i^{l-1} \cdot \sigma'\left(z_j^l\right)$$

Back Propagation - preliminaries

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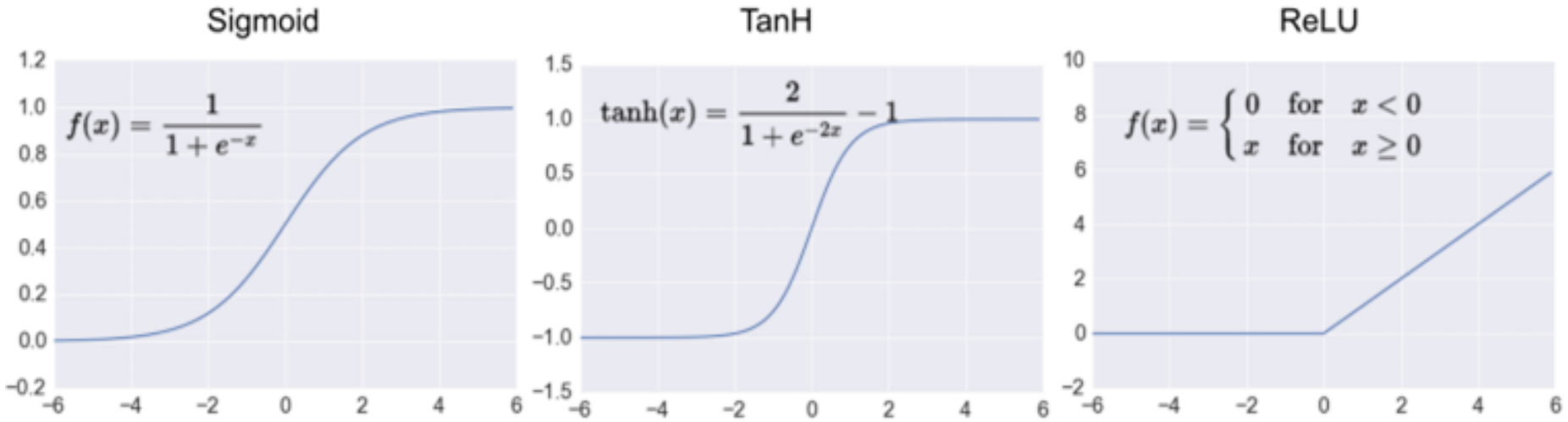
Easy!

$$x_i^{l-1} \cdot \sigma'(z_j^l)$$

Derivatives of common activations are easy!



$$\sigma'(z) = \sigma(z)\left(1 - \sigma(z)\right)$$

$$\tanh'(z) = 1 - \tanh^2(z)$$

1 for z>0, 0 otherwise

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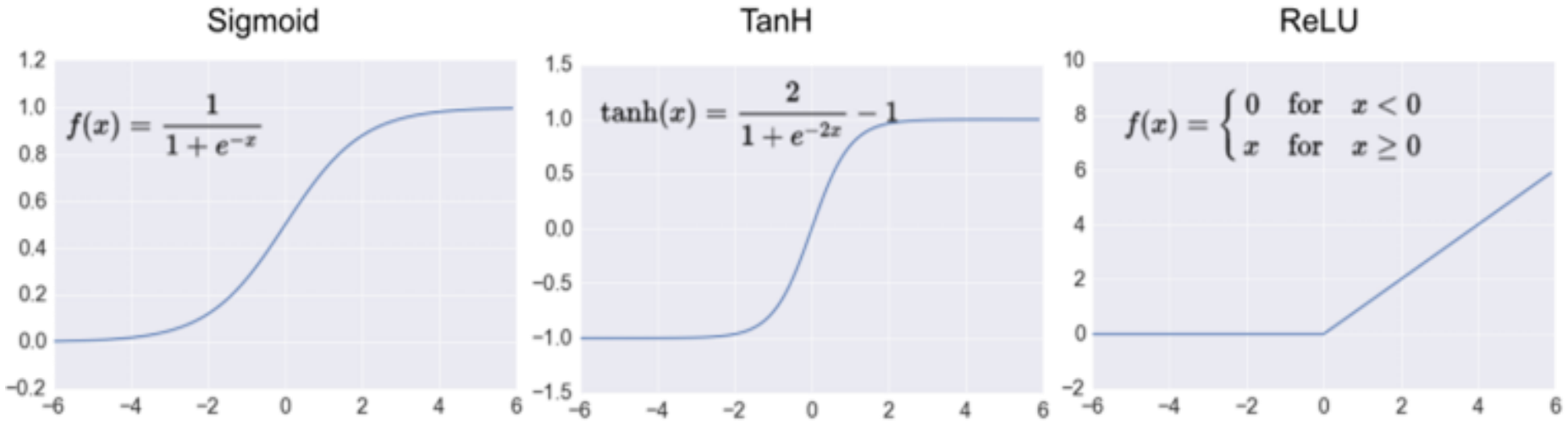

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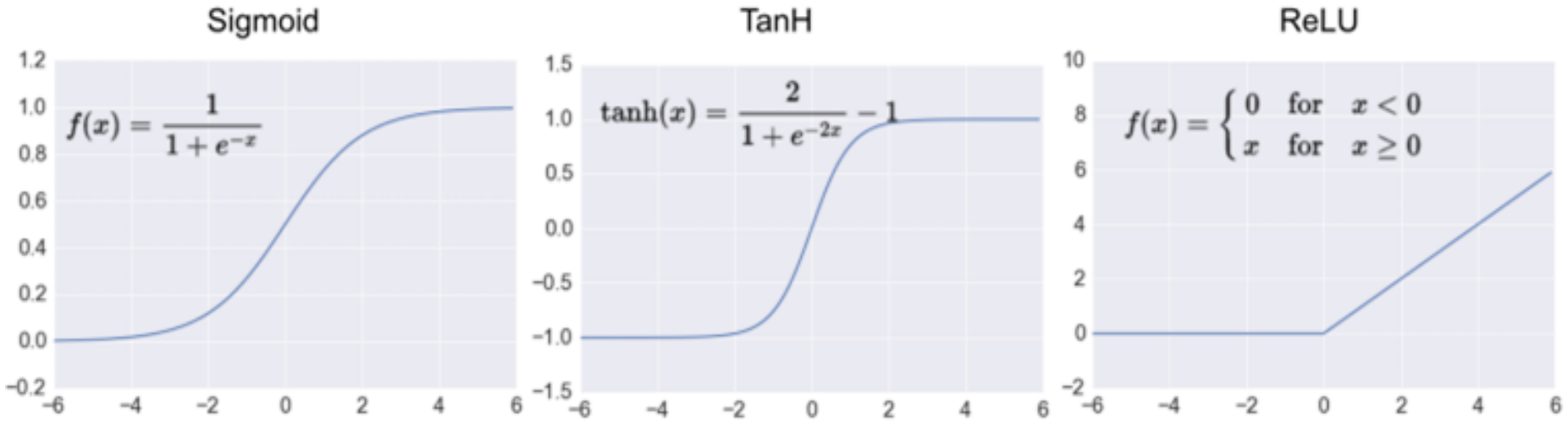
$\triangleq g_j^l$
Obtained by backprop

Easy!
$x_i^{l-1} \cdot \sigma'(z_j^l)$

Derivatives of common activations are easy!



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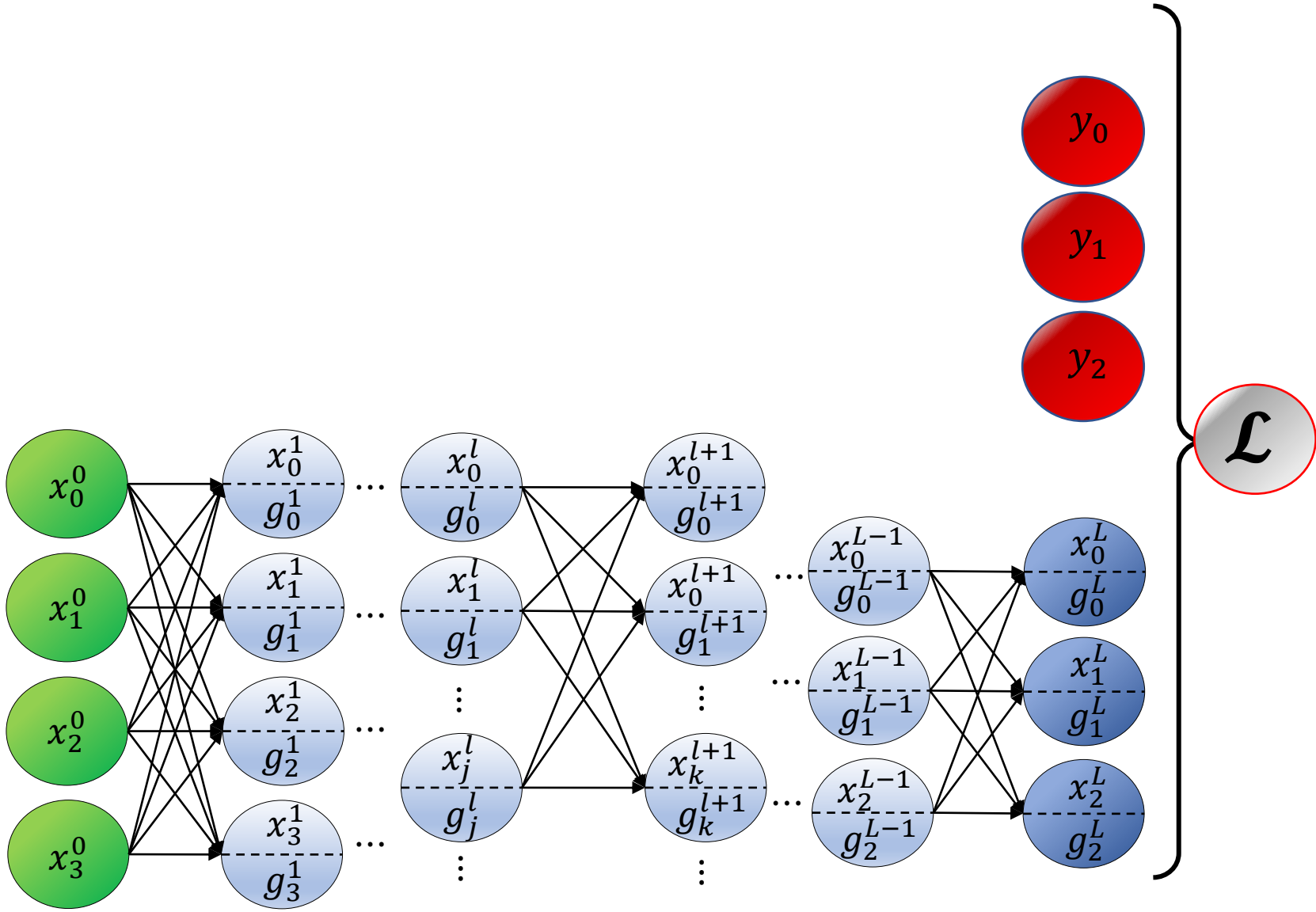

Back Propagation

$$g_j^l \triangleq \frac{\partial \mathcal{L}}{\partial x_j^l}$$

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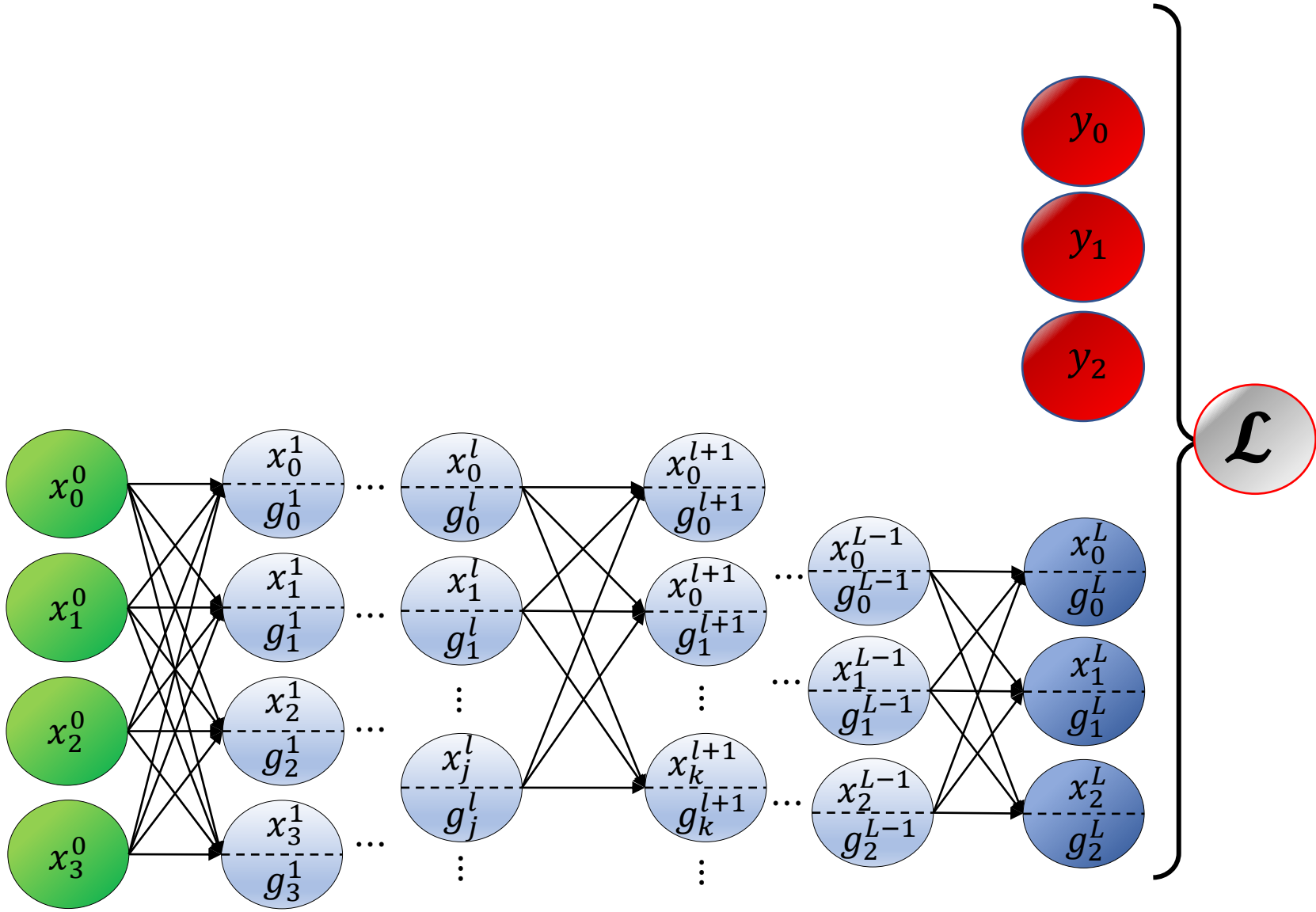

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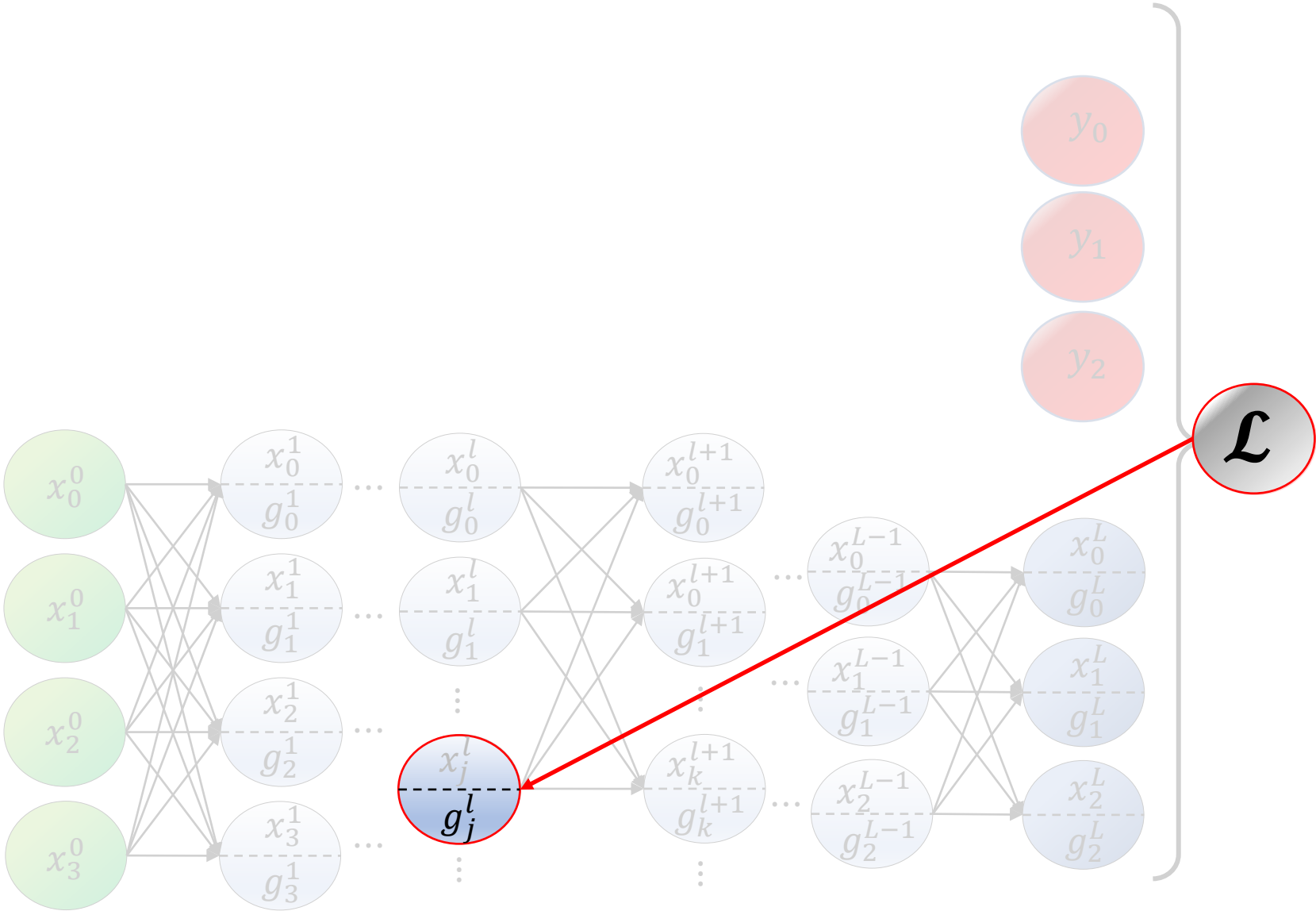

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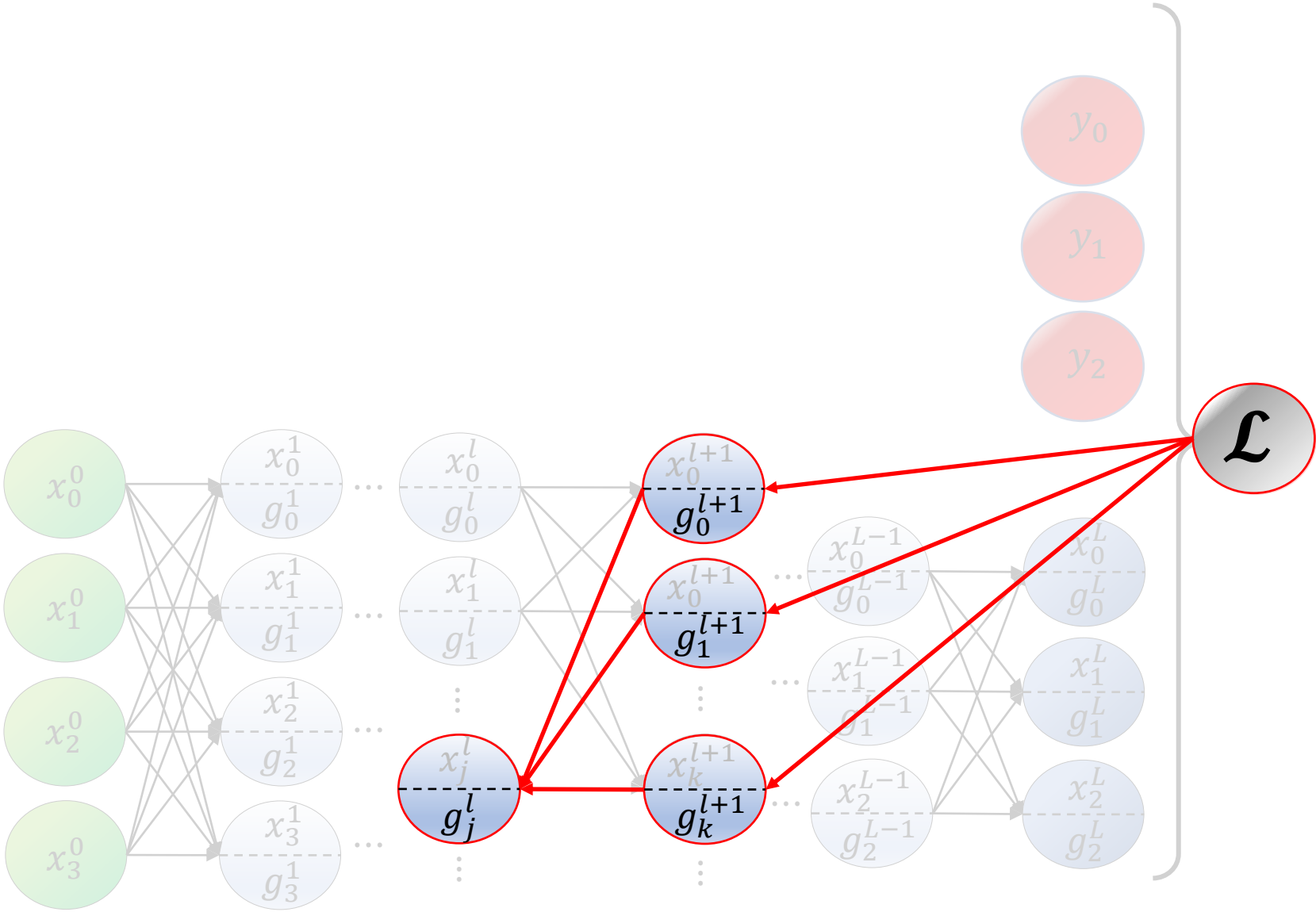

Back Propagation

$$g_j^l \triangleq \frac{\partial \mathcal{L}}{\partial x_j^l}$$

$$= \sum_k \frac{\partial \mathcal{L}}{\partial x_k^{l+1}} \cdot \frac{\partial x_k^{l+1}}{\partial x_j^l}$$

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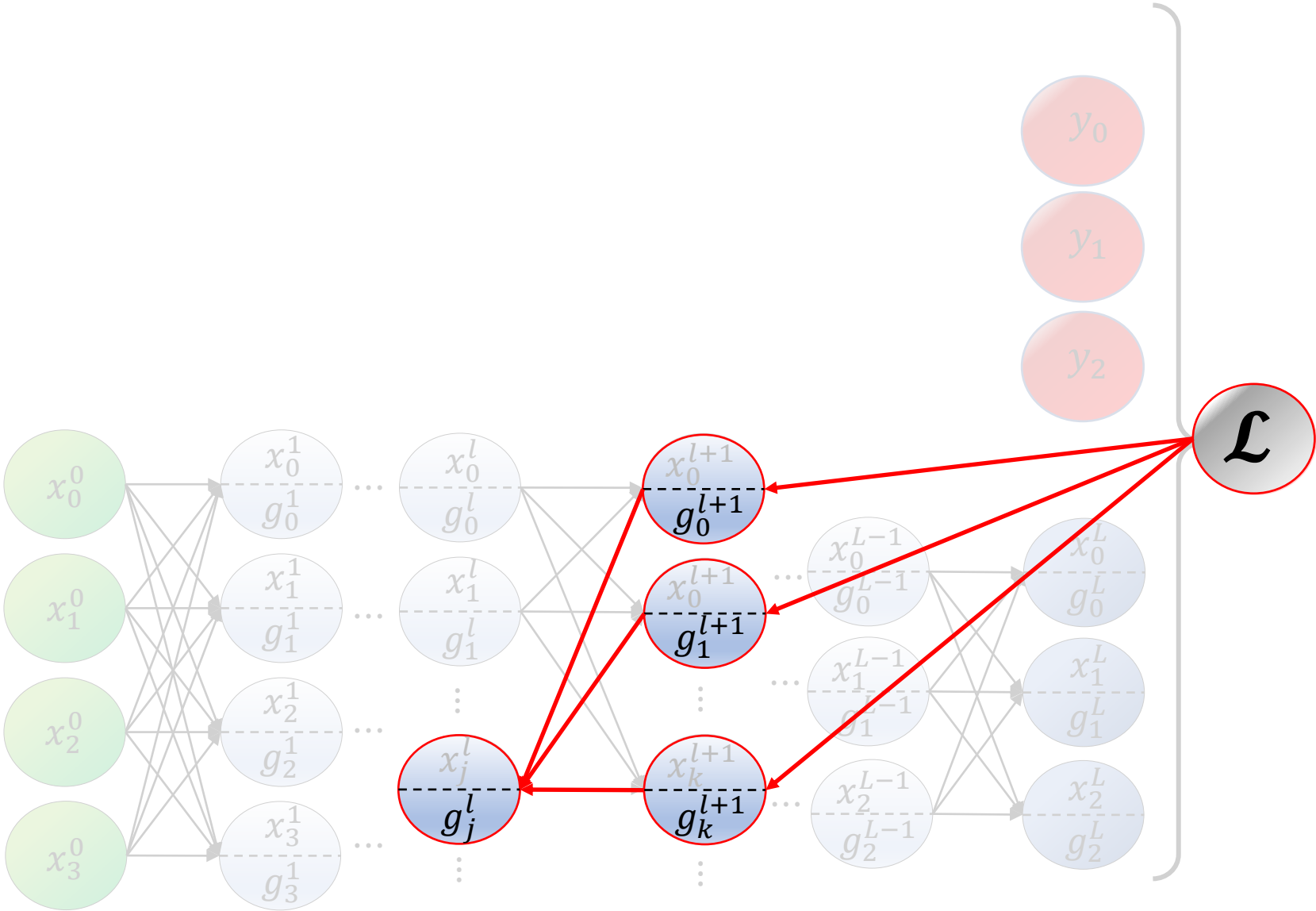

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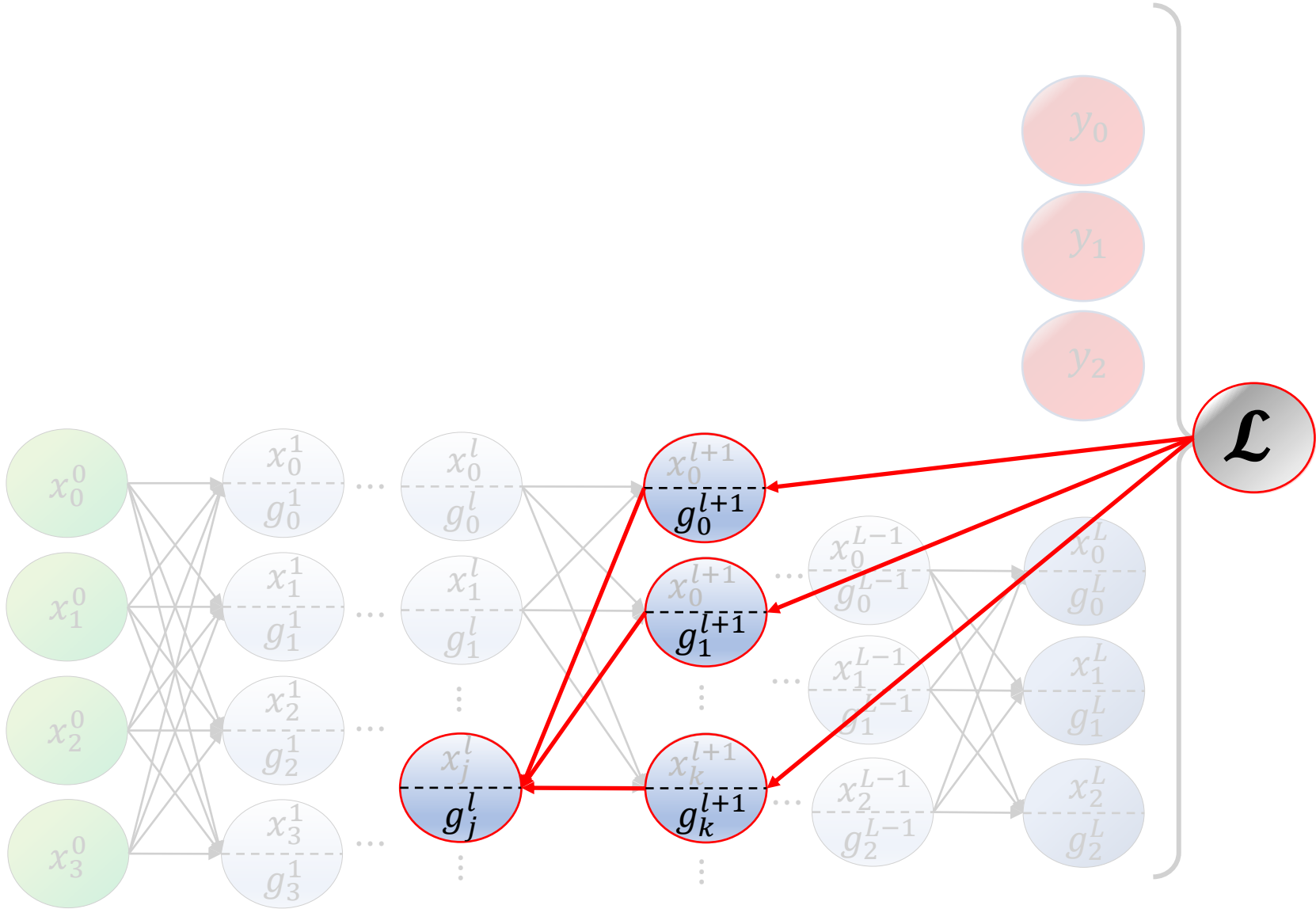

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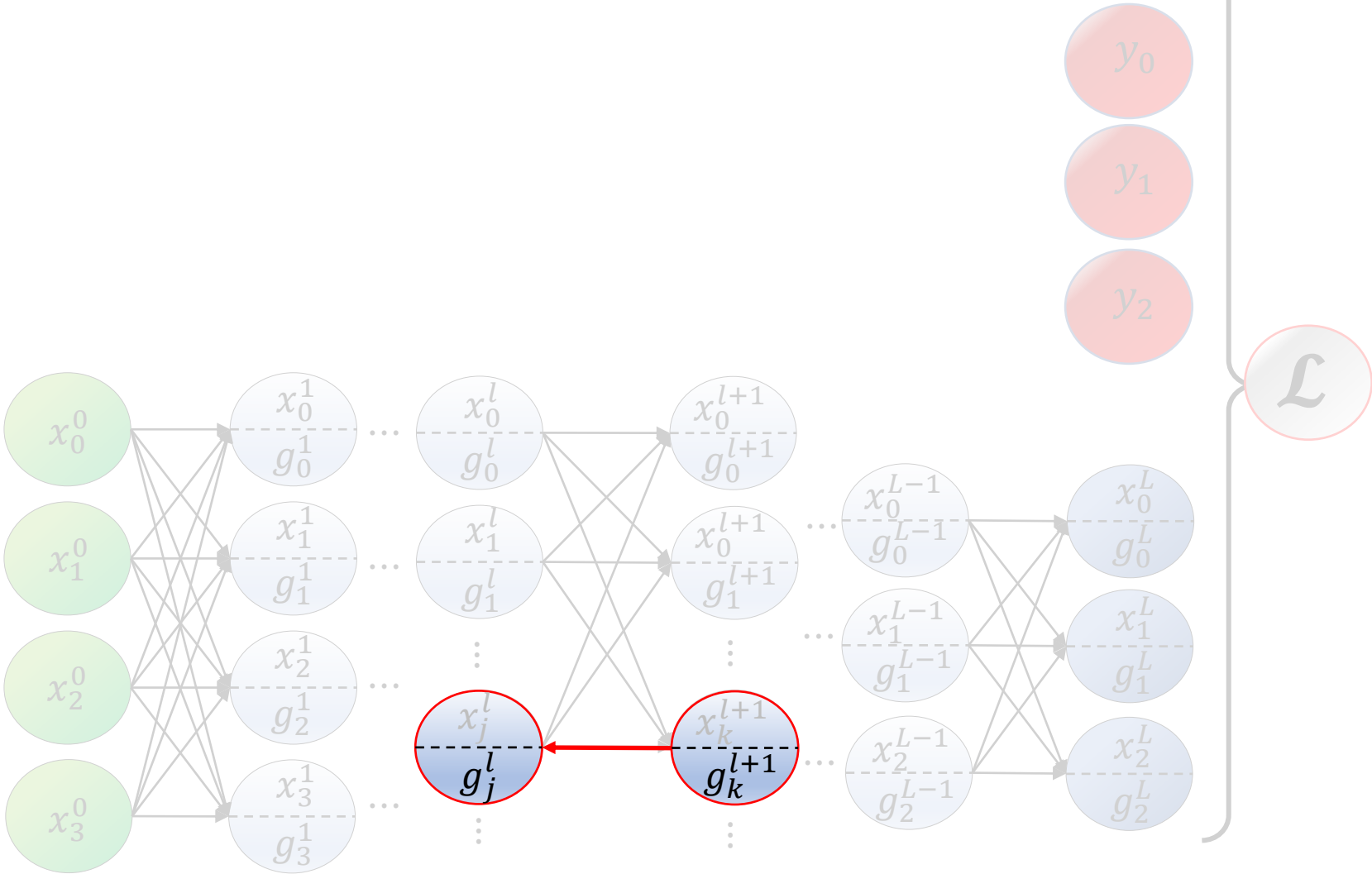

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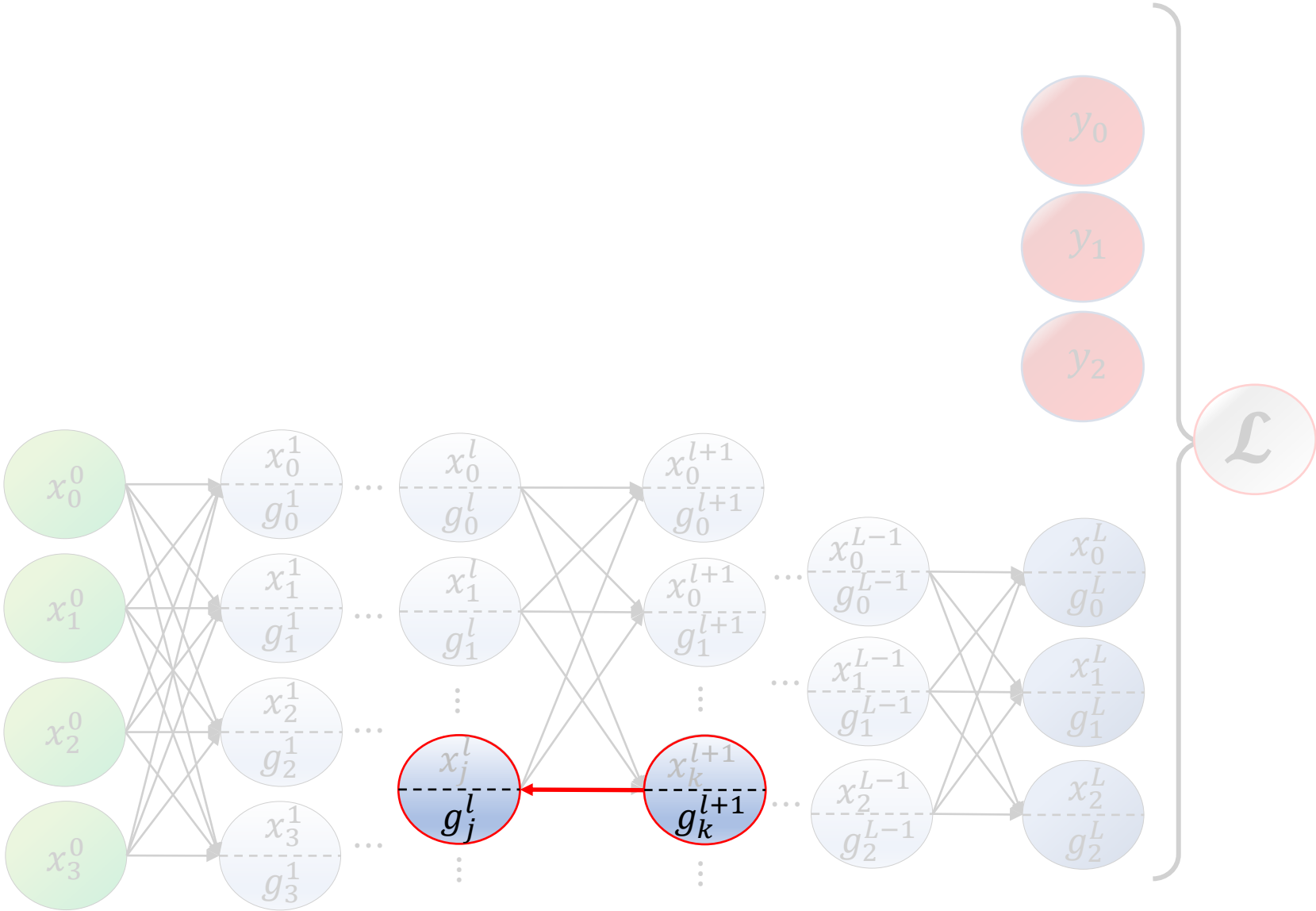
$$w_{jk}^{l+1} \cdot \sigma'\left(z_k^{l+1}\right)$$

$$x_j^l = \sigma\left(\sum_i w_{ij}^l \cdot x_i^{l-1} + b_j\right)$$

$$z_j^l$$



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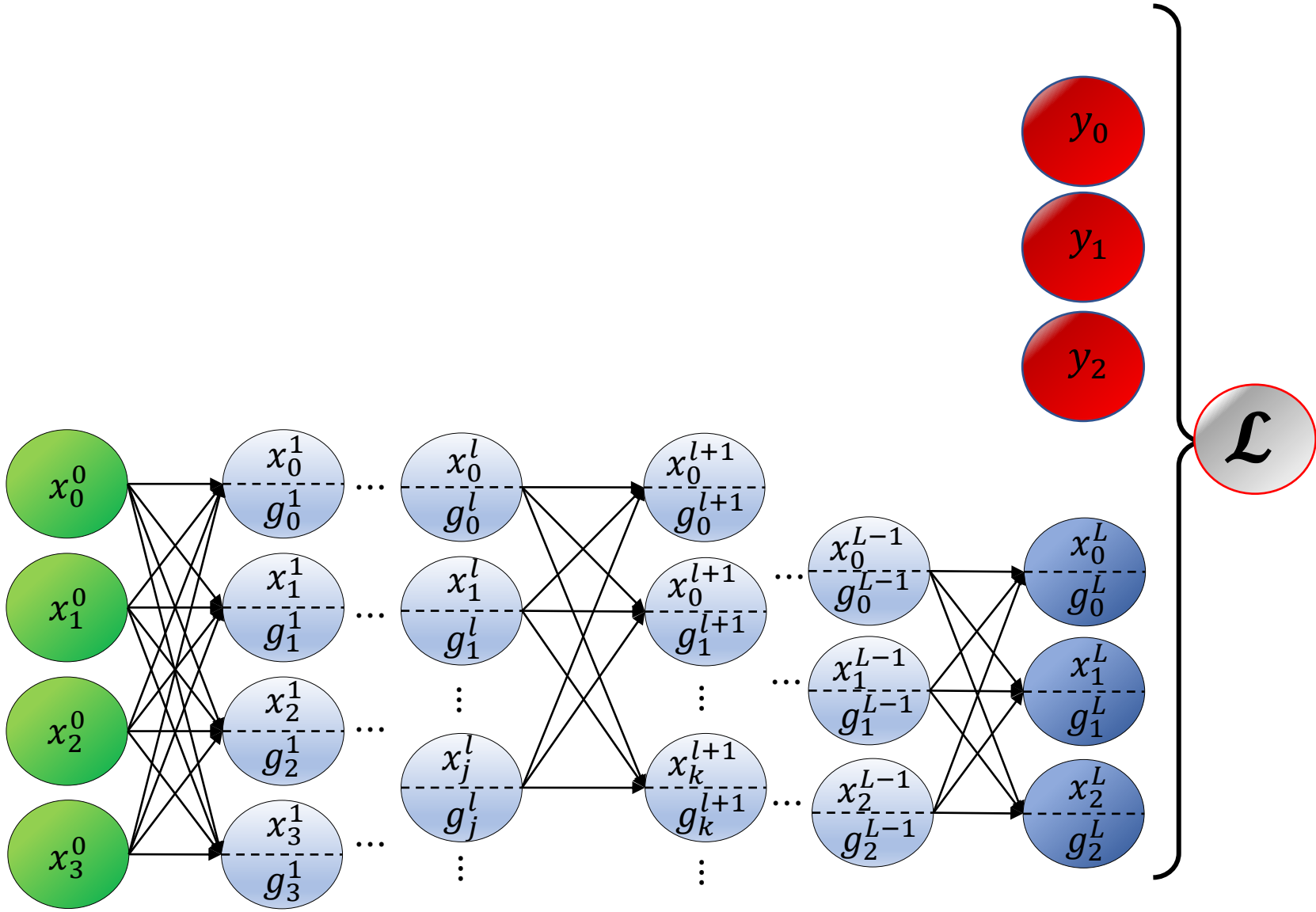

Back Propagation

$$g_j^l \triangleq \frac{\partial \mathcal{L}}{\partial x_j^l}$$

$$= \sum_k \boxed{\frac{\partial \mathcal{L}}{\partial x_k^{l+1}}} \cdot \boxed{\frac{\partial x_k^{l+1}}{\partial x_j^l}}$$

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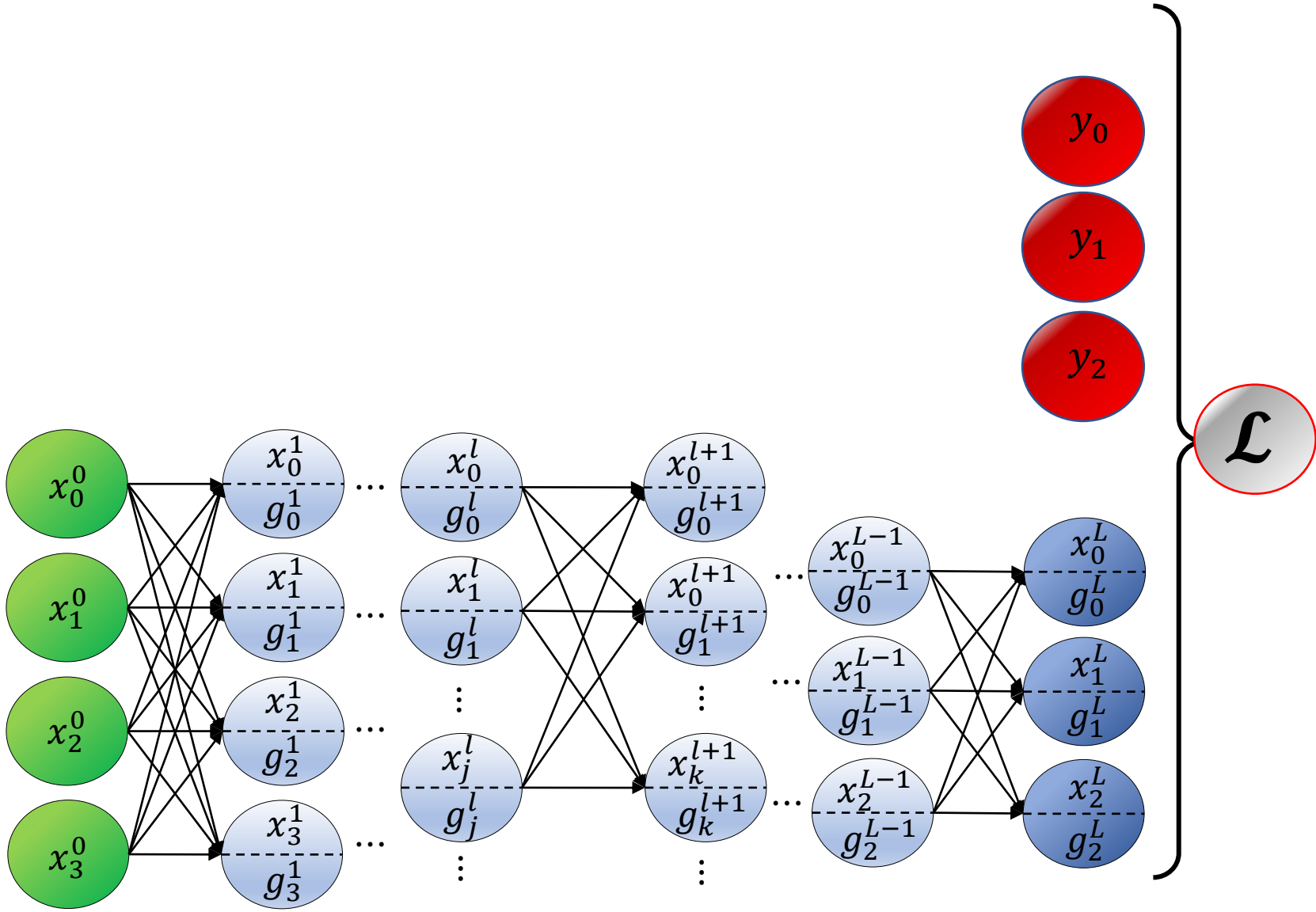

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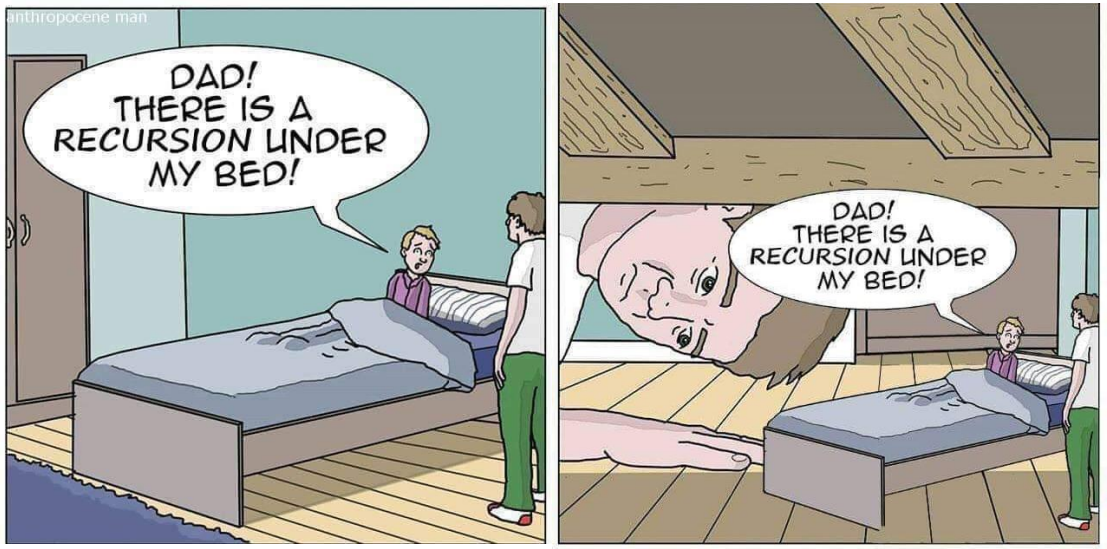

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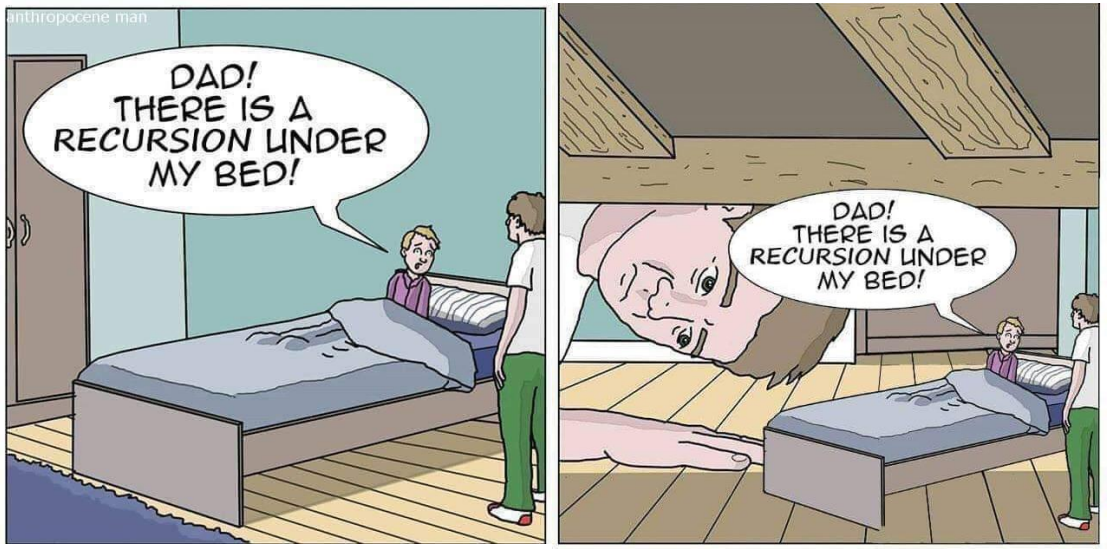

Stopping criterion:

$$g_j^L = \frac{\partial \mathcal{L}}{\partial x_j^L}$$

Back Propagation

$$g_j^l \triangleq \frac{\partial \mathcal{L}}{\partial x_j^l}$$

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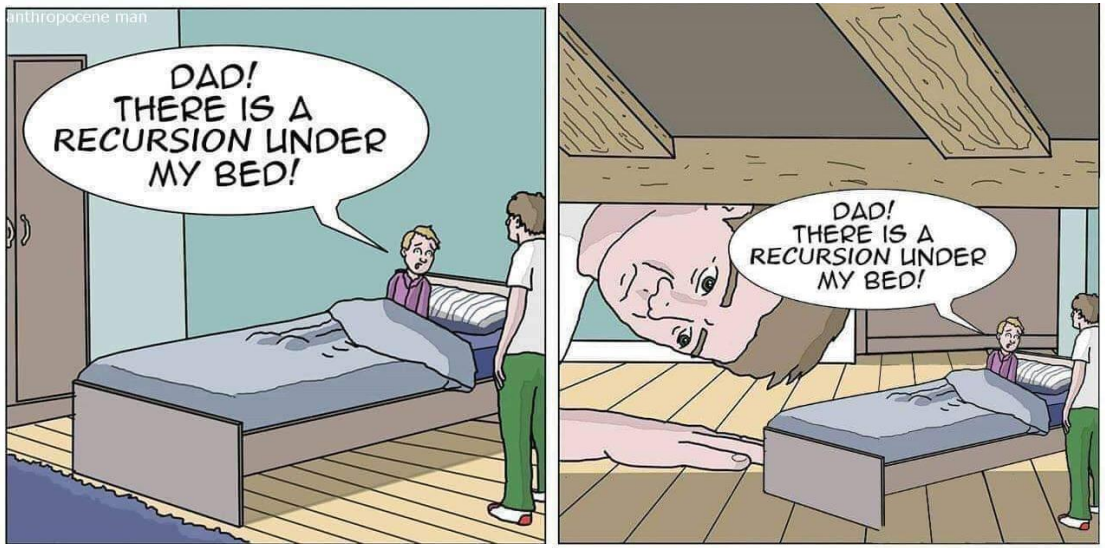

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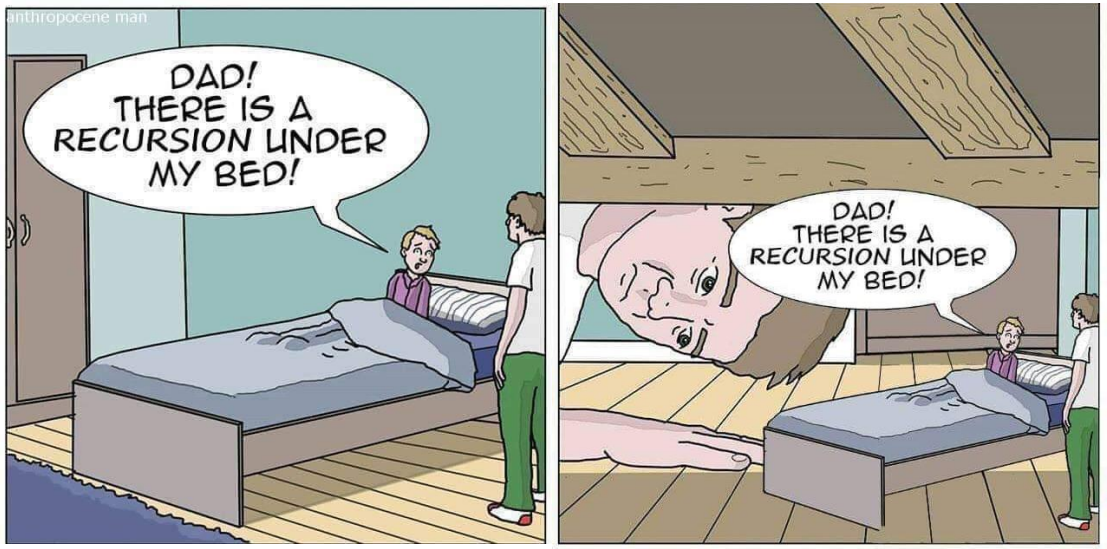

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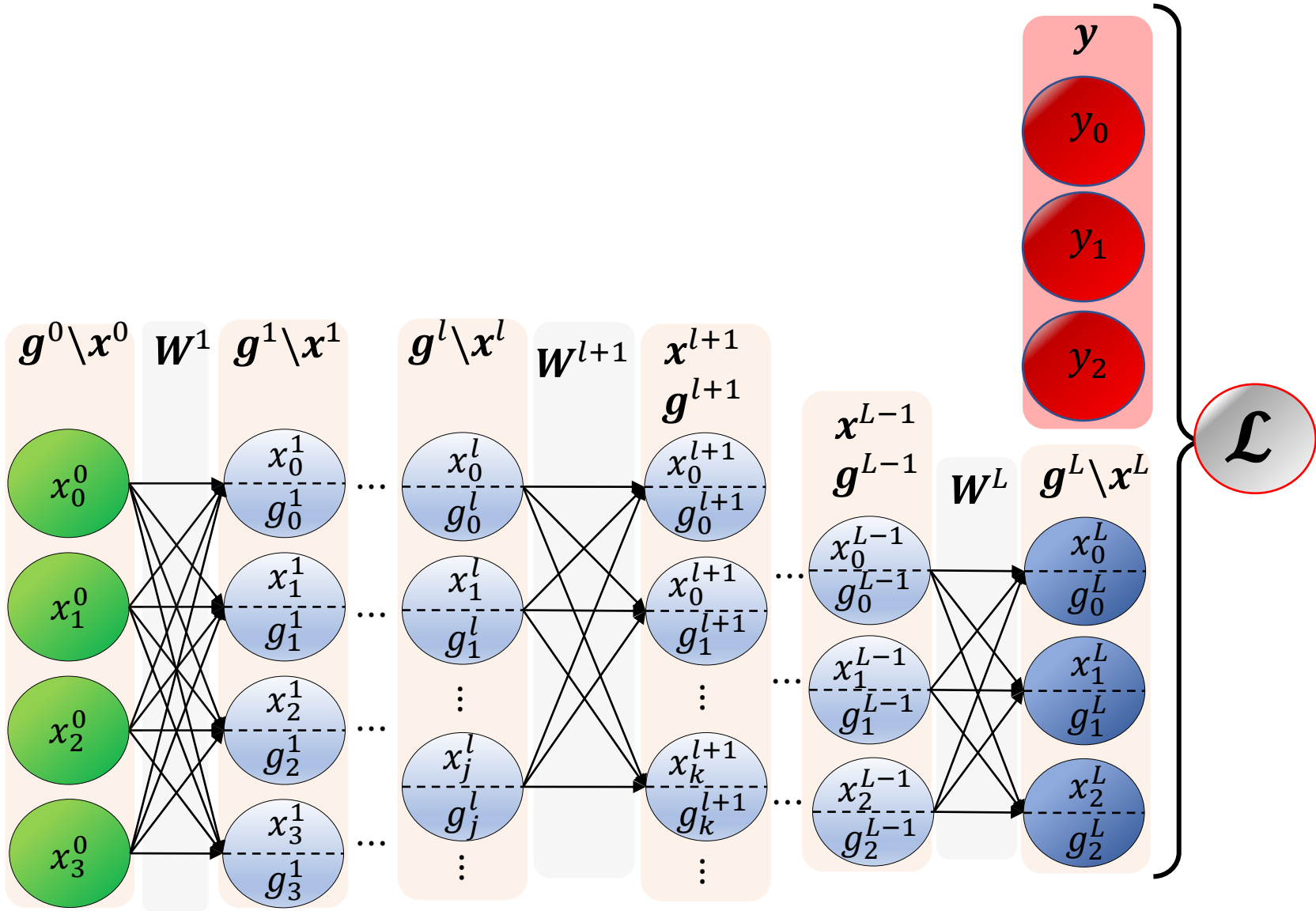

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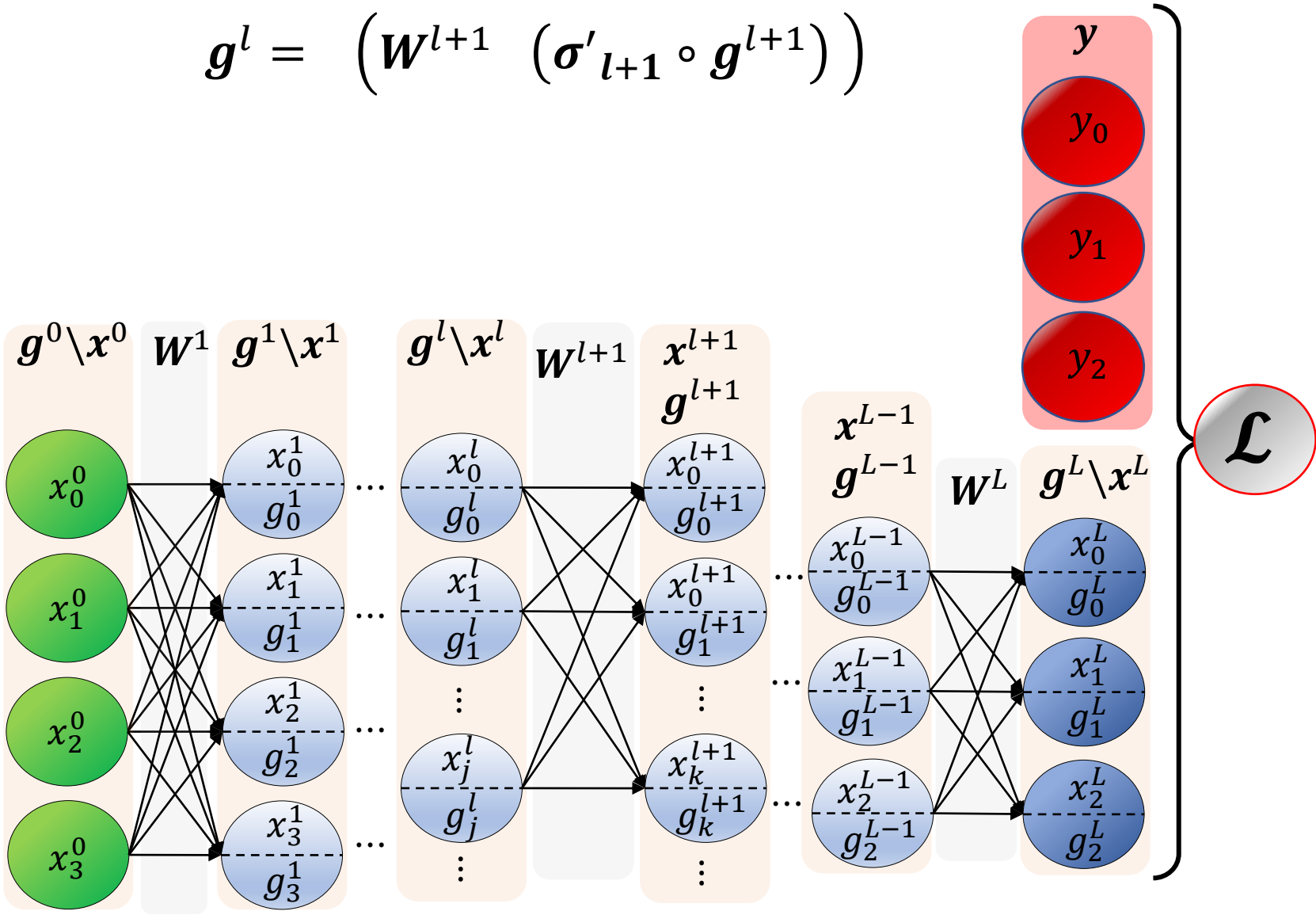

Back Propagation

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Linear activation

Stopping criterion:

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Linear activation

Not Transposed

Stopping criterion:

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Linear activation

Not Transposed

Multiply by activation derivative

Stopping criterion:

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Back Propagation

$$g_j^l \triangleq \frac{\partial \mathcal{L}}{\partial x_j^l}$$

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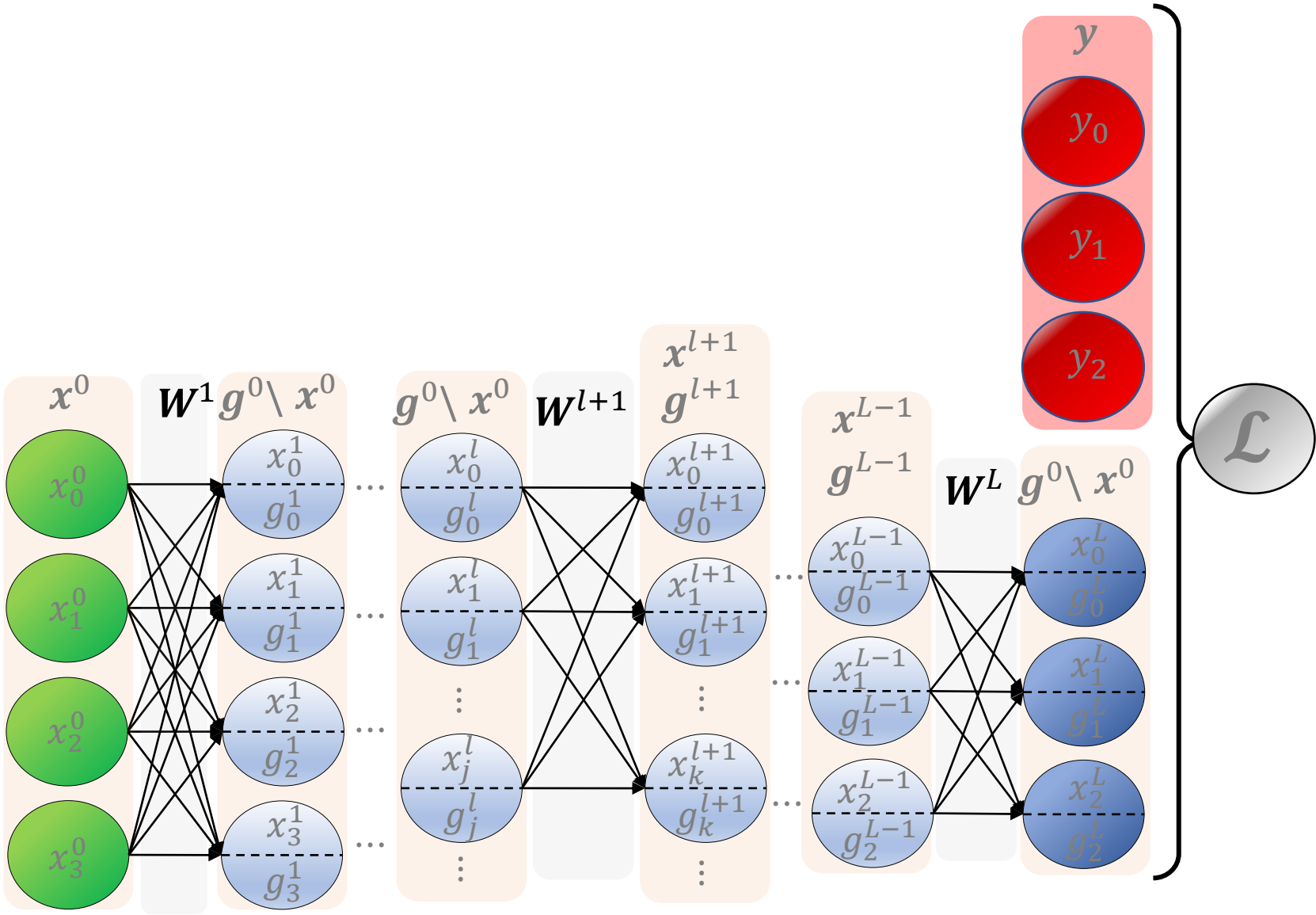
Linear activation

Not Transposed

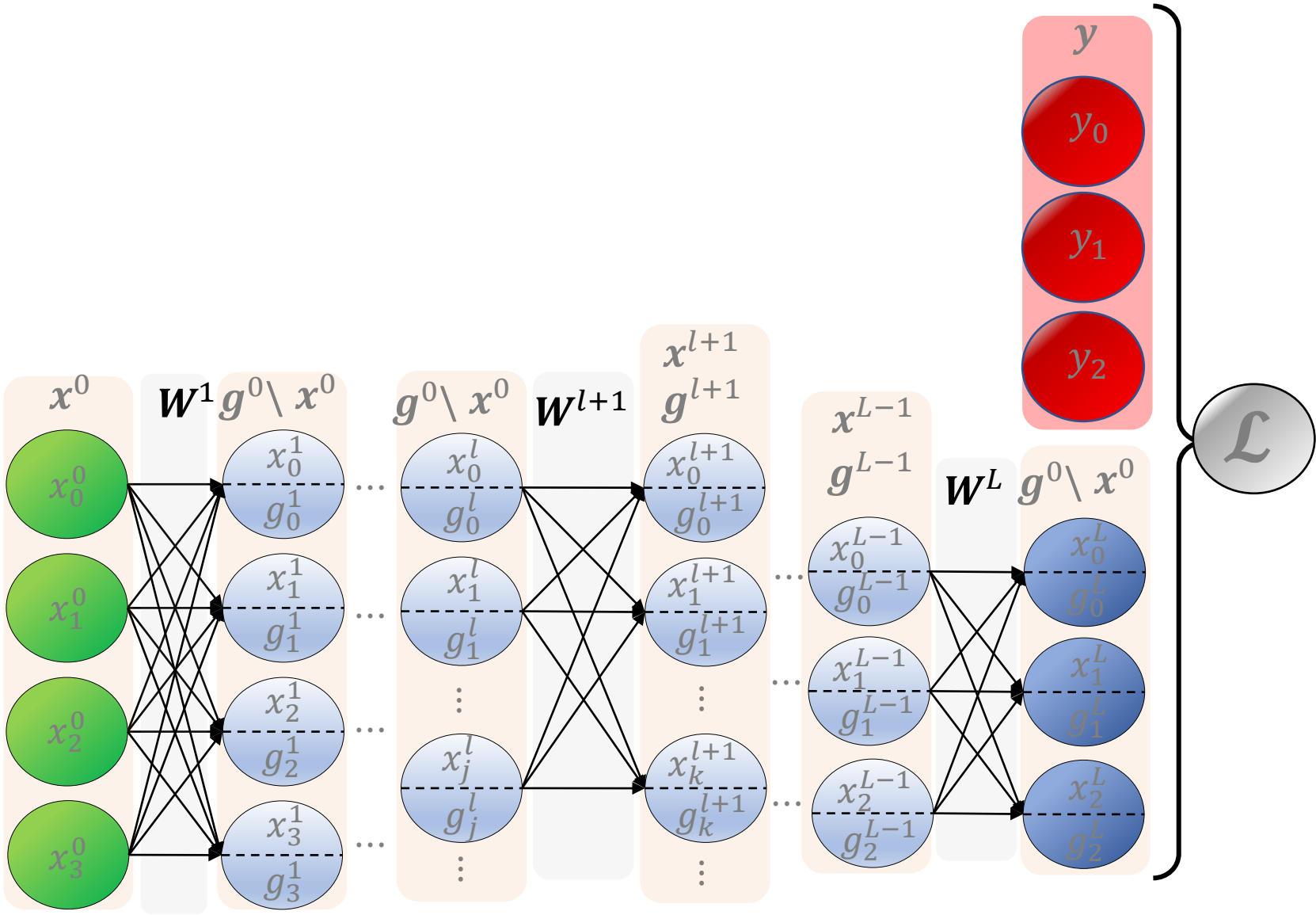
Multiply by activation derivative

No bias

Back Propagation

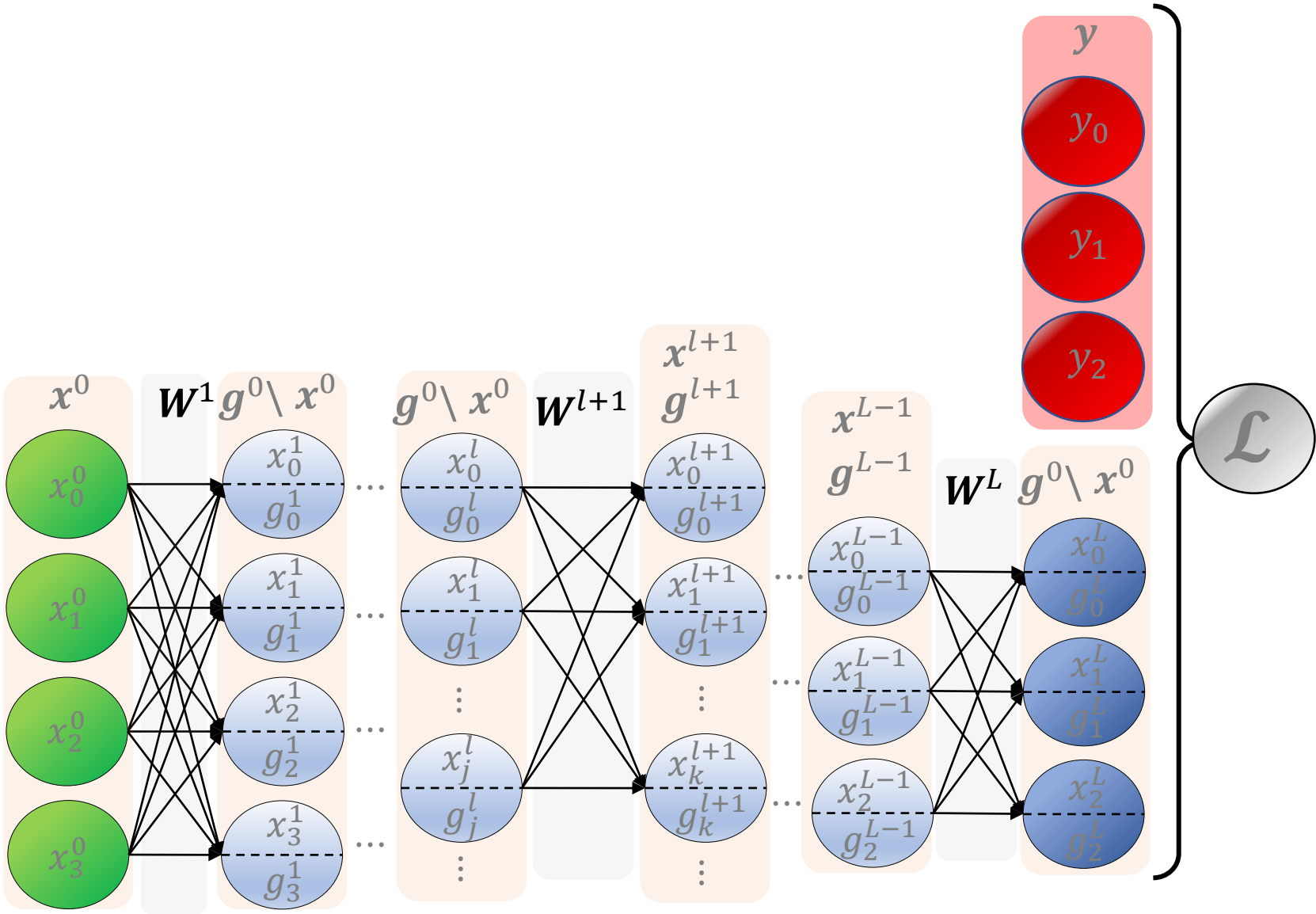


Back Propagation



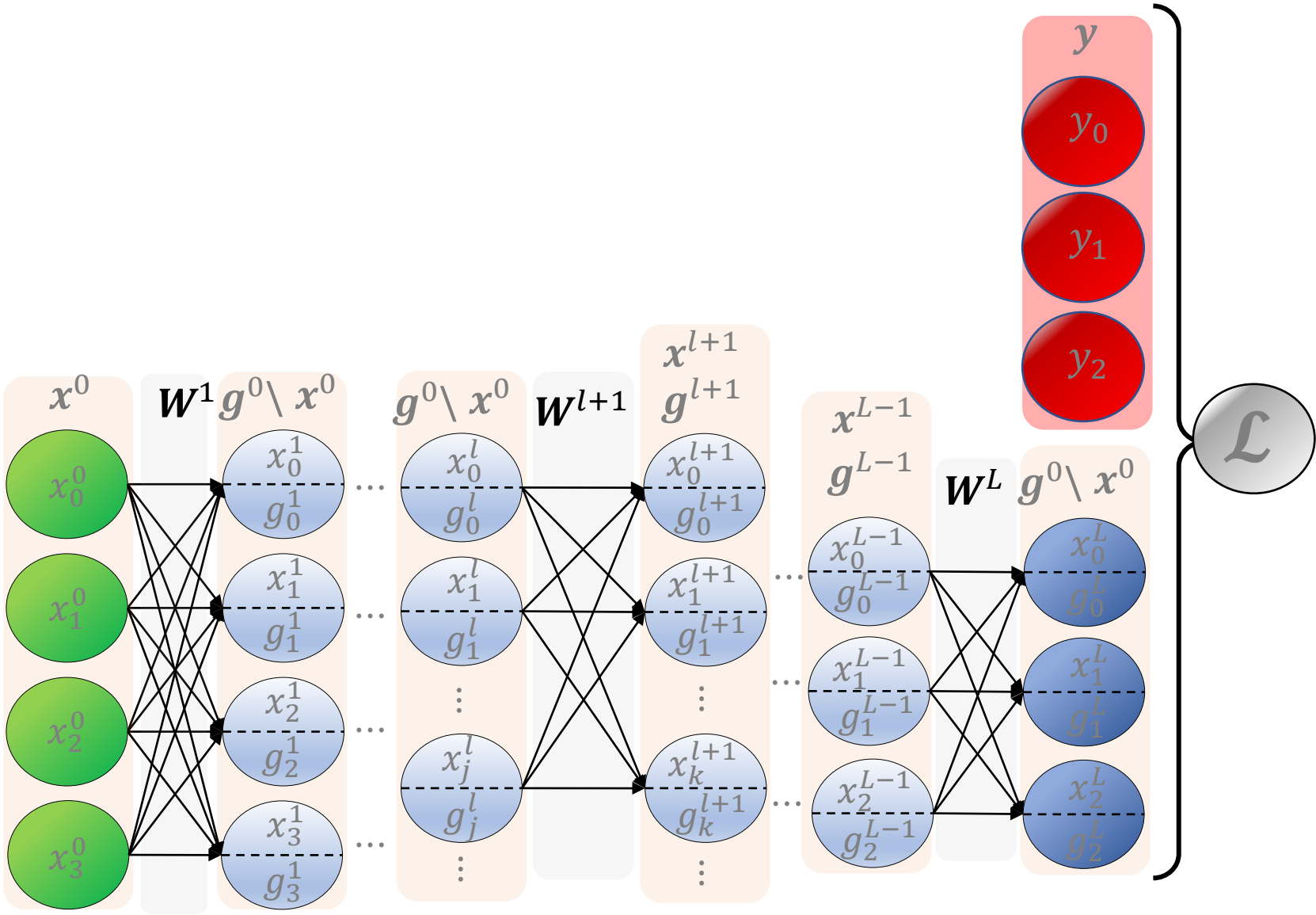
- Initialize weights

Back Propagation



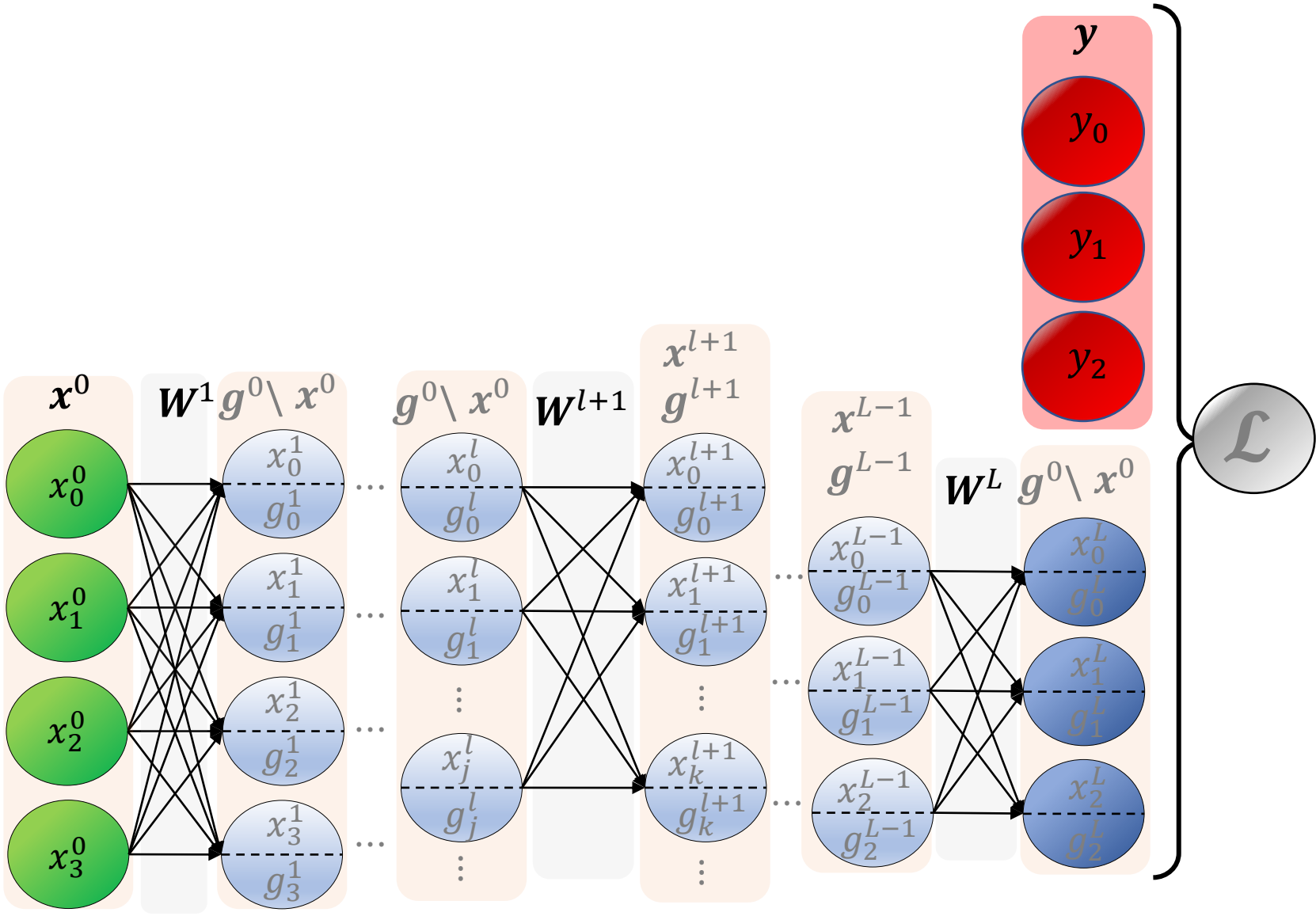
- Initialize weights
- Repeat until convergence:

Back Propagation



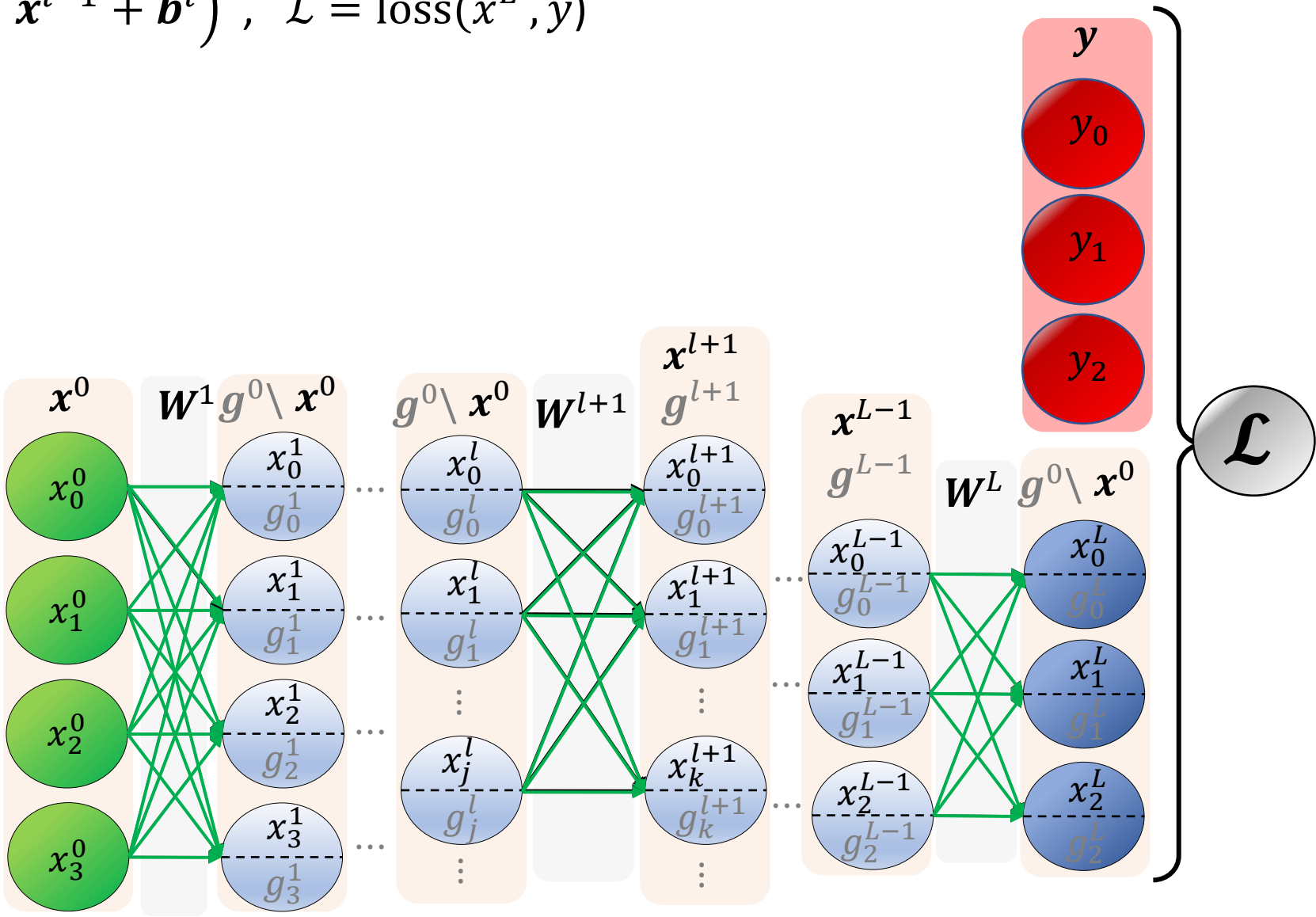
- Initialize weights
- Repeat until convergence:
  1. Sample a batch from the data: $\{(\boldsymbol{x_i}, \boldsymbol{y_i}) \dots\}$

Back Propagation



- Initialize weights
- Repeat until convergence:
  1. Sample a batch from the data: $\{(\boldsymbol{x_i}, \boldsymbol{y_i}) \dots\}$
  2. Forward pass: $\boldsymbol{x}^l = \boldsymbol{\sigma}\left(\boldsymbol{W}^{l^T}\boldsymbol{x}^{l-1} + \boldsymbol{b}^l\right)$, $\mathcal{L} = \text{loss}(x^L, y)$

Back Propagation



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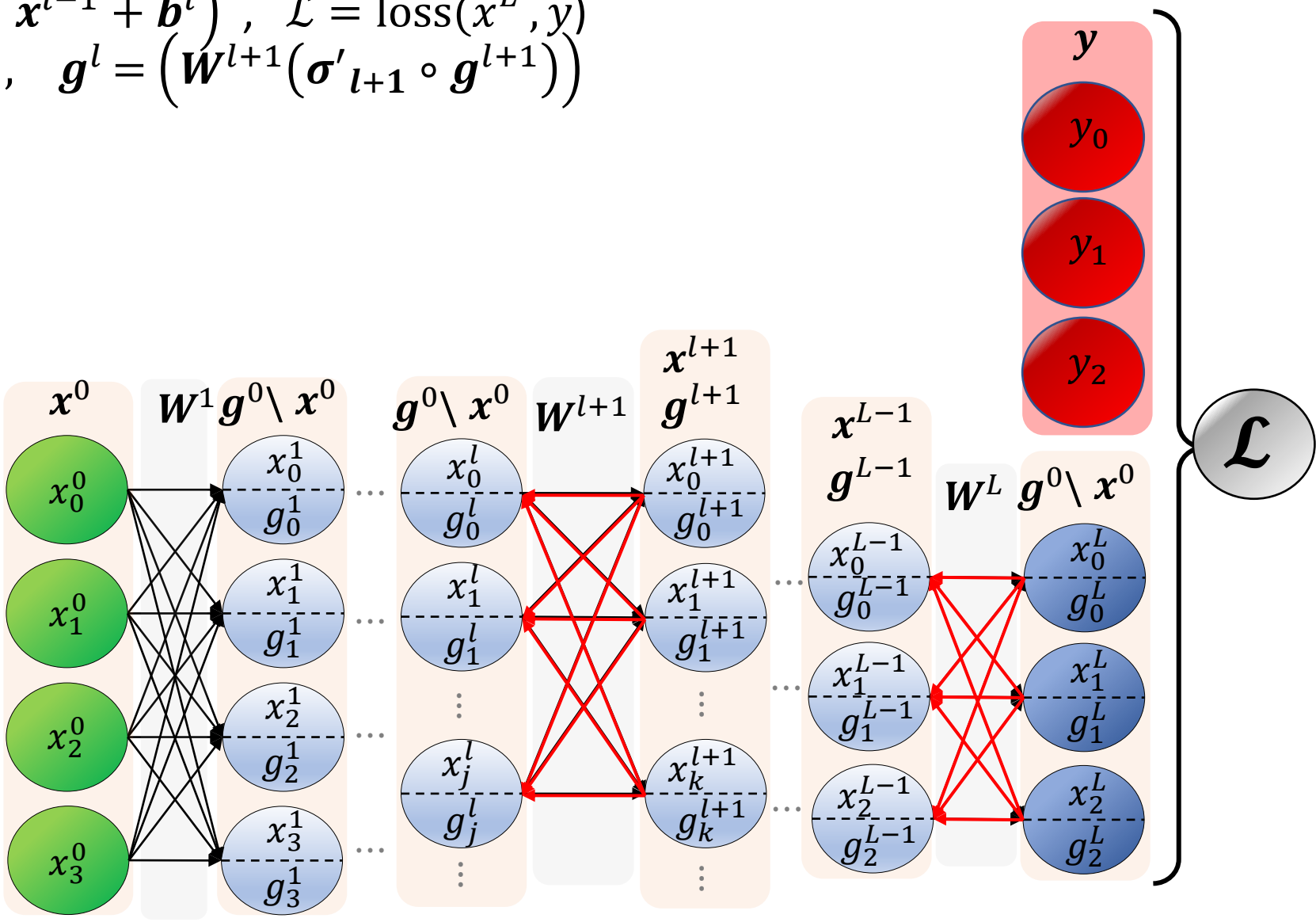
Back Propagation



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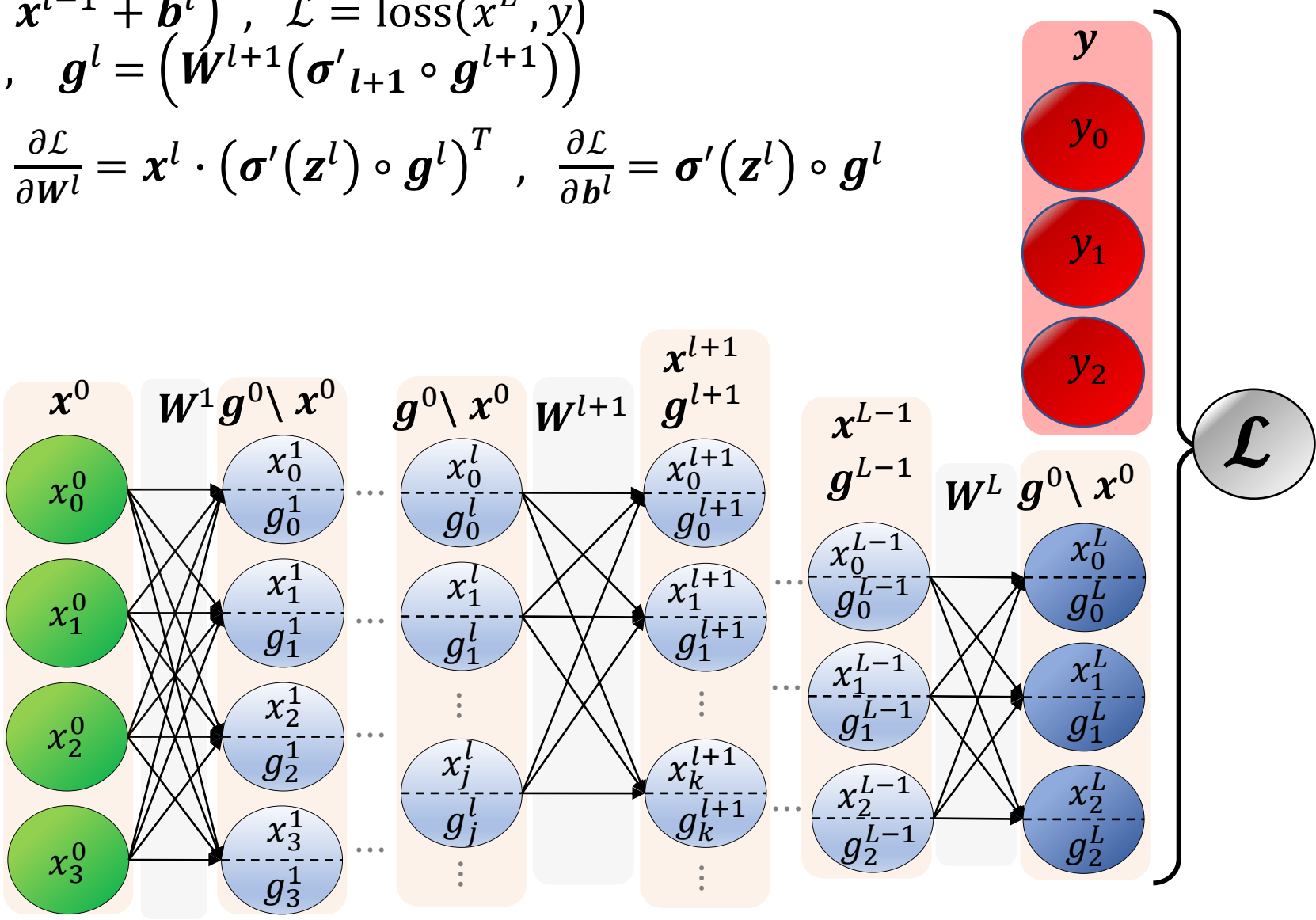
  4. Calculate weights gradient: $\frac{\partial \mathcal{L}}{\partial \boldsymbol{W}^l} = \boldsymbol{x}^l \cdot \left(\boldsymbol{\sigma'}\left(\boldsymbol{z}^l\right) \circ \boldsymbol{g}^l\right)^T$, $\frac{\partial \mathcal{L}}{\partial \boldsymbol{b}^l} = \boldsymbol{\sigma'}\left(\boldsymbol{z}^l\right) \circ \boldsymbol{g}^l$

Back Propagation



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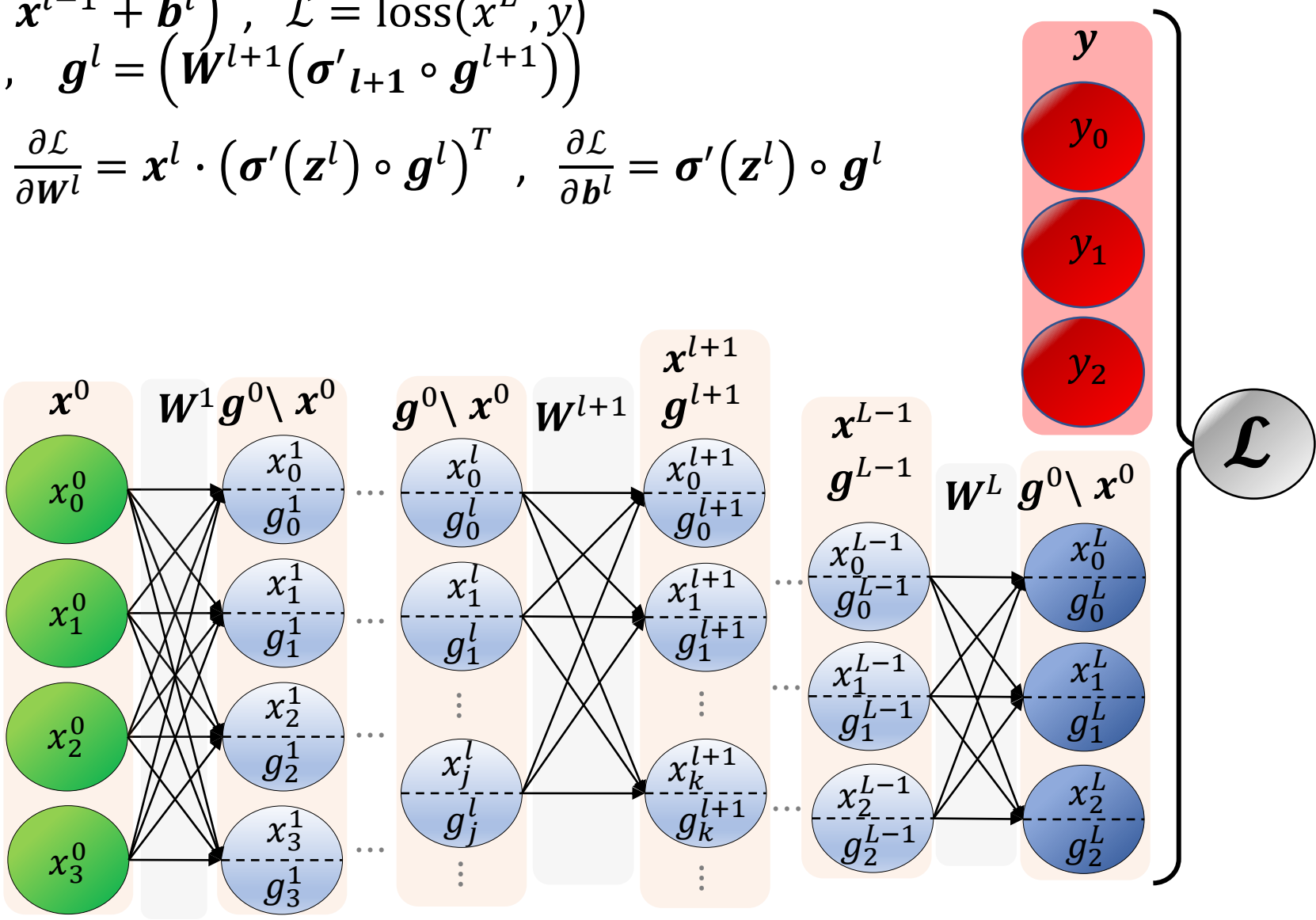
Back Propagation



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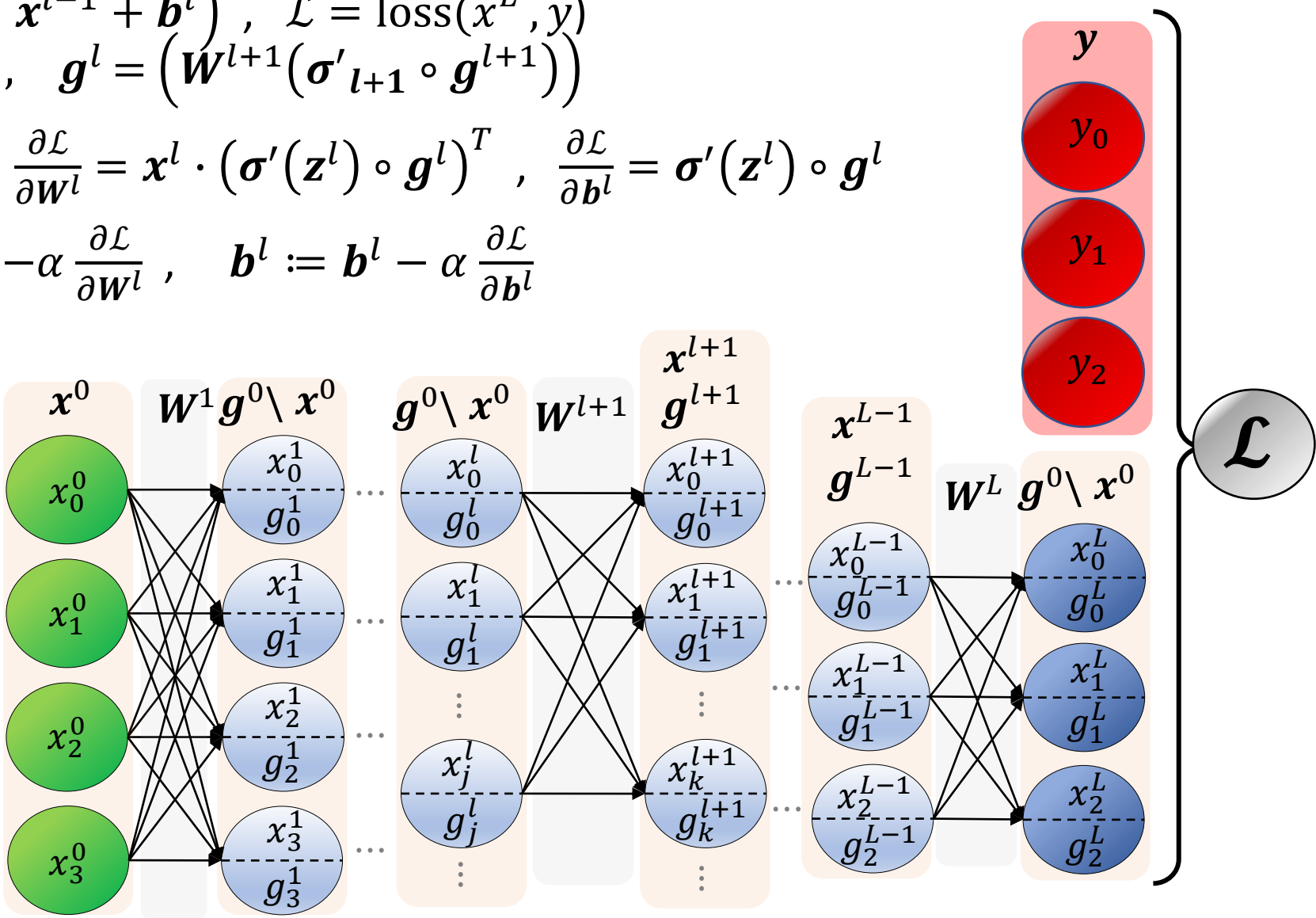
  5. Update weights: $\boldsymbol{W}^l := \boldsymbol{W}^l - \alpha \frac{\partial \mathcal{L}}{\partial \boldsymbol{W}^l}$, $\boldsymbol{b}^l := \boldsymbol{b}^l - \alpha \frac{\partial \mathcal{L}}{\partial \boldsymbol{b}^l}$

Back Propagation



Let's get more generic

Yann LeCun

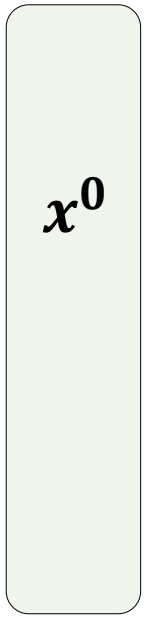
January 5, 2018 ·

OK, Deep Learning has outlived its usefulness as a buzz-phrase.
Deep Learning est mort. Vive Differentiable Programming!

Yeah, Differentiable Programming is little more than a rebranding of the modern collection Deep Learning techniques, the same way Deep Learning was a rebranding of the modern incarnations of neural nets with more than two layers.

But the important point is that people are now building a new kind of software by assembling networks of parameterized functional blocks and by training them from examples using some form of gradient-based optimization.

Let's get more generic



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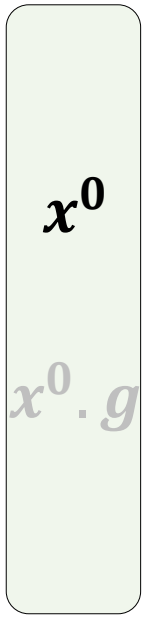
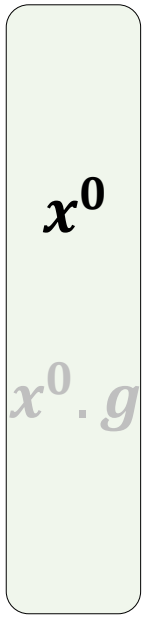


Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}(\boldsymbol{x}^{l})$$



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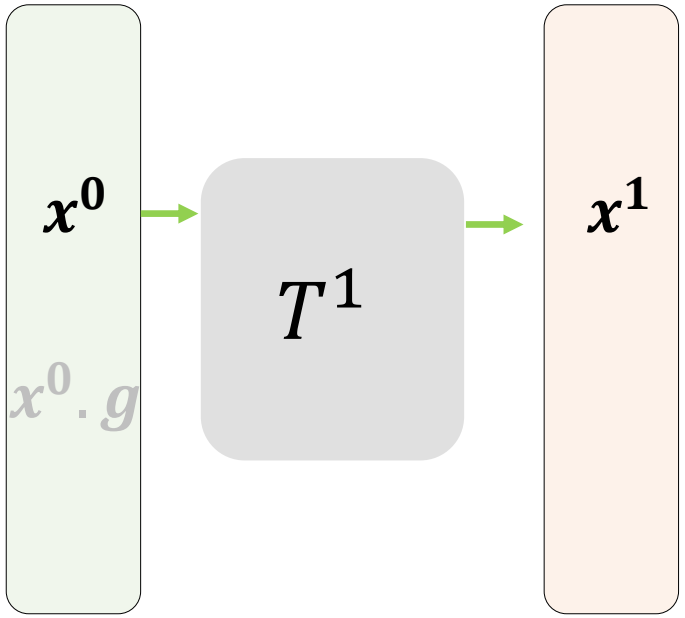
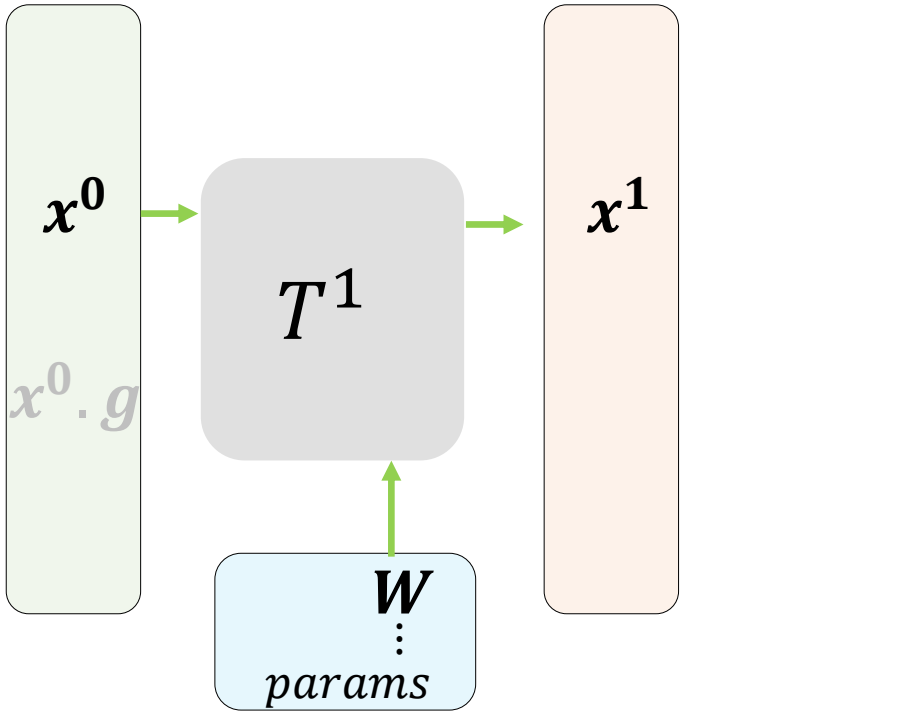


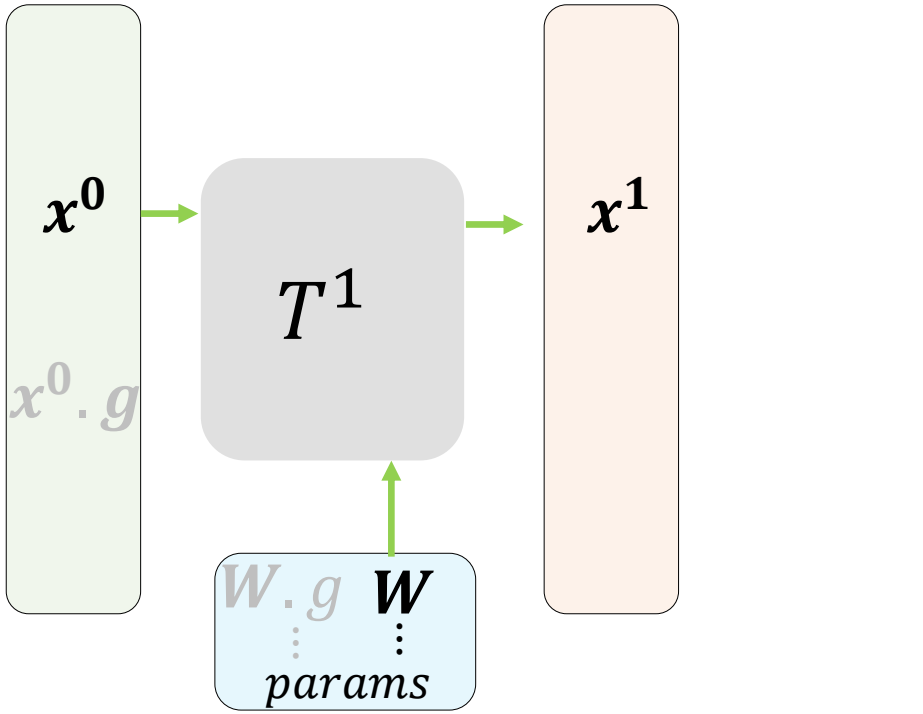
Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}(\boldsymbol{x}^{l}; T^{l+1}.params)$$



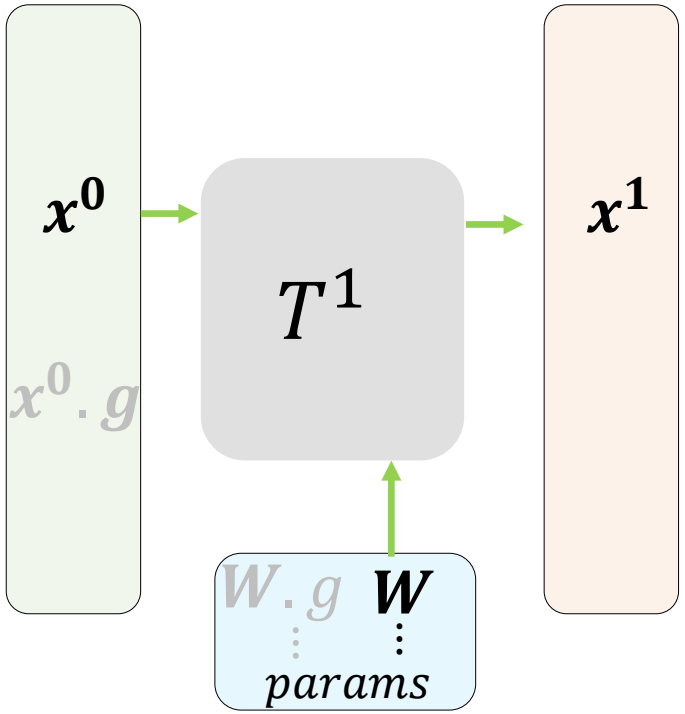
Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}(\boldsymbol{x}^{l}; T^{l+1}.params)$$



Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}(\boldsymbol{x}^{l}; T^{l+1}.params)$$



$$\boldsymbol{x}^{l}.grad = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l}}$$

Let's get more generic

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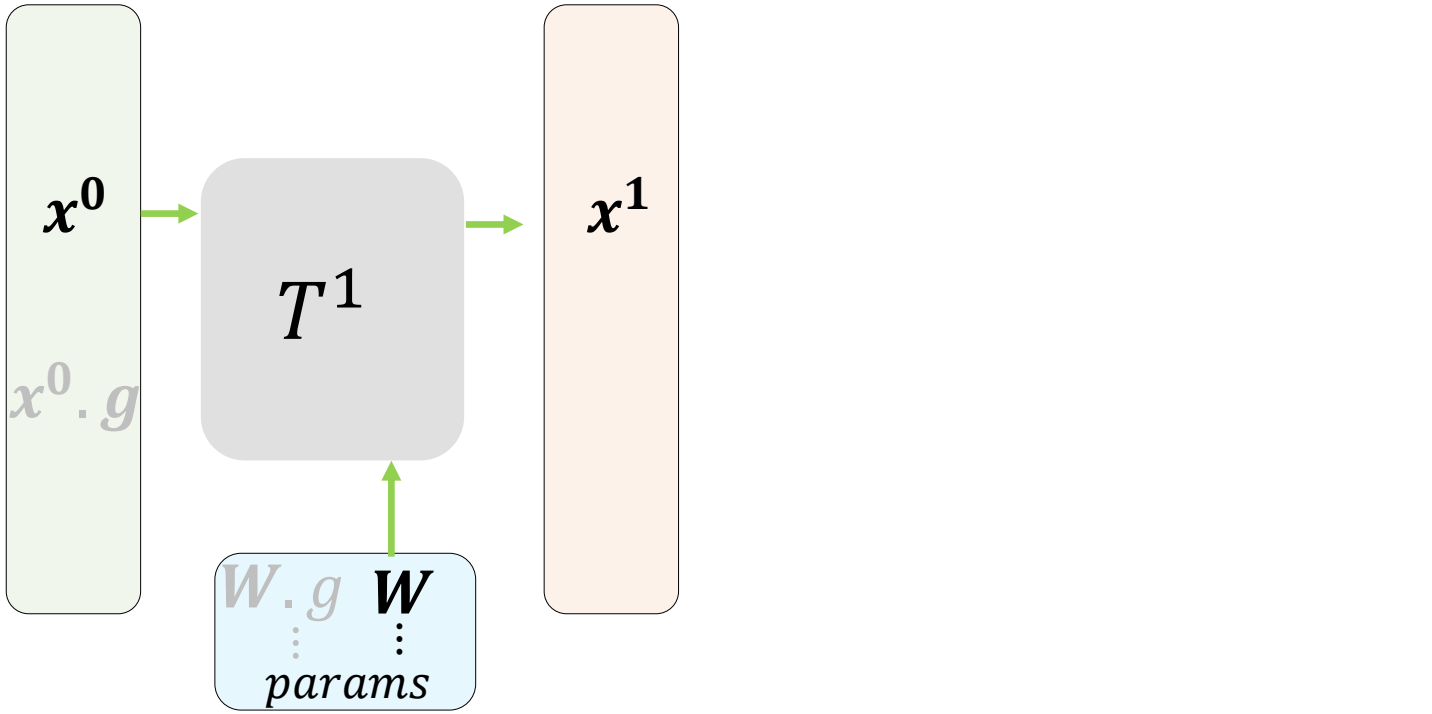
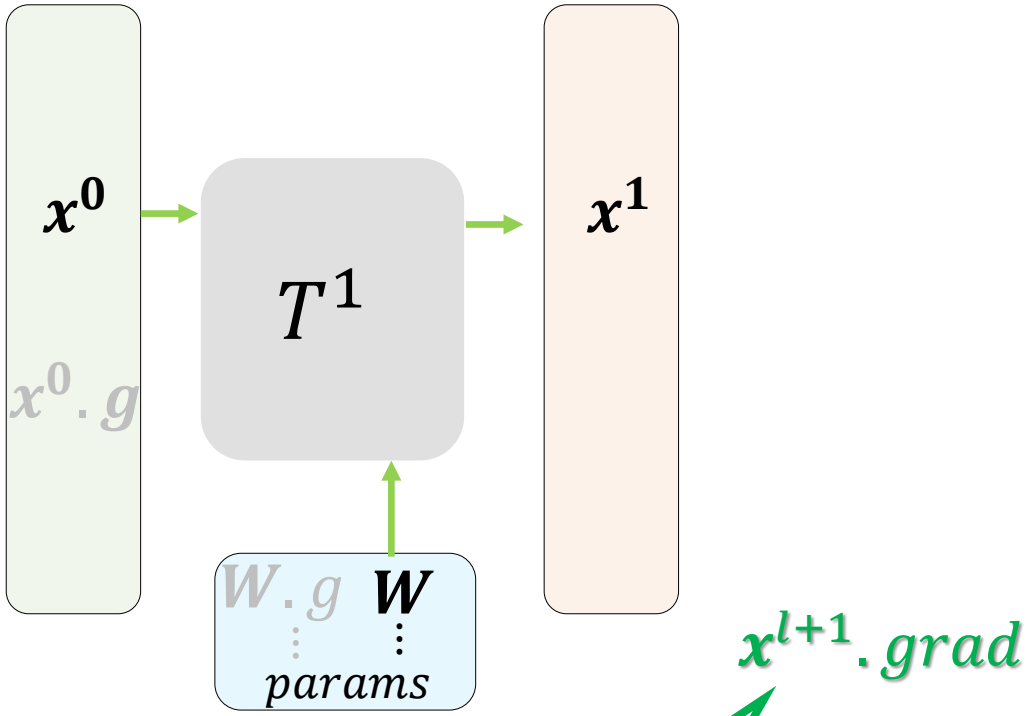


$$\boldsymbol{x}^{l}.grad = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l}} = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l+1}} \cdot \frac{\partial \boldsymbol{x}^{l+1}}{\partial \boldsymbol{x}^{l}}$$

Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}(\boldsymbol{x}^{l}; T^{l+1}.params)$$

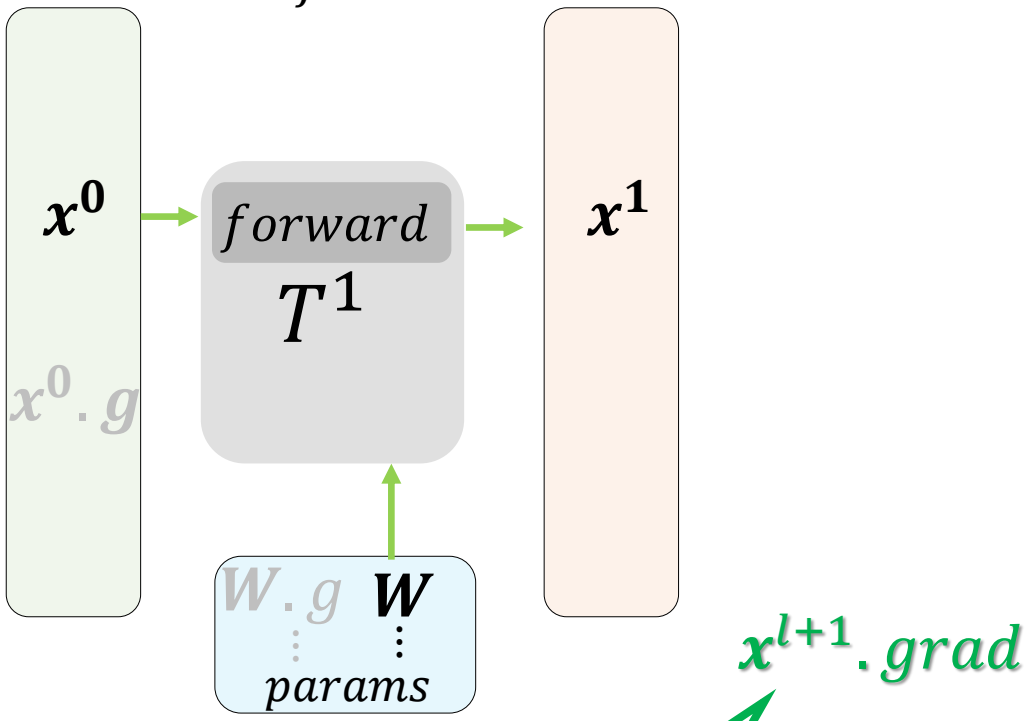


$x^{l+1}.grad$

$$\boldsymbol{x}^{l}.grad = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l}} = \boxed{\frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l+1}}} \cdot \frac{\partial \boldsymbol{x}^{l+1}}{\partial \boldsymbol{x}^{l}}$$

Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}_{forward}\left(\boldsymbol{x}^{l}; T^{l+1}.params\right)$$

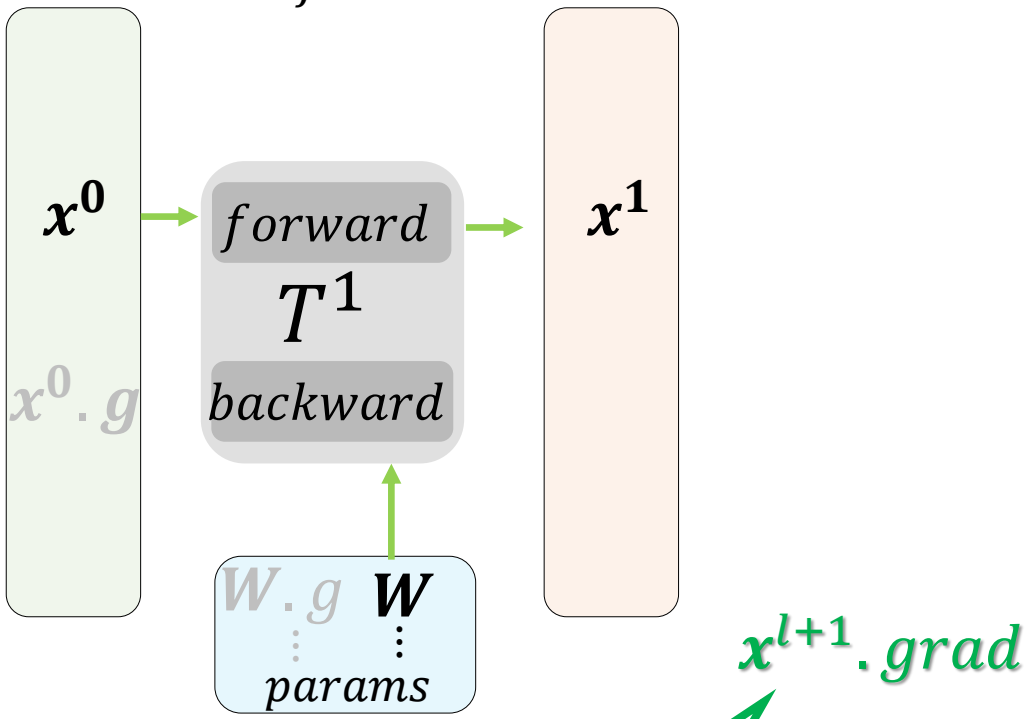


$$\boldsymbol{x}^{l+1}.grad$$

$$\boldsymbol{x}^{l}.grad = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l}} = \boxed{\frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l+1}}} \cdot \frac{\partial \boldsymbol{x}^{l+1}}{\partial \boldsymbol{x}^{l}}$$

Let's get more generic

$$\boldsymbol{x}^{l+1} = T^{l+1}_{forward}\left(\boldsymbol{x}^{l}; T^{l+1}.params\right)$$



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$$\boldsymbol{x}^{l}.grad = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l}} = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}^{l+1}} \cdot \frac{\partial \boldsymbol{x}^{l+1}}{\partial \boldsymbol{x}^{l}} = T^{l+1}_{backward}\left(\boldsymbol{x}^{l+1}.grad;\quad T^{l+1}.params,\quad \boldsymbol{x}^{l}\right)$$

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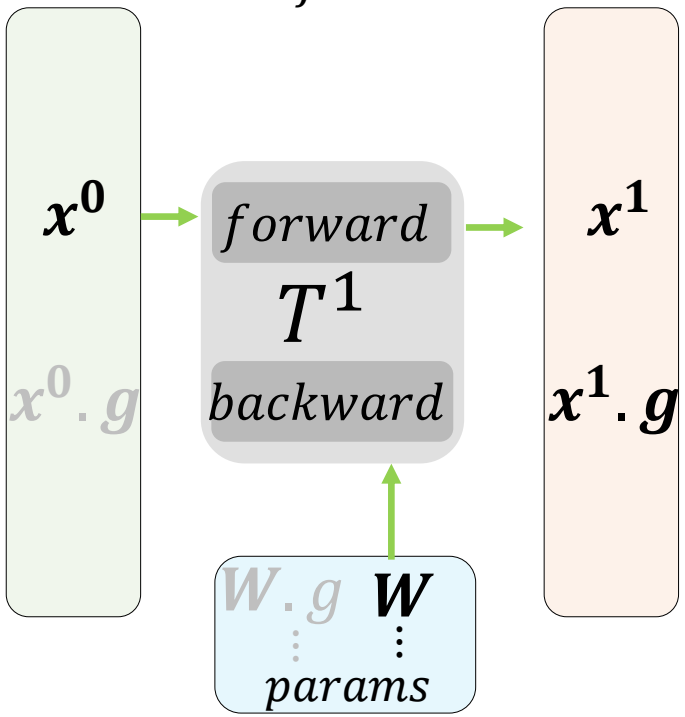

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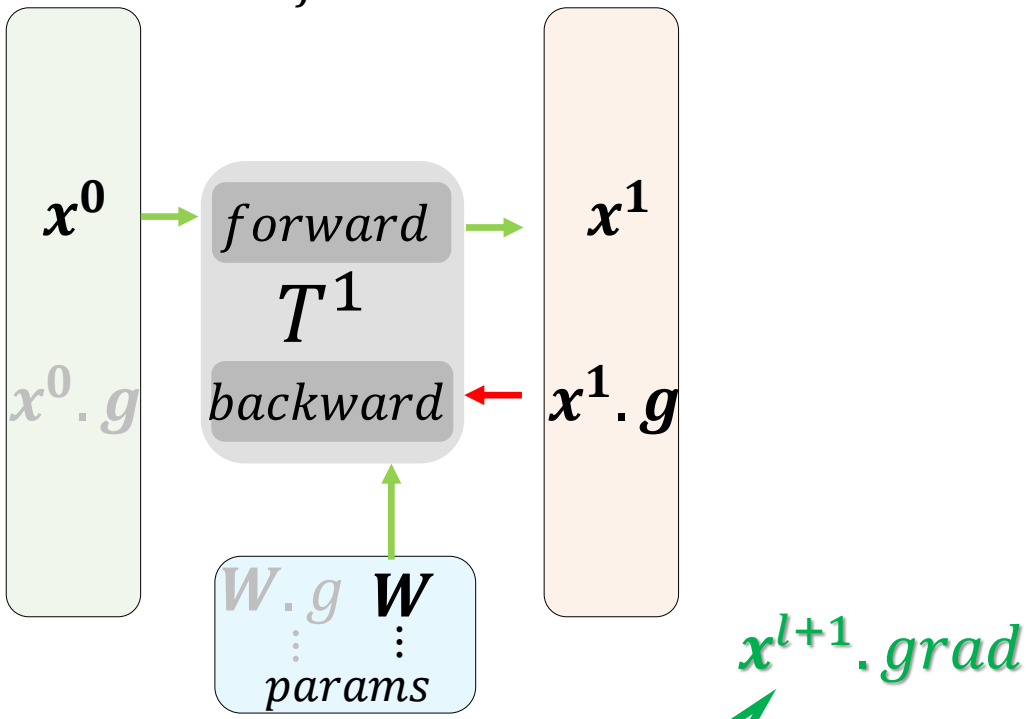

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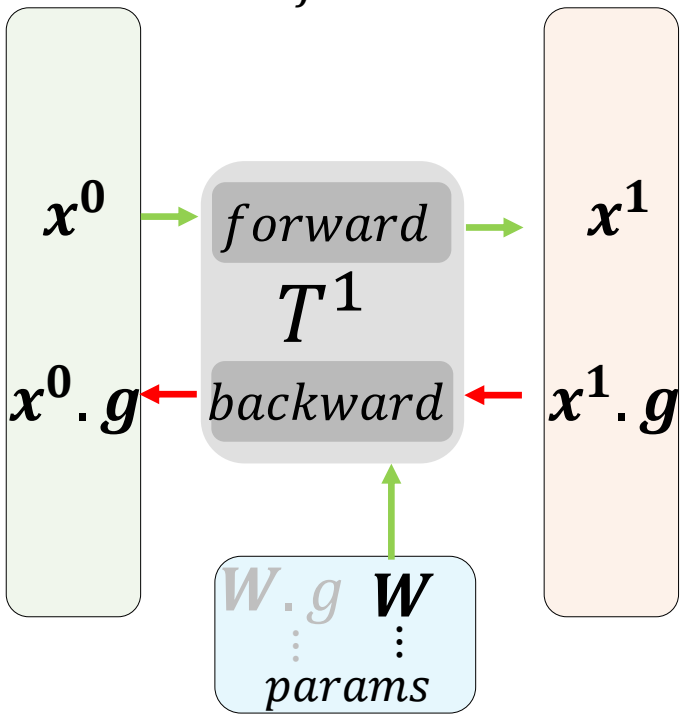

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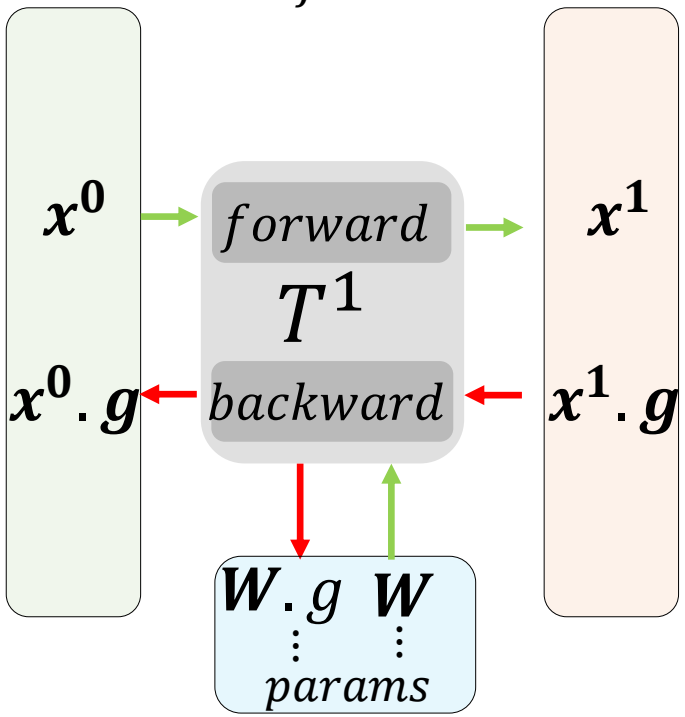

$$\begin{pmatrix} \boldsymbol{x}^{l}.grad \\ T^{l+1}.params.grad \end{pmatrix} = T^{l+1}_{backward}\left(\boldsymbol{x}^{l+1}.grad;\quad T^{l+1}.params,\quad \boldsymbol{x}^{l}\right)$$

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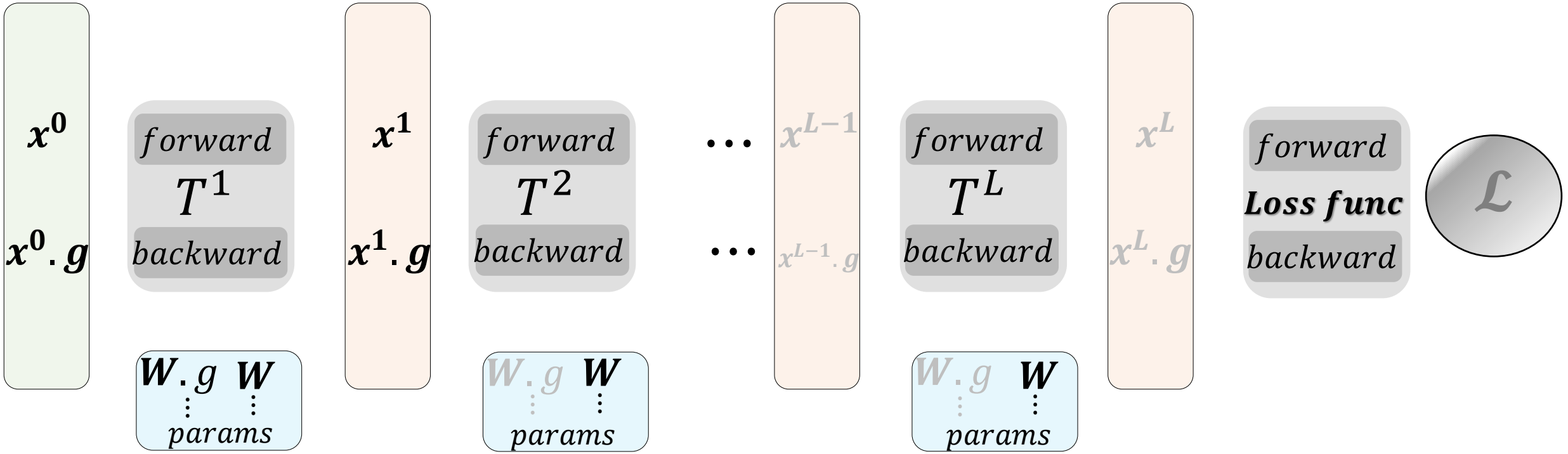

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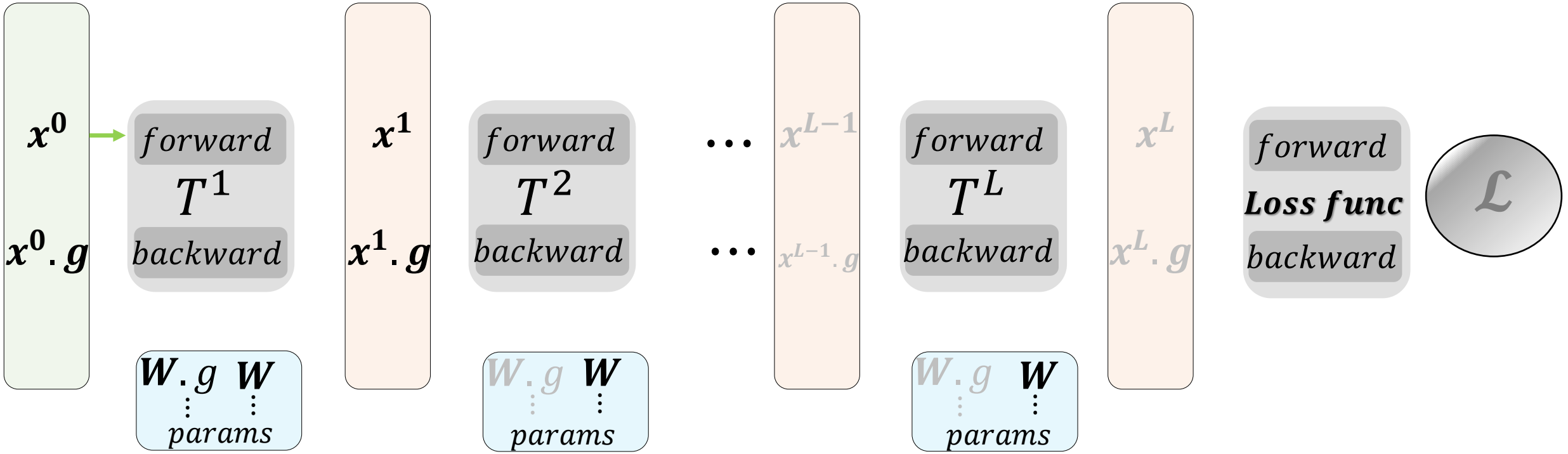

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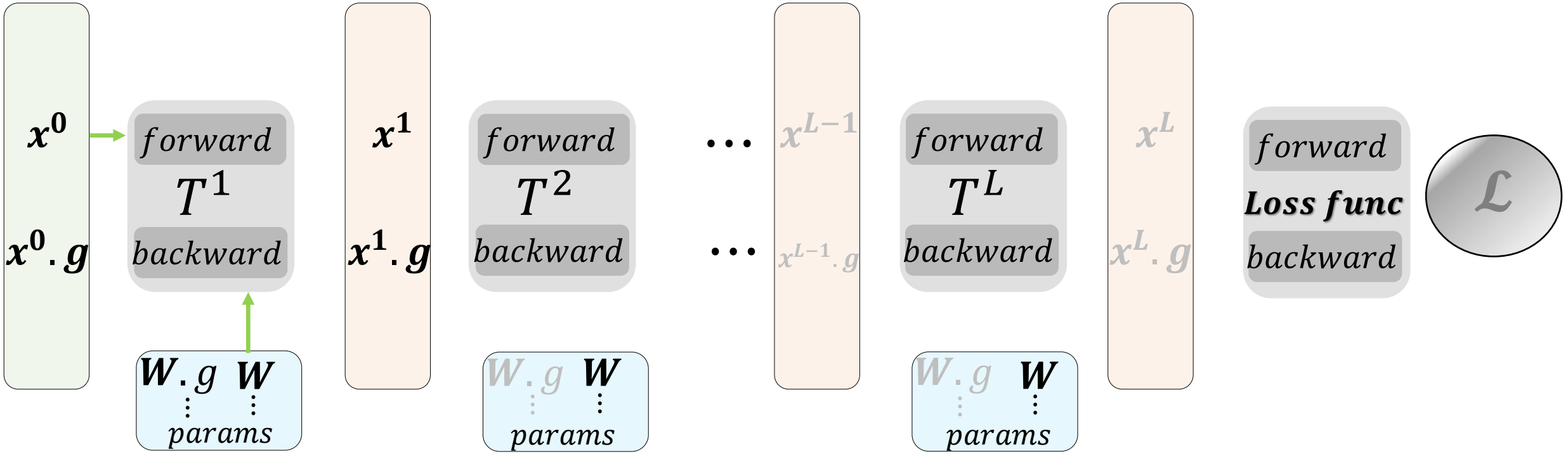

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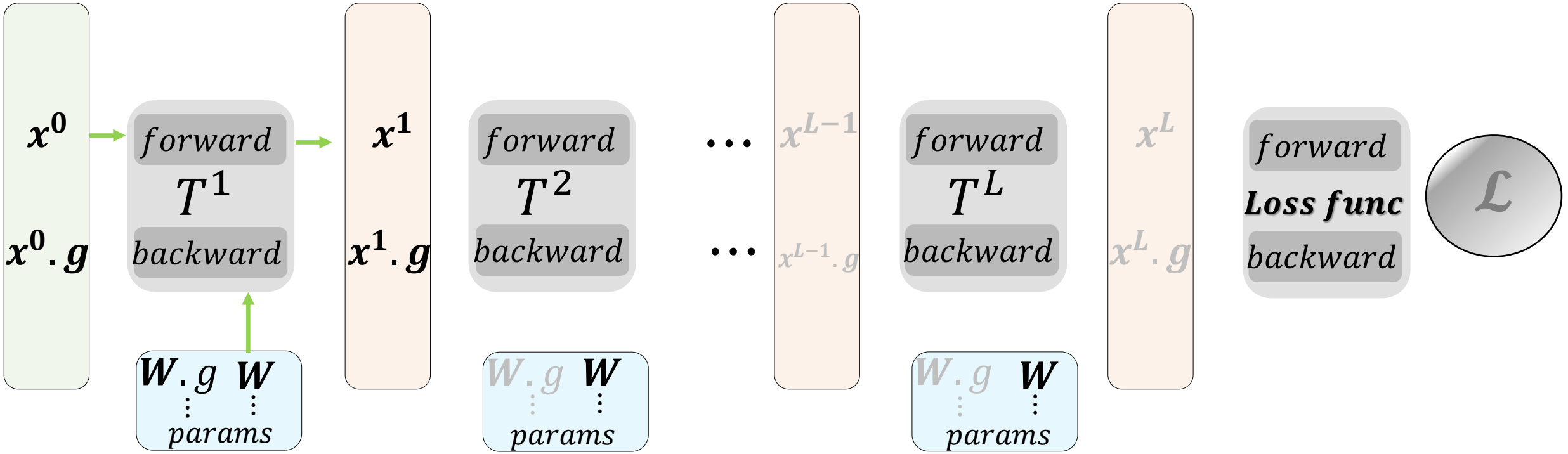

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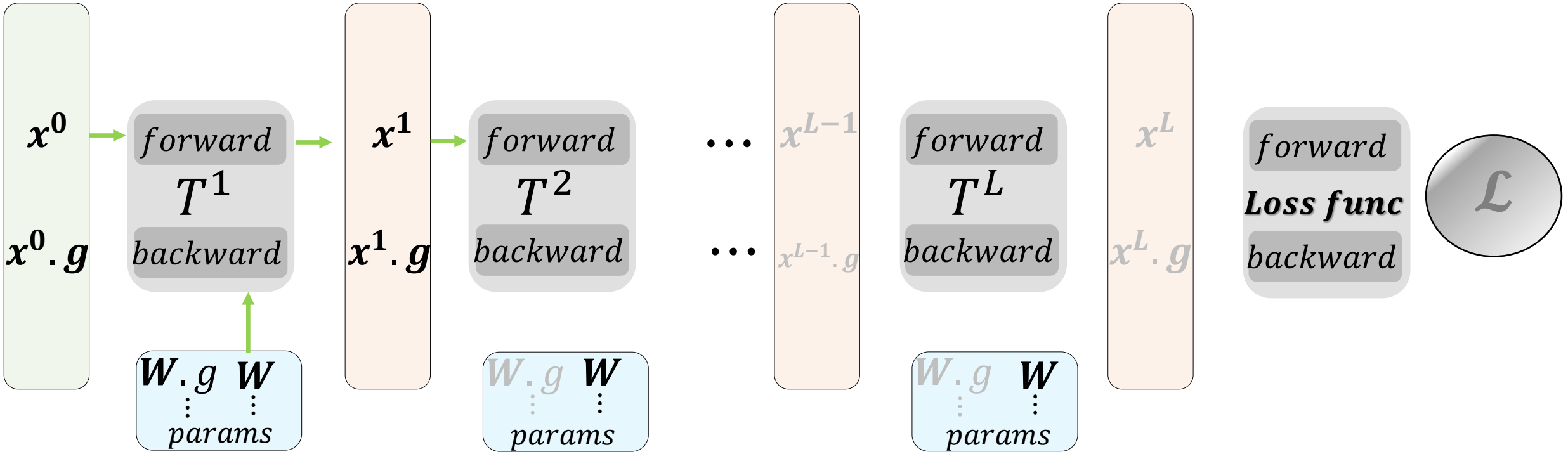

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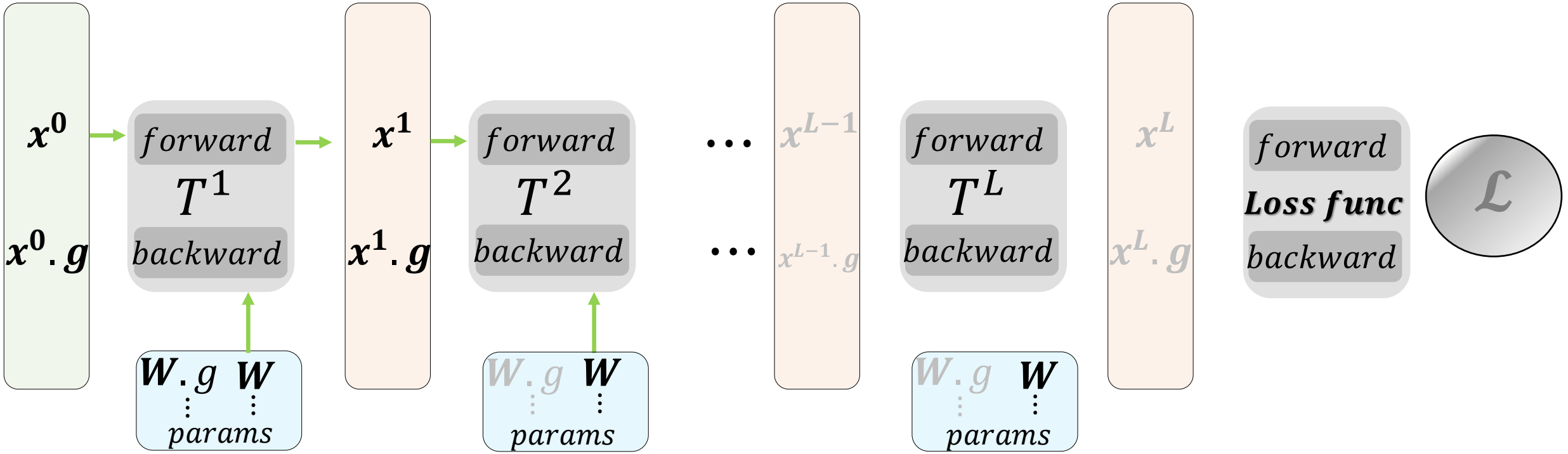

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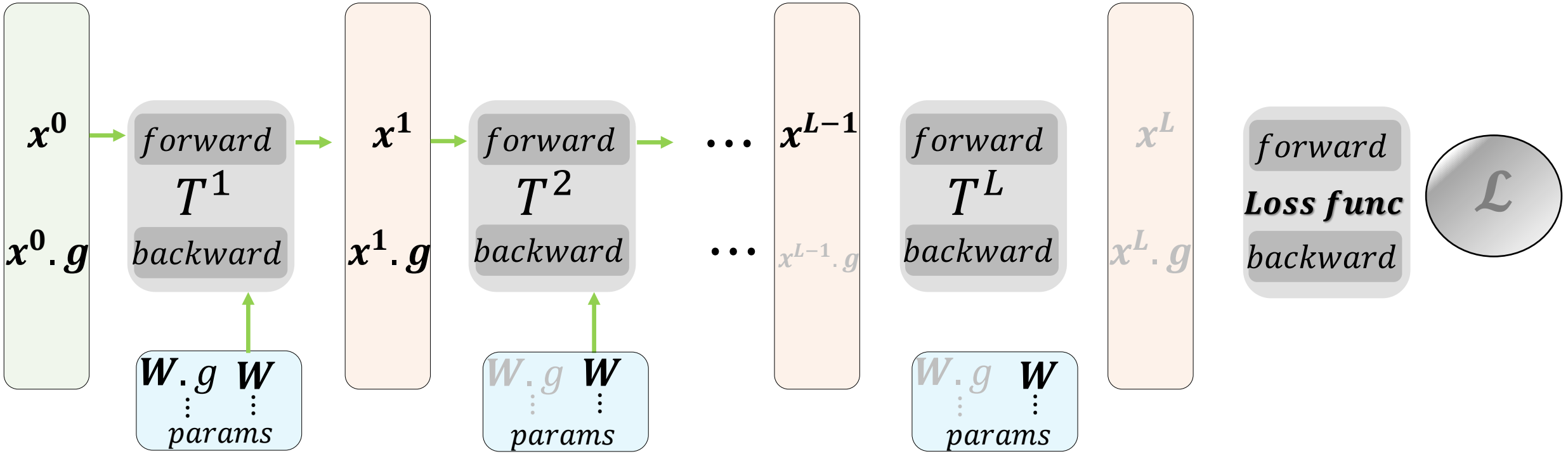

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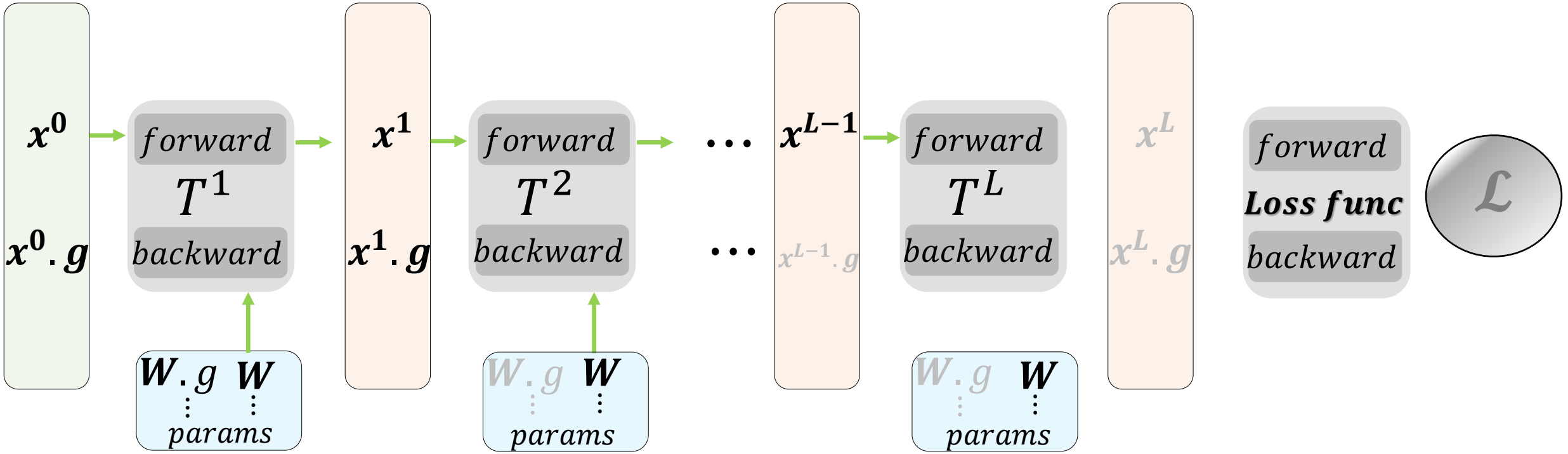

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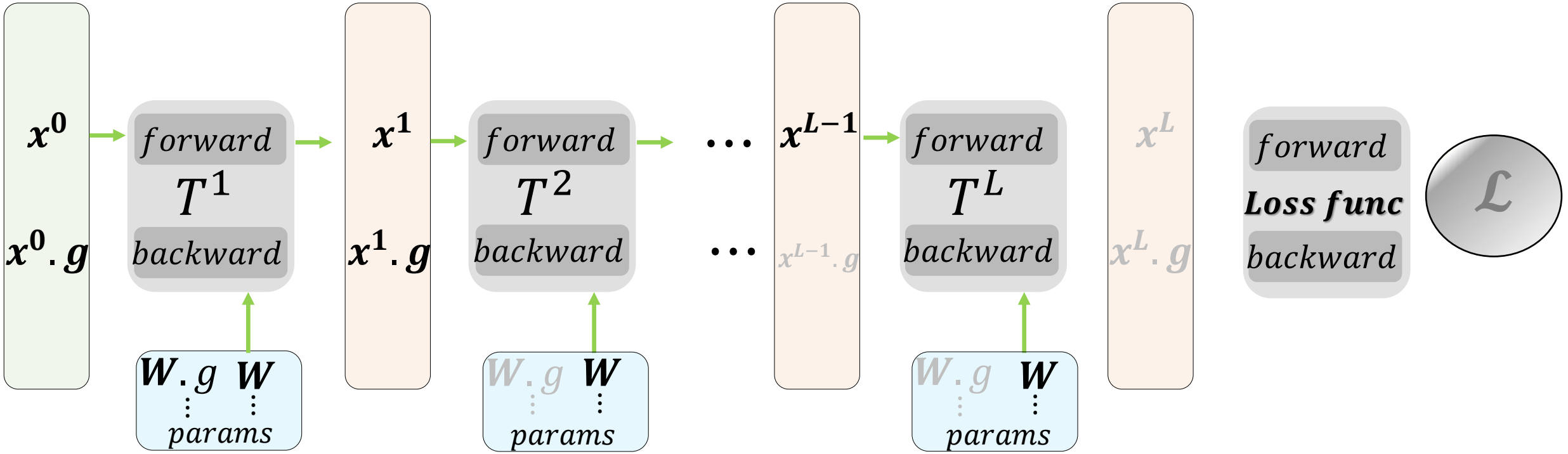

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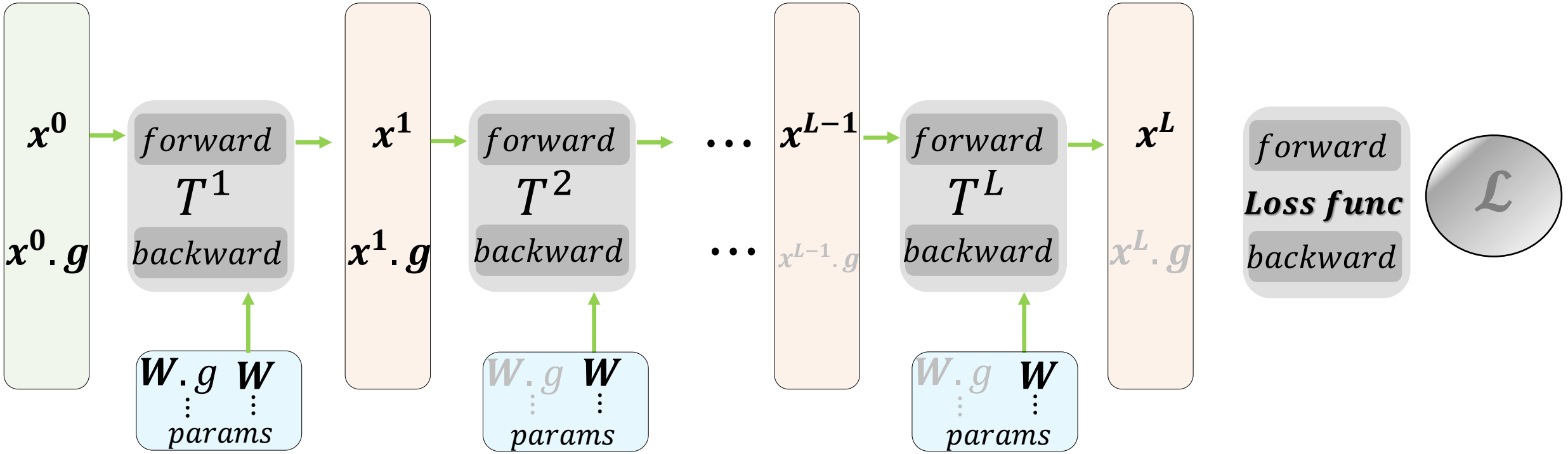

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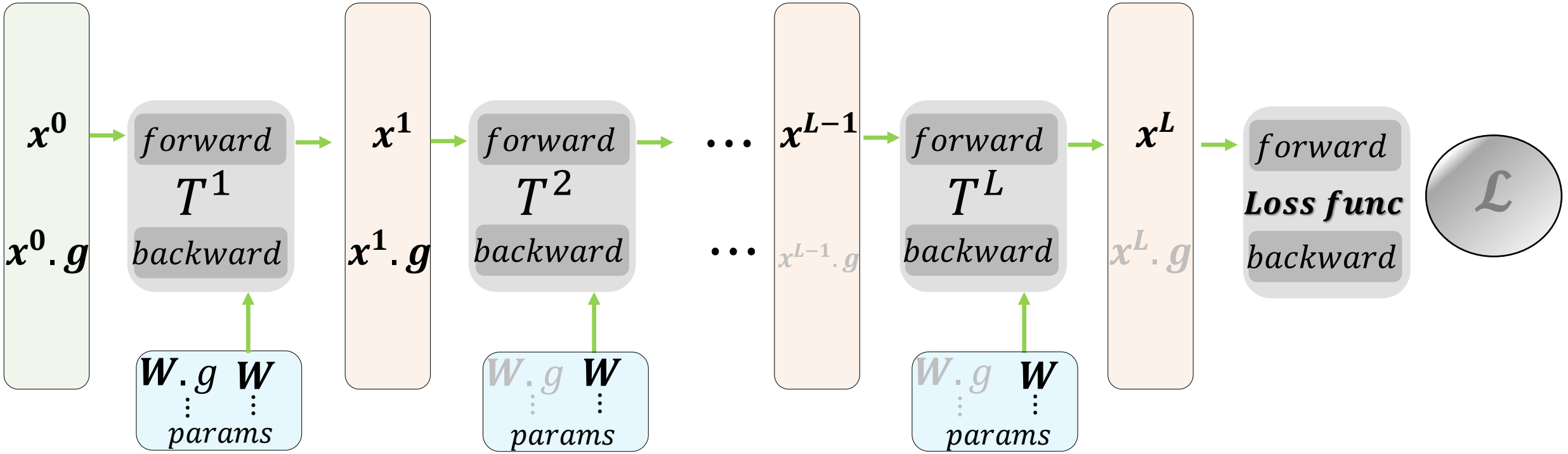

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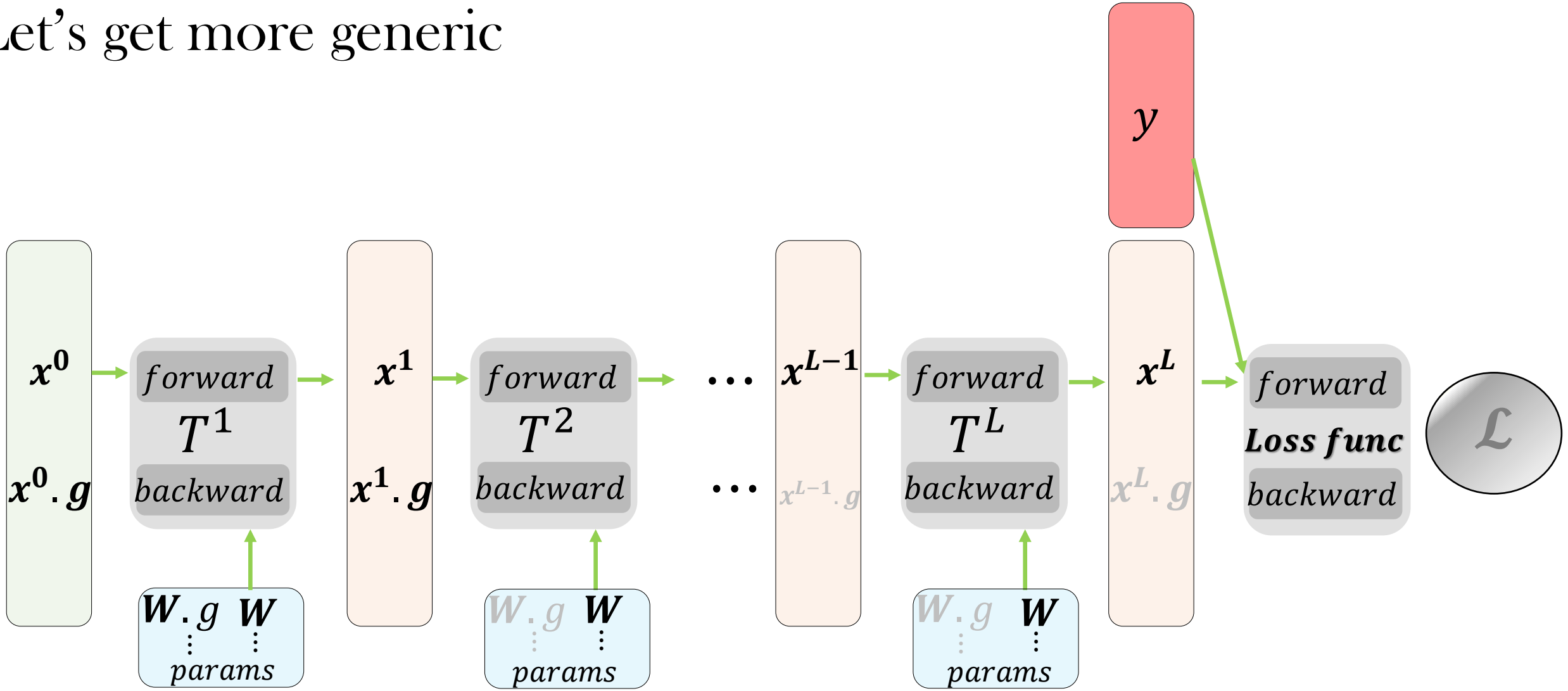

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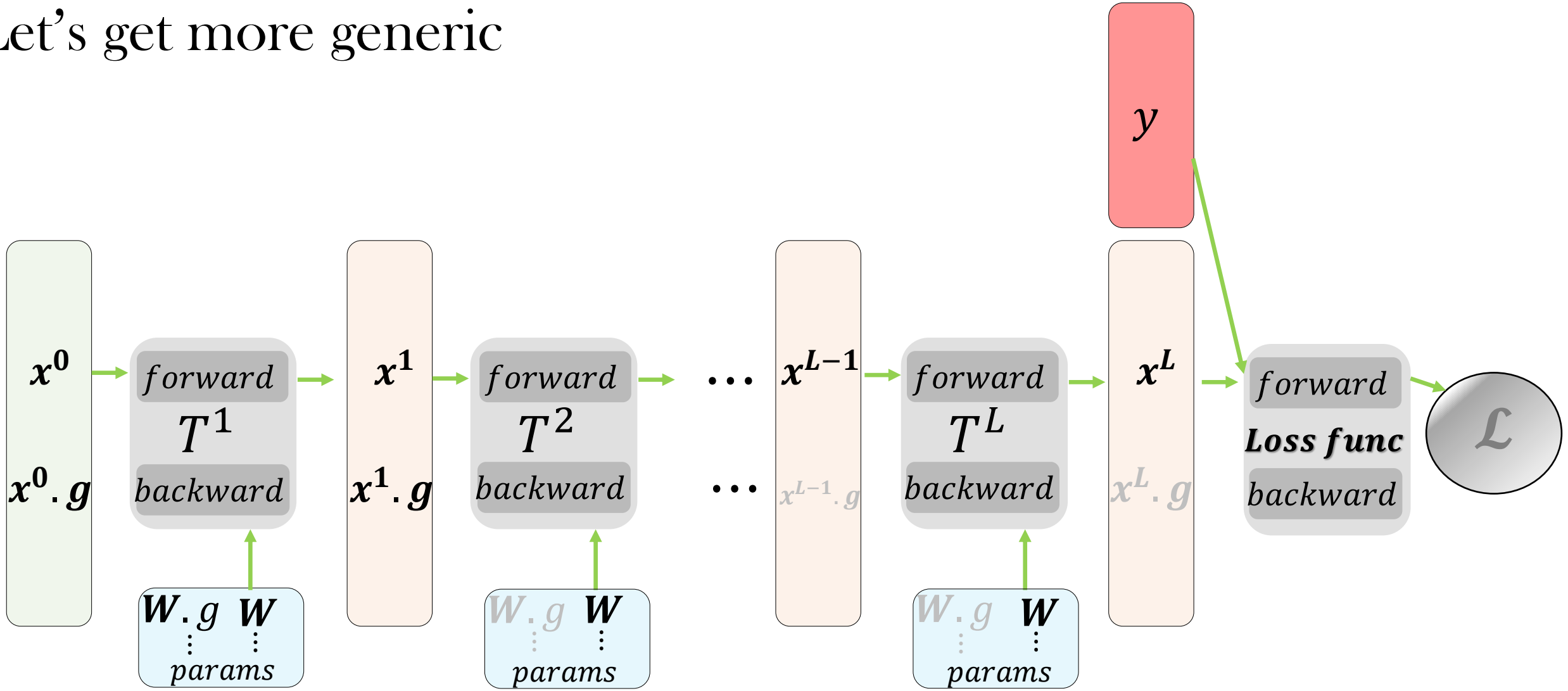

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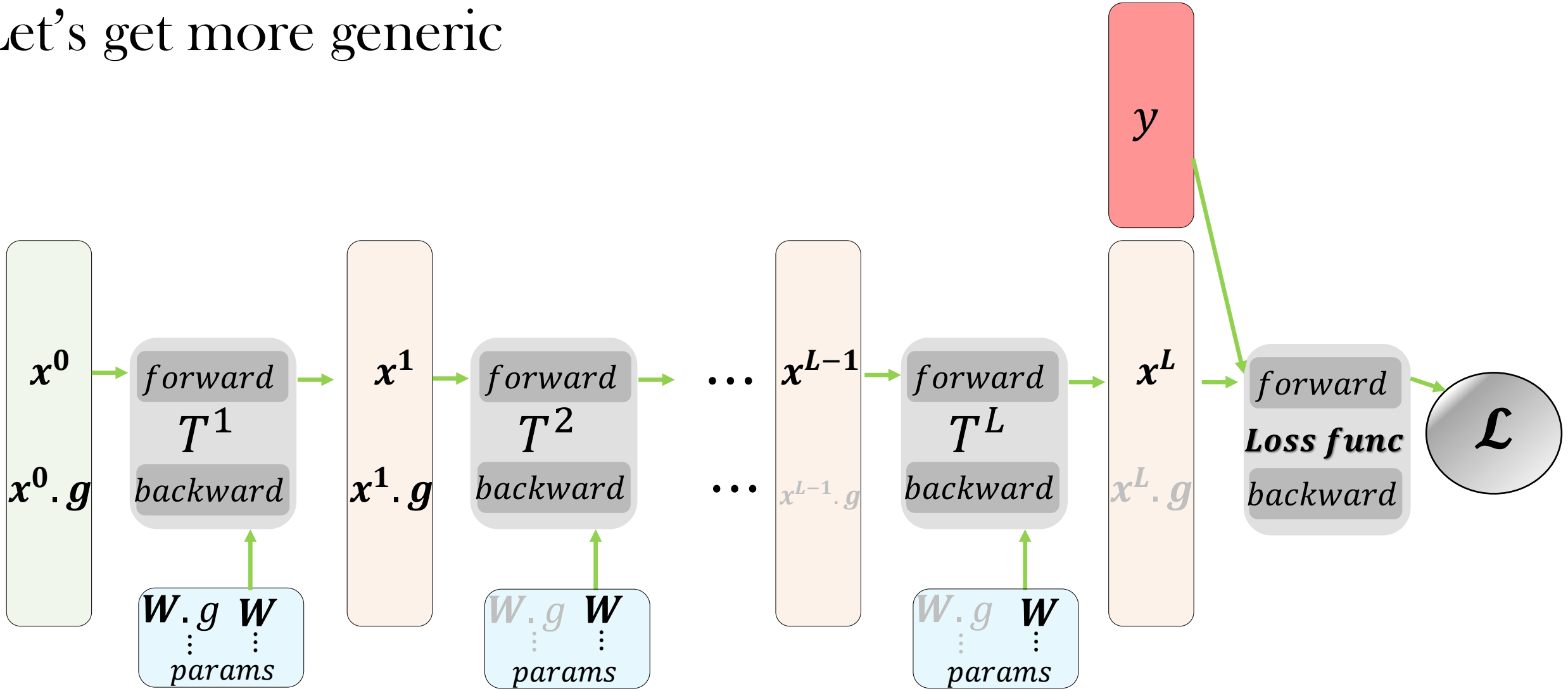

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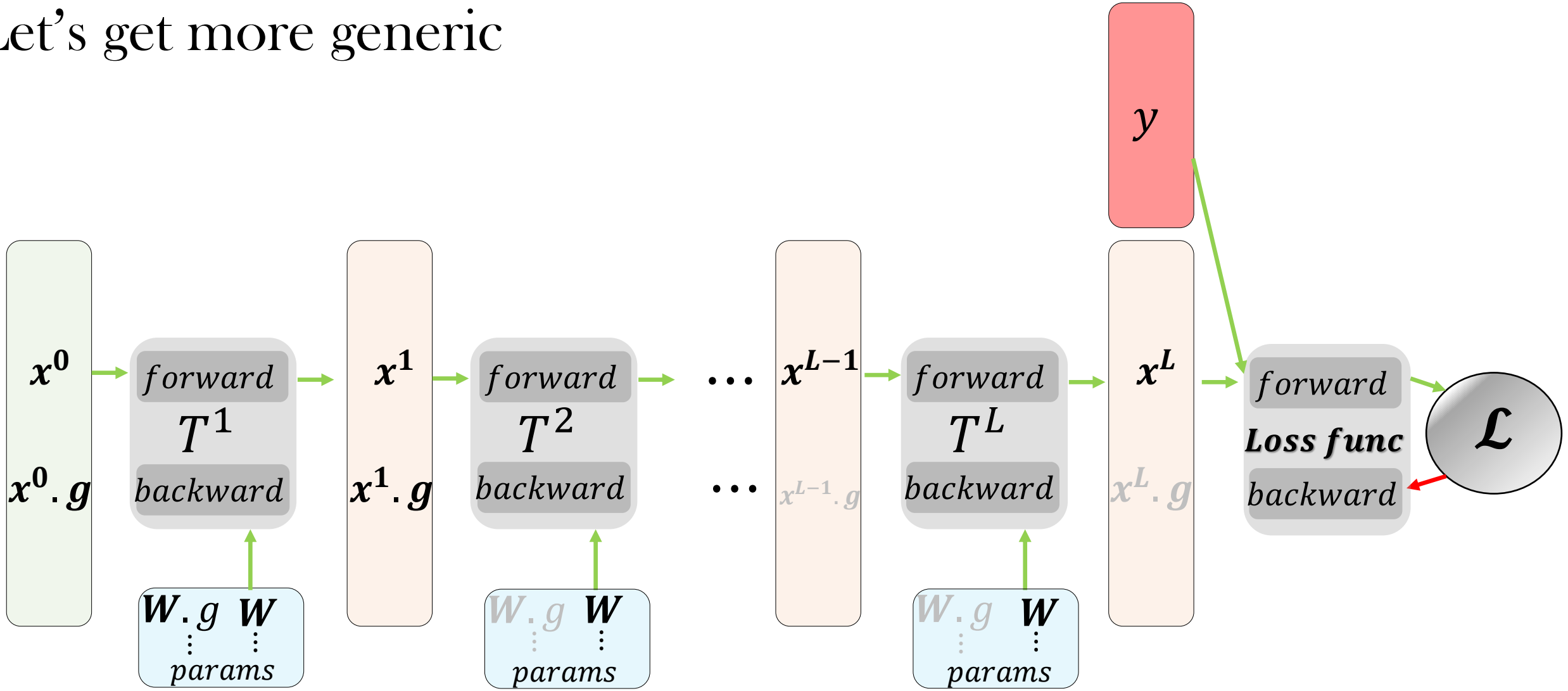

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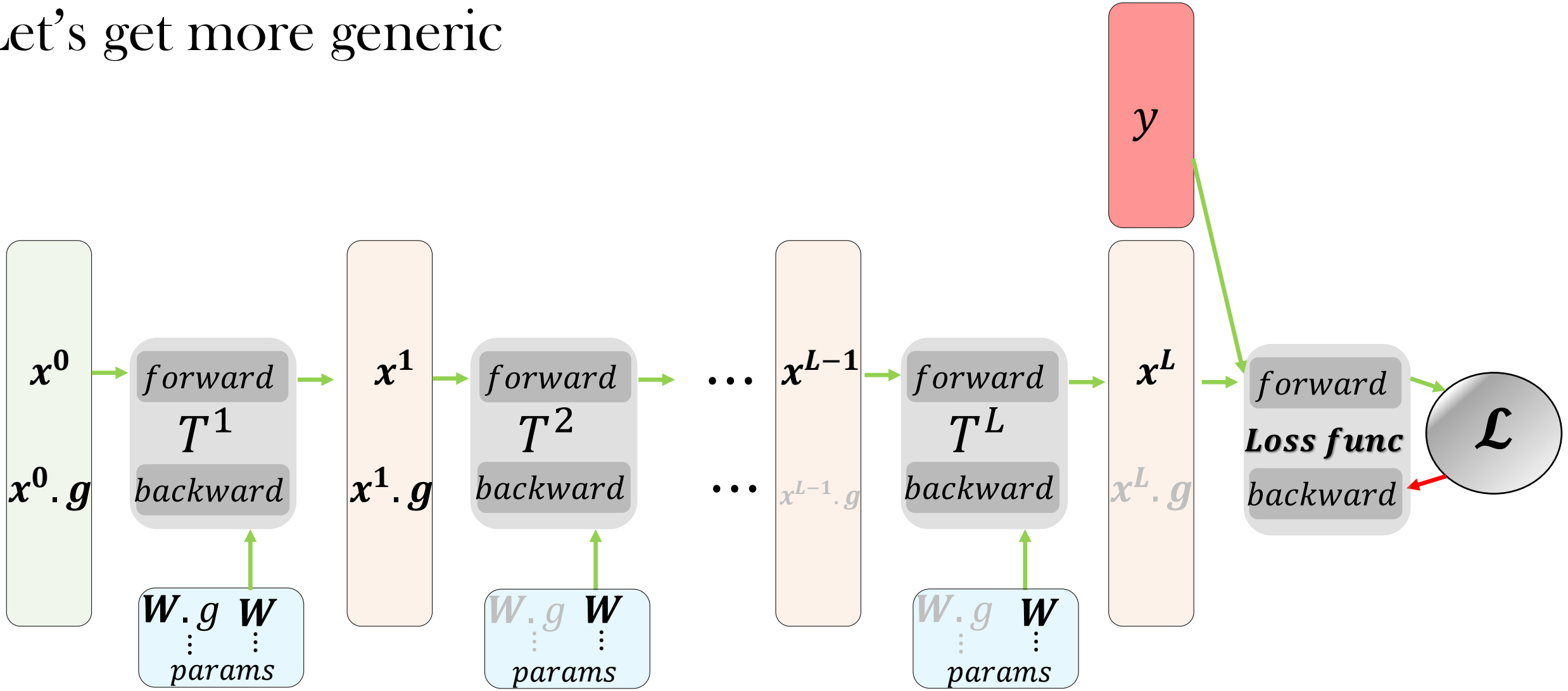

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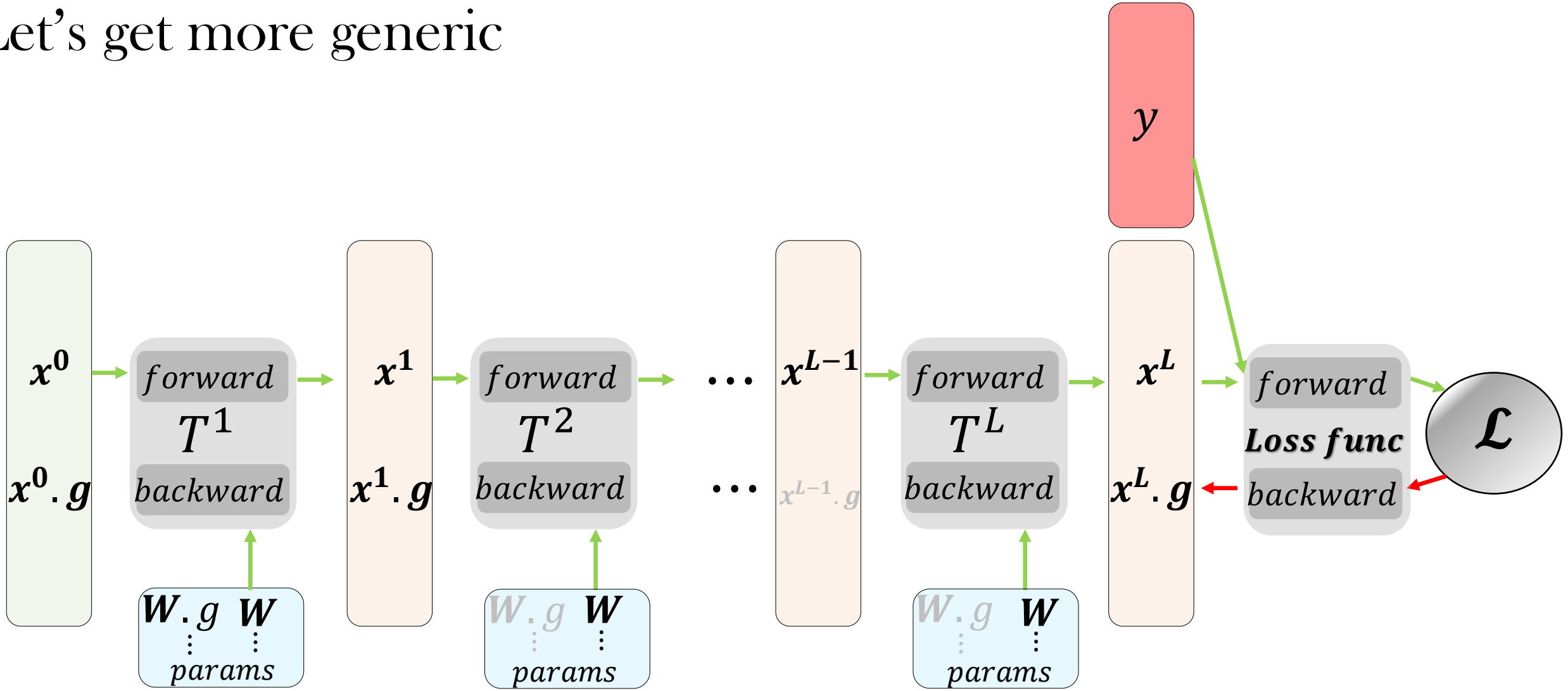

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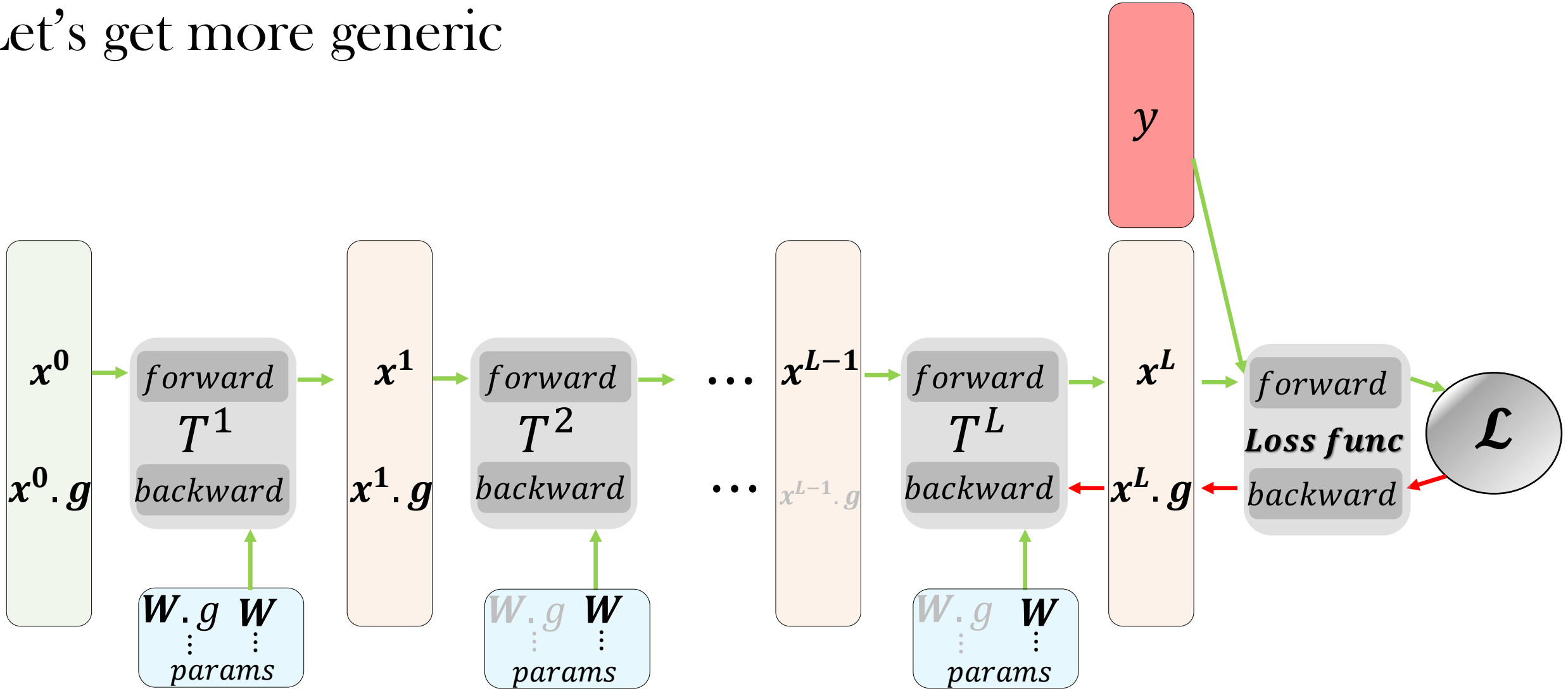

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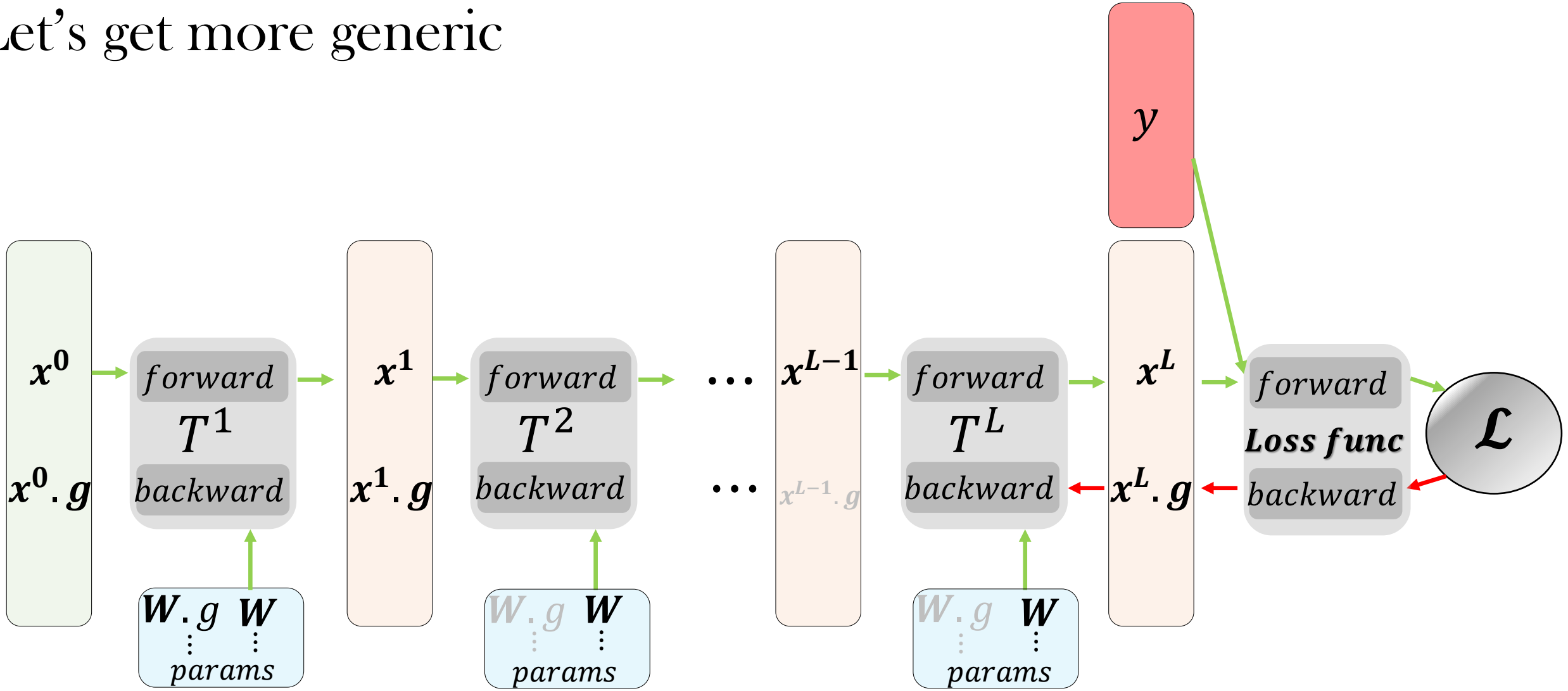

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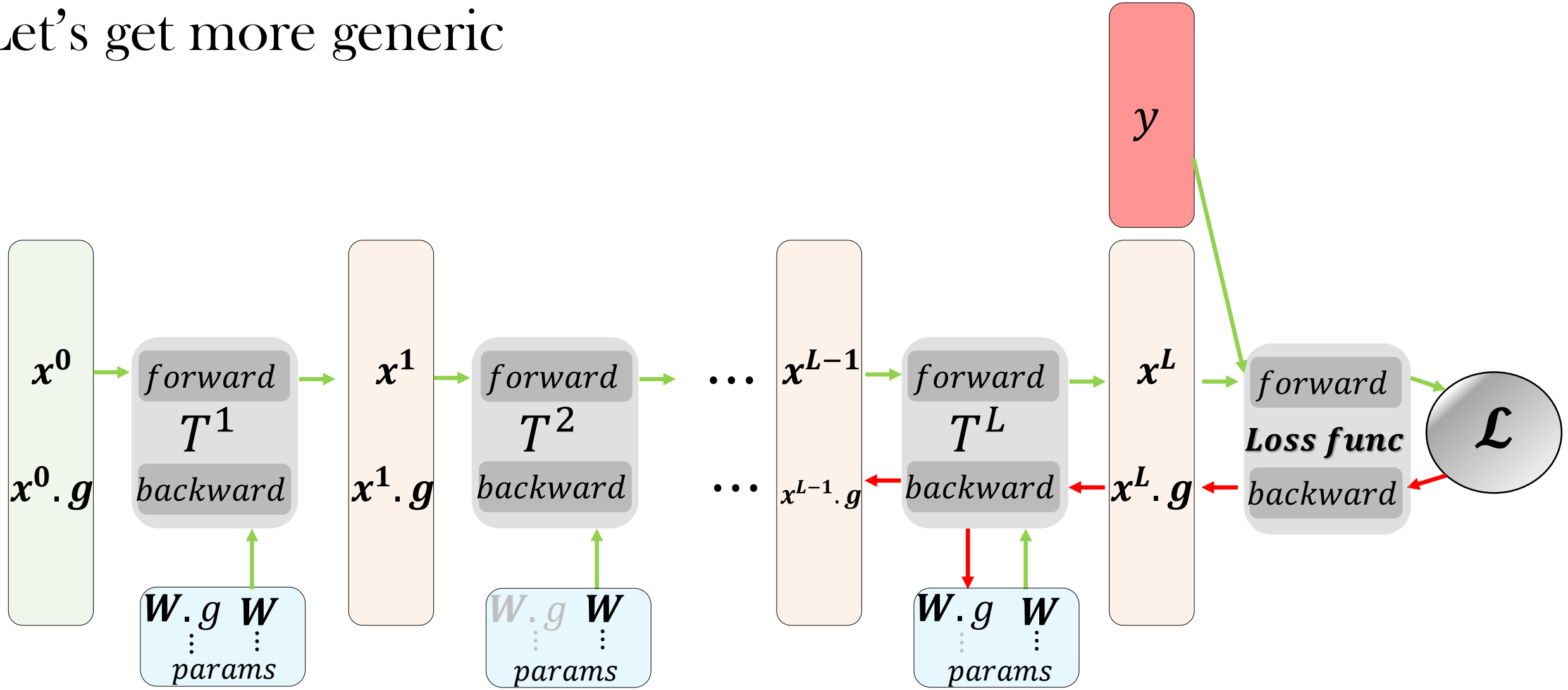

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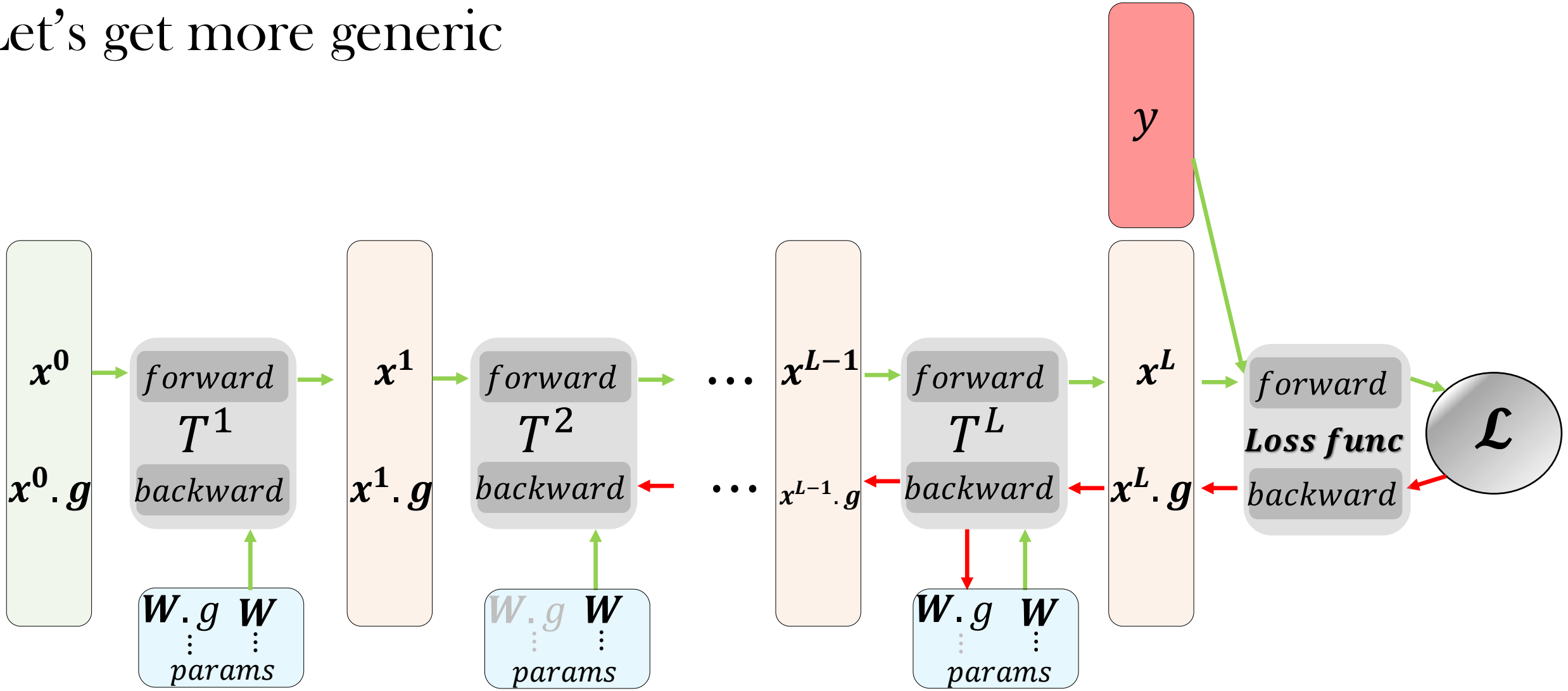

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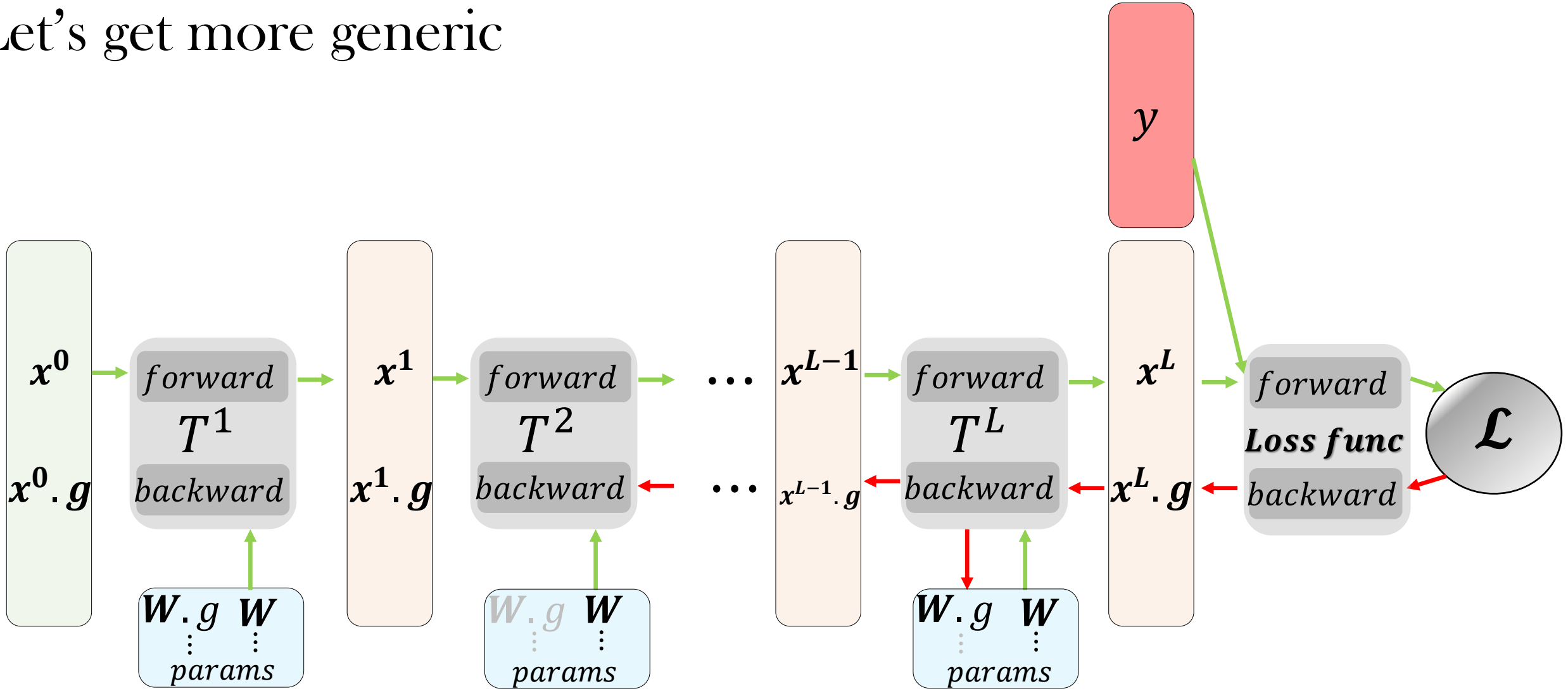

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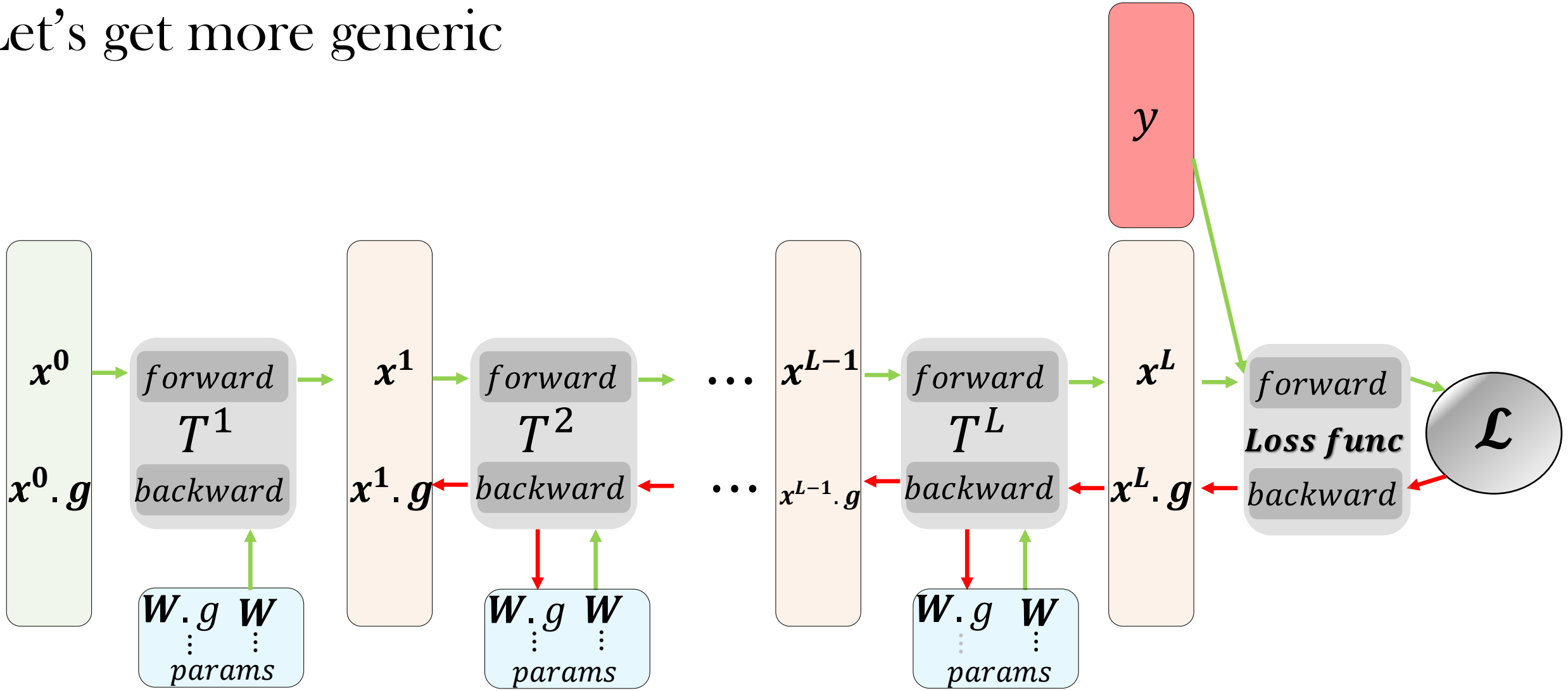

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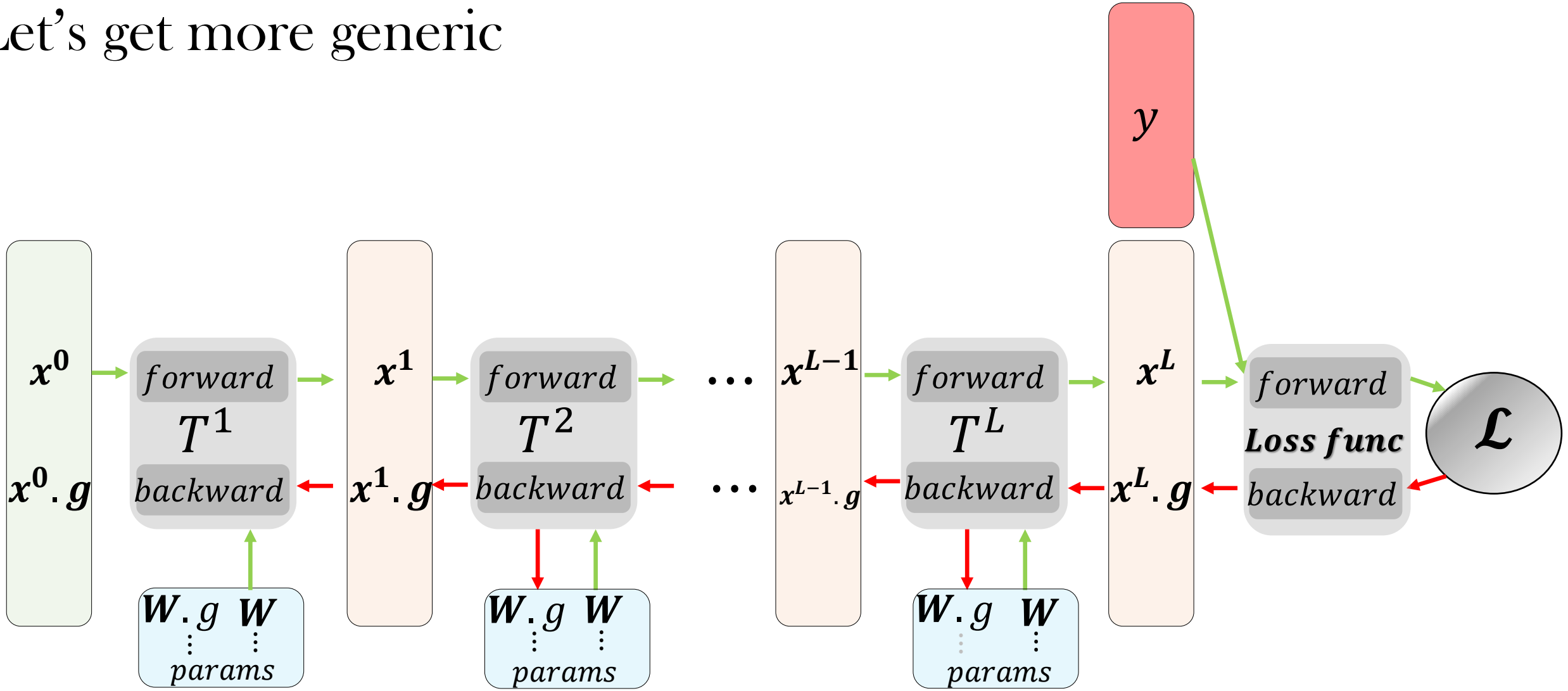

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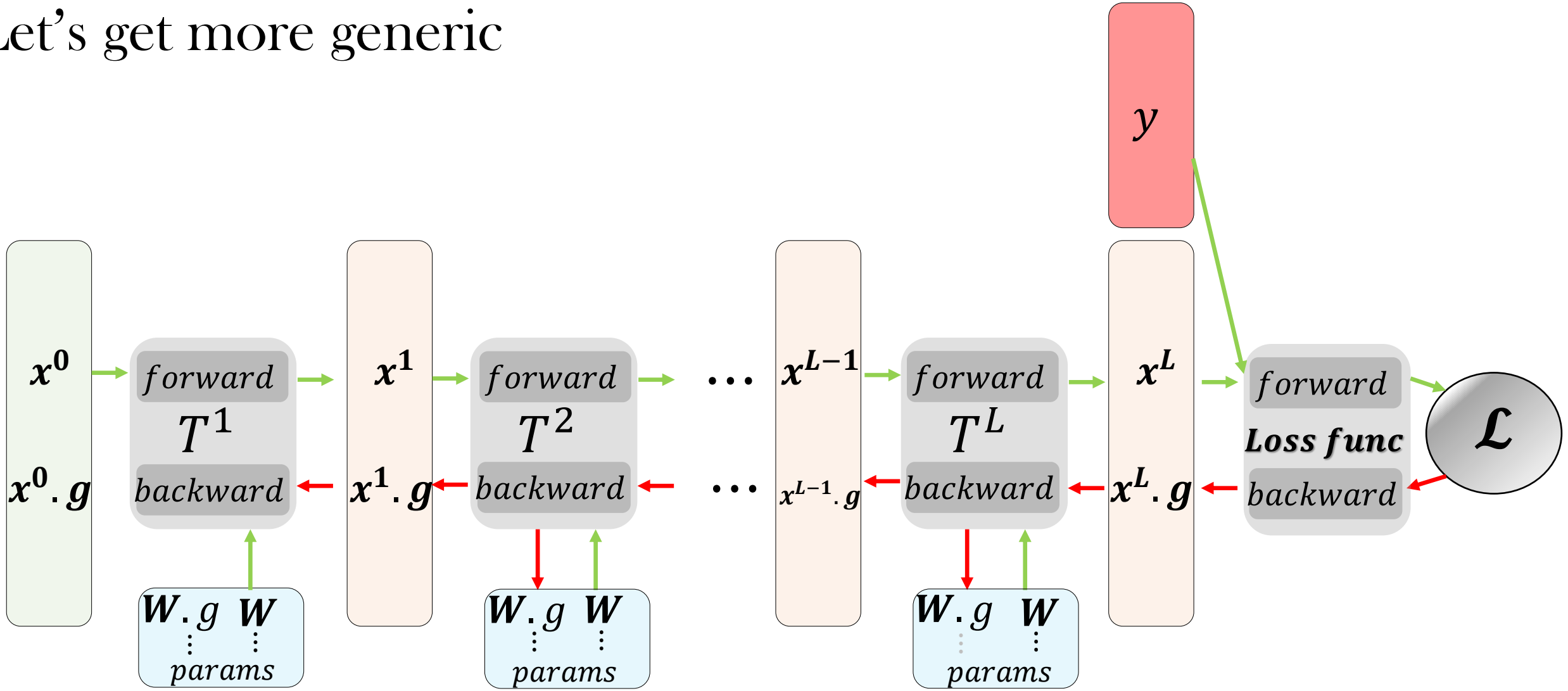

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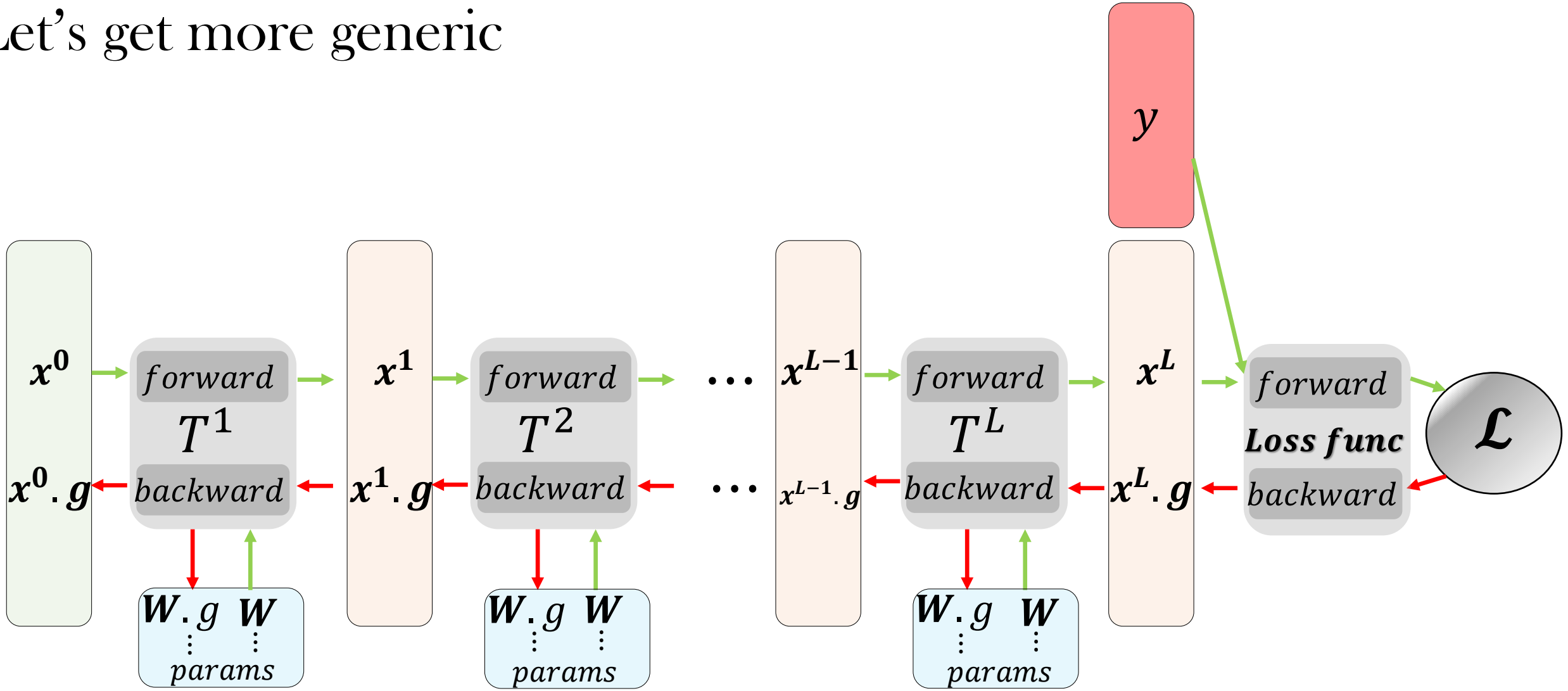

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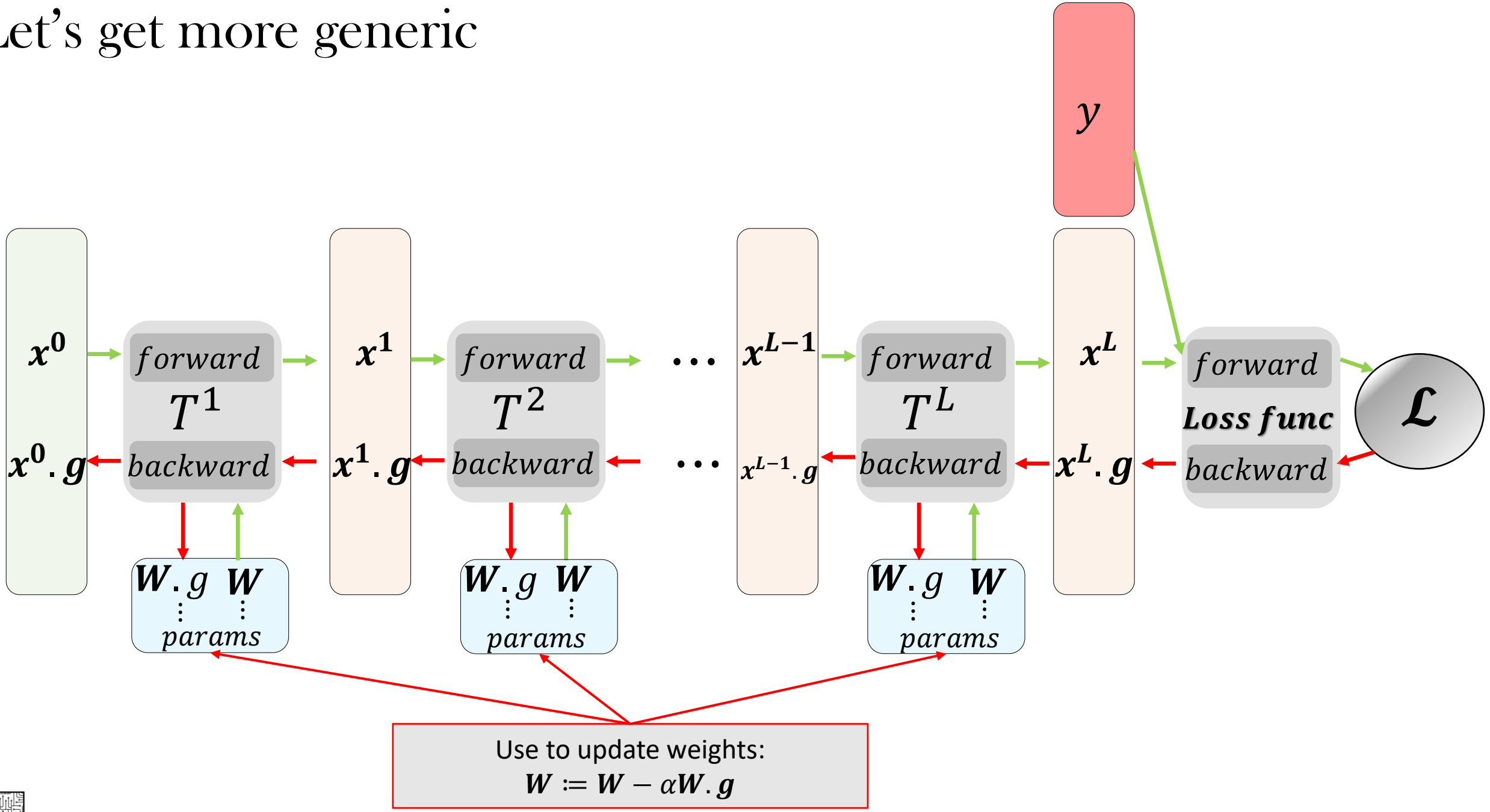
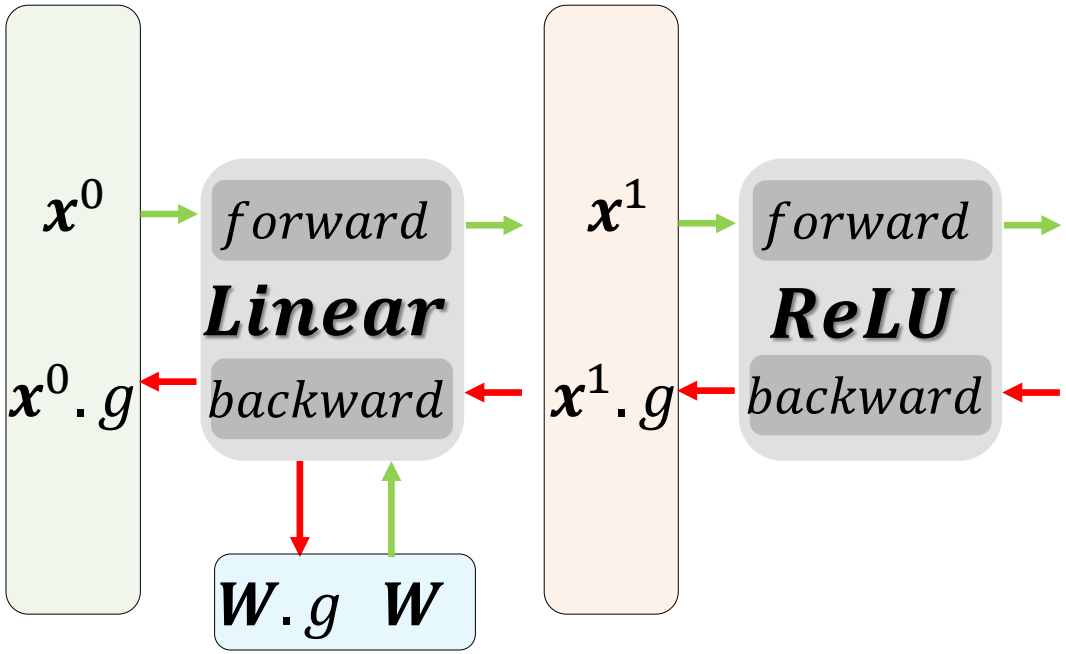
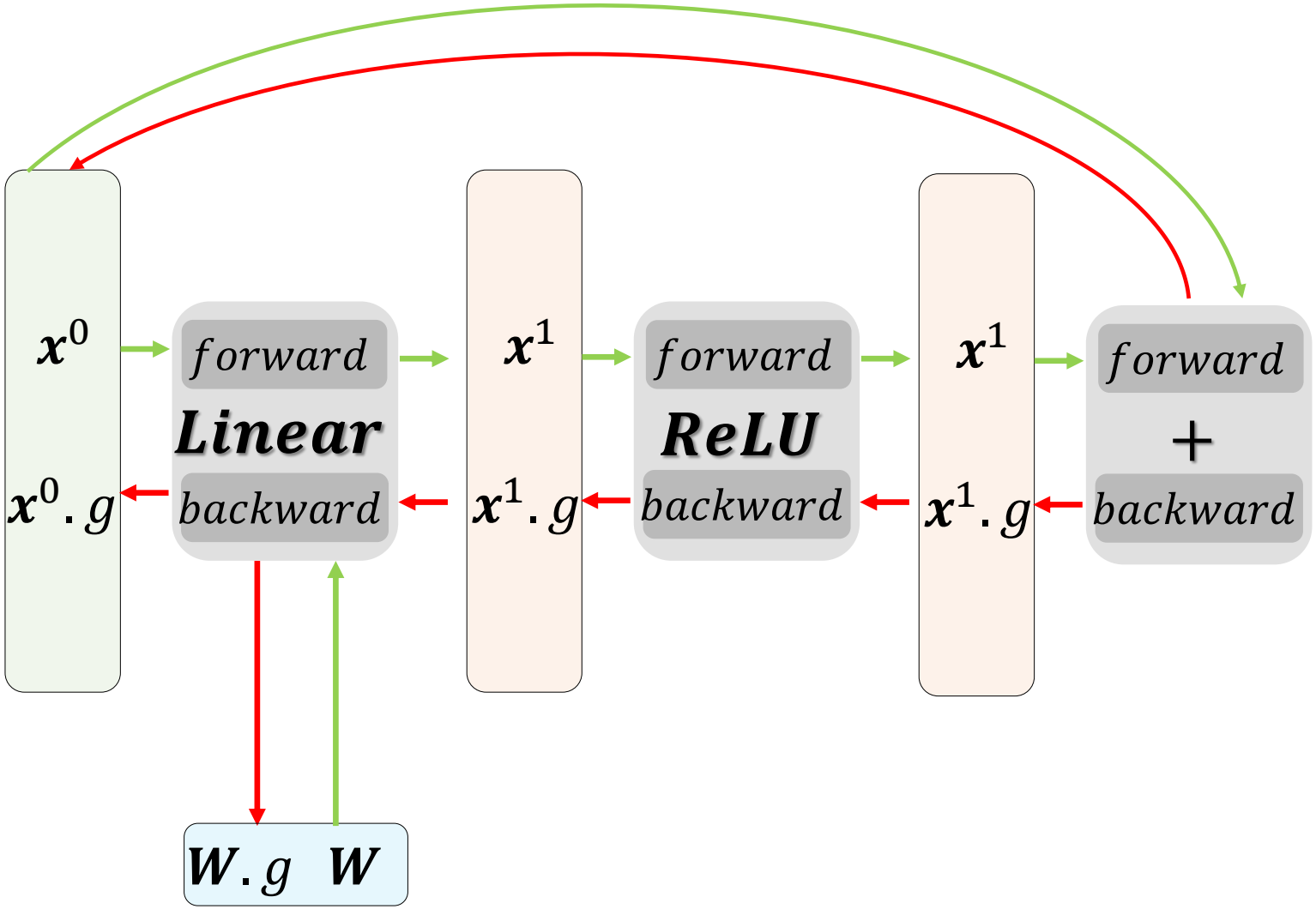


Example: Standard layer



BTW : You can backprop any DAG!



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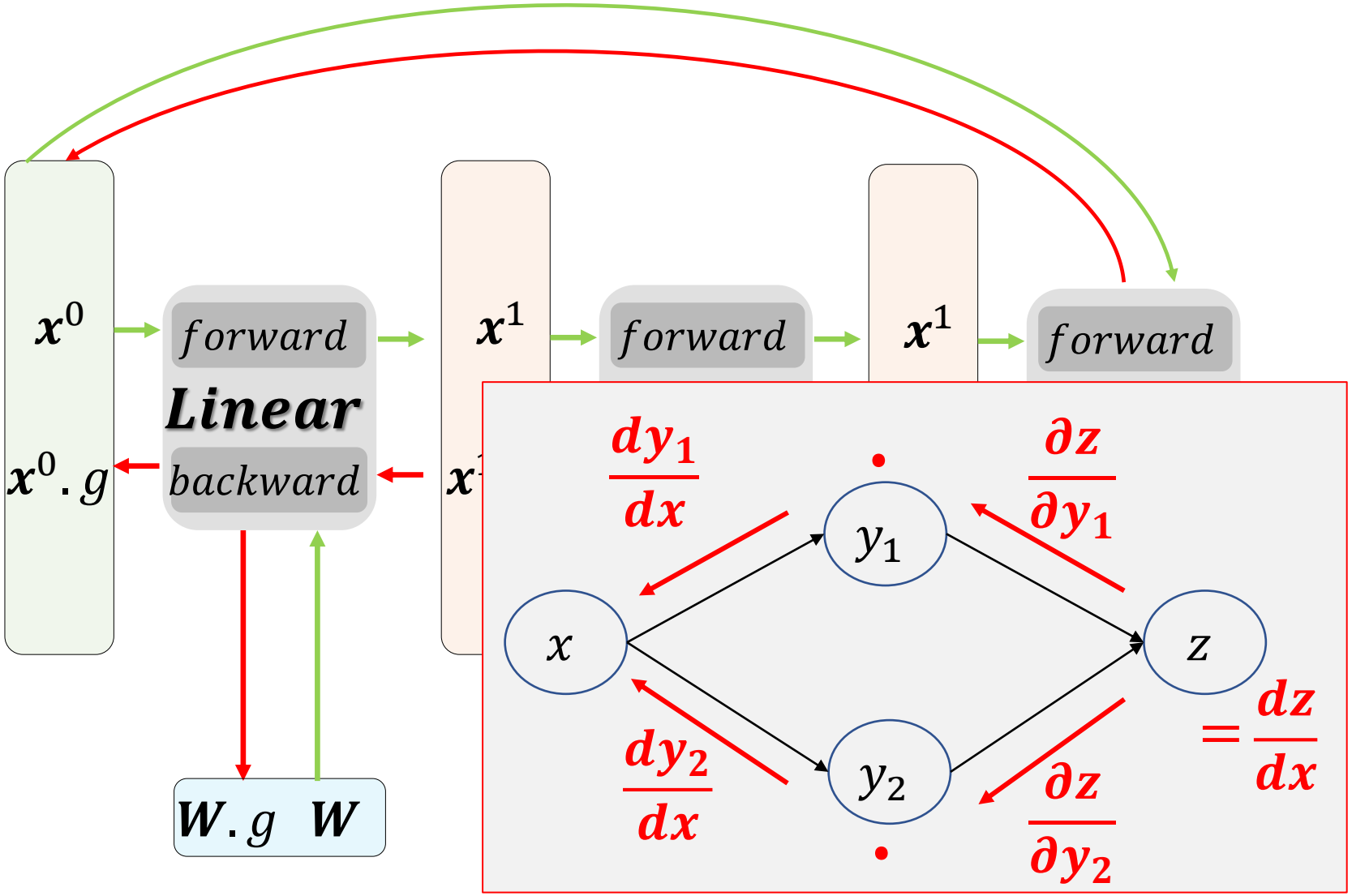

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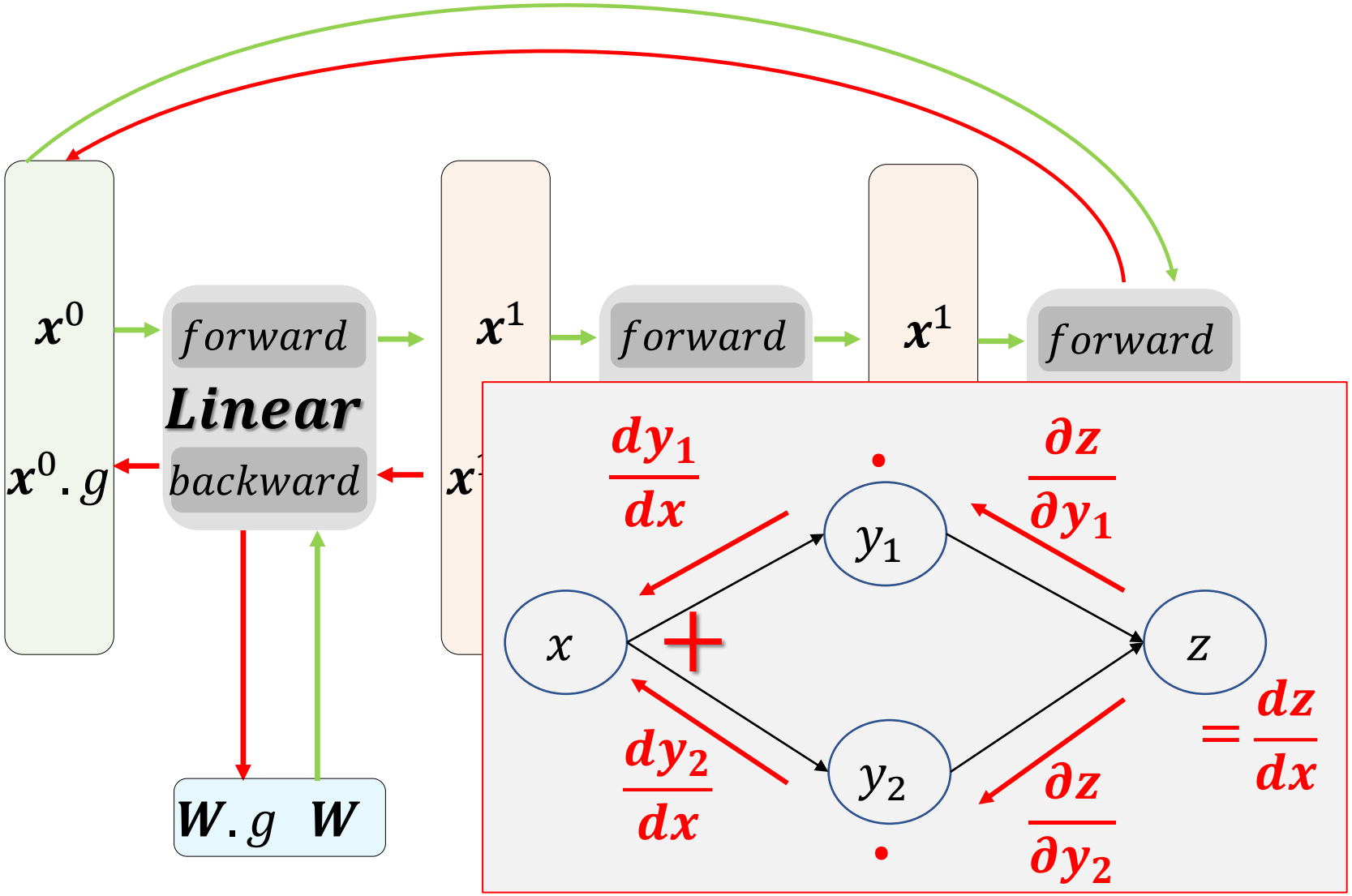

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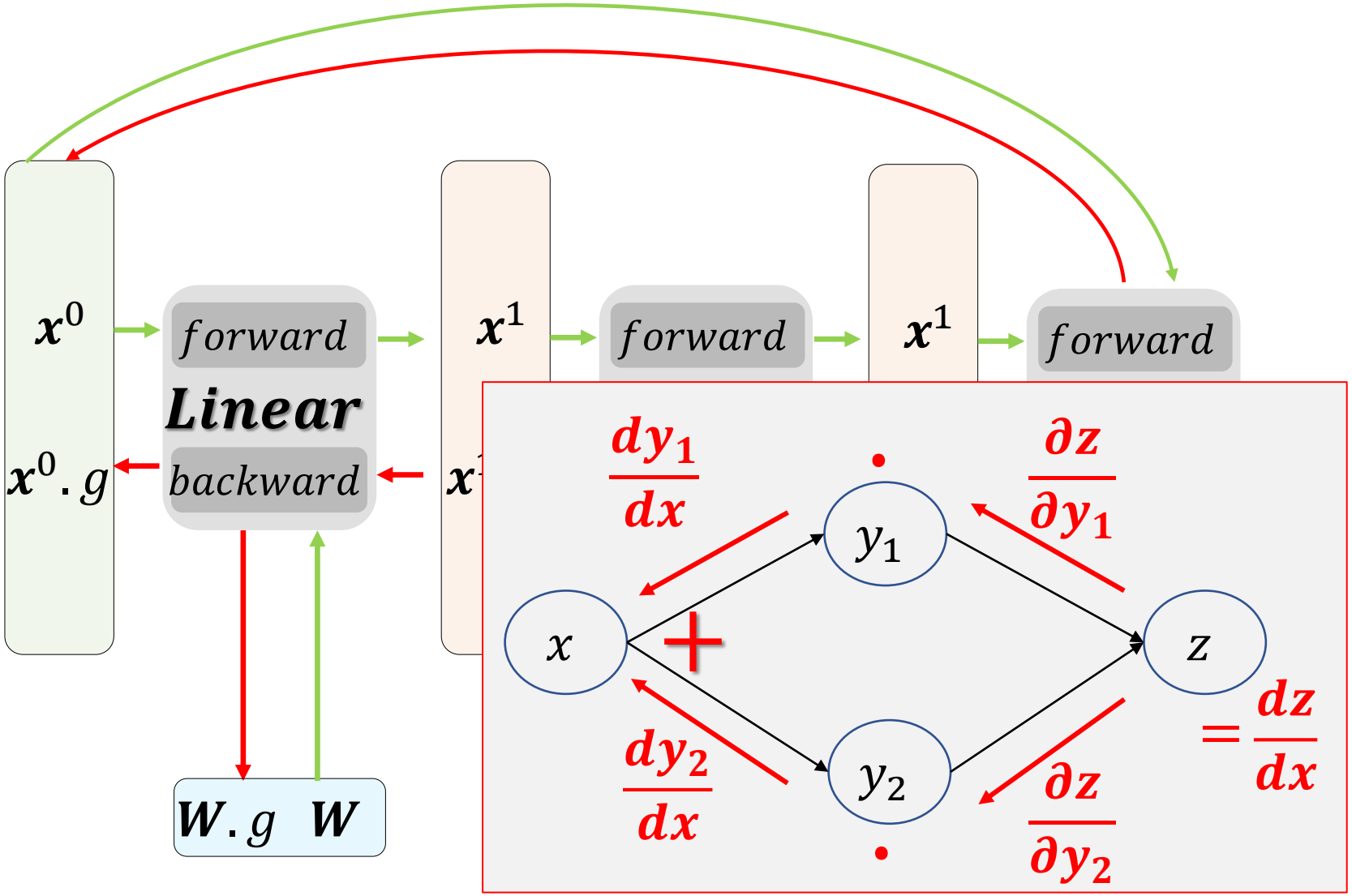

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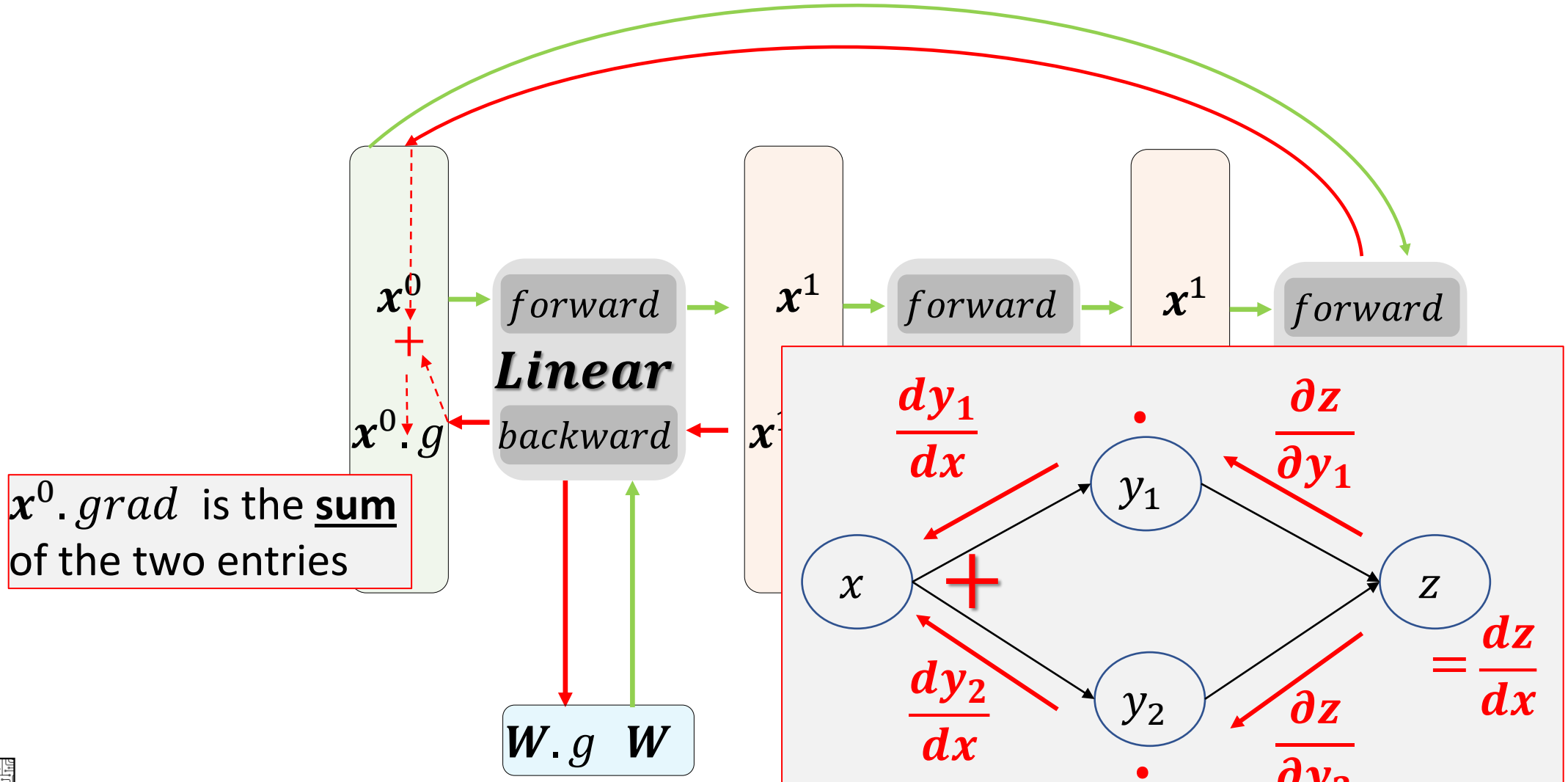

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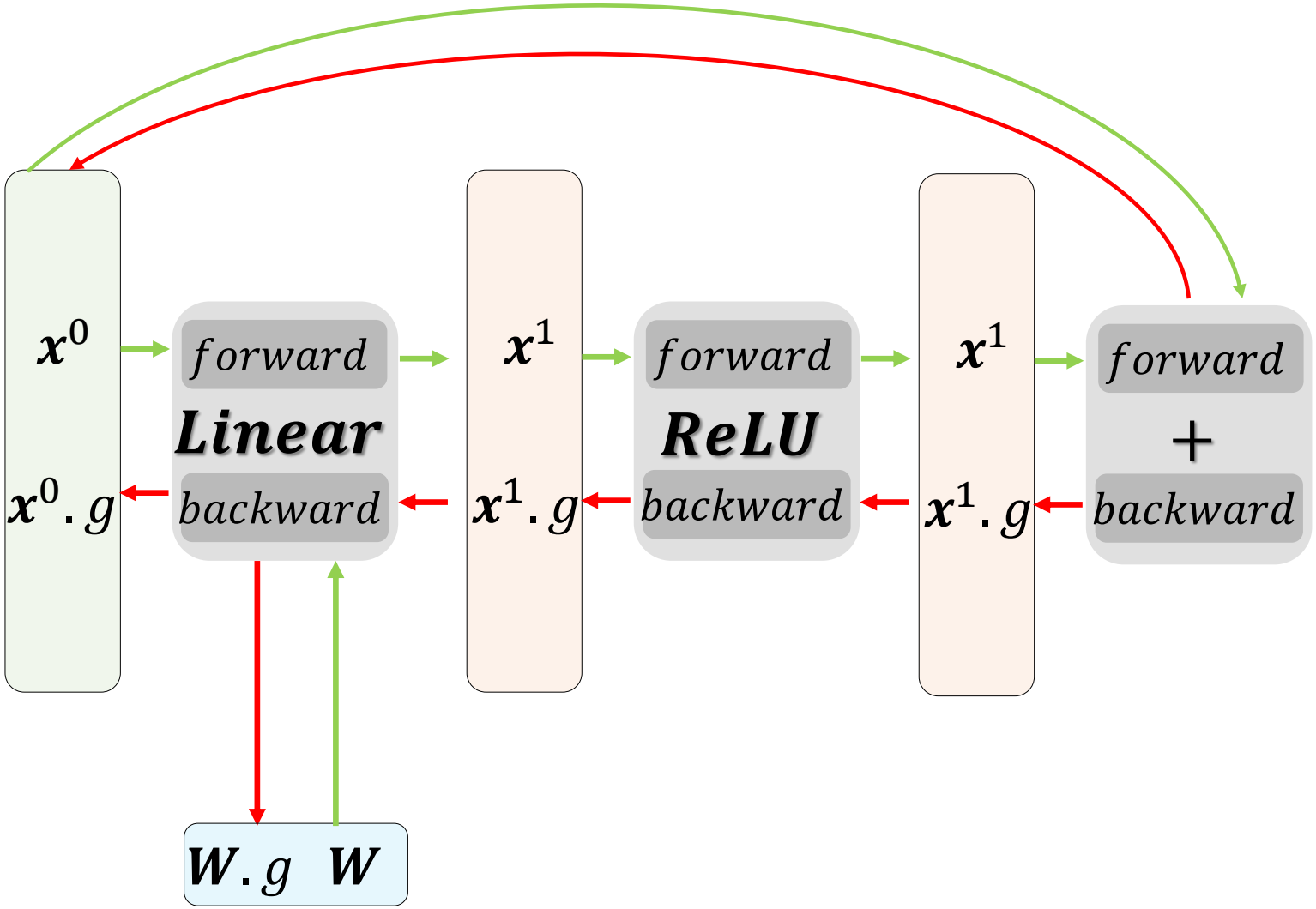
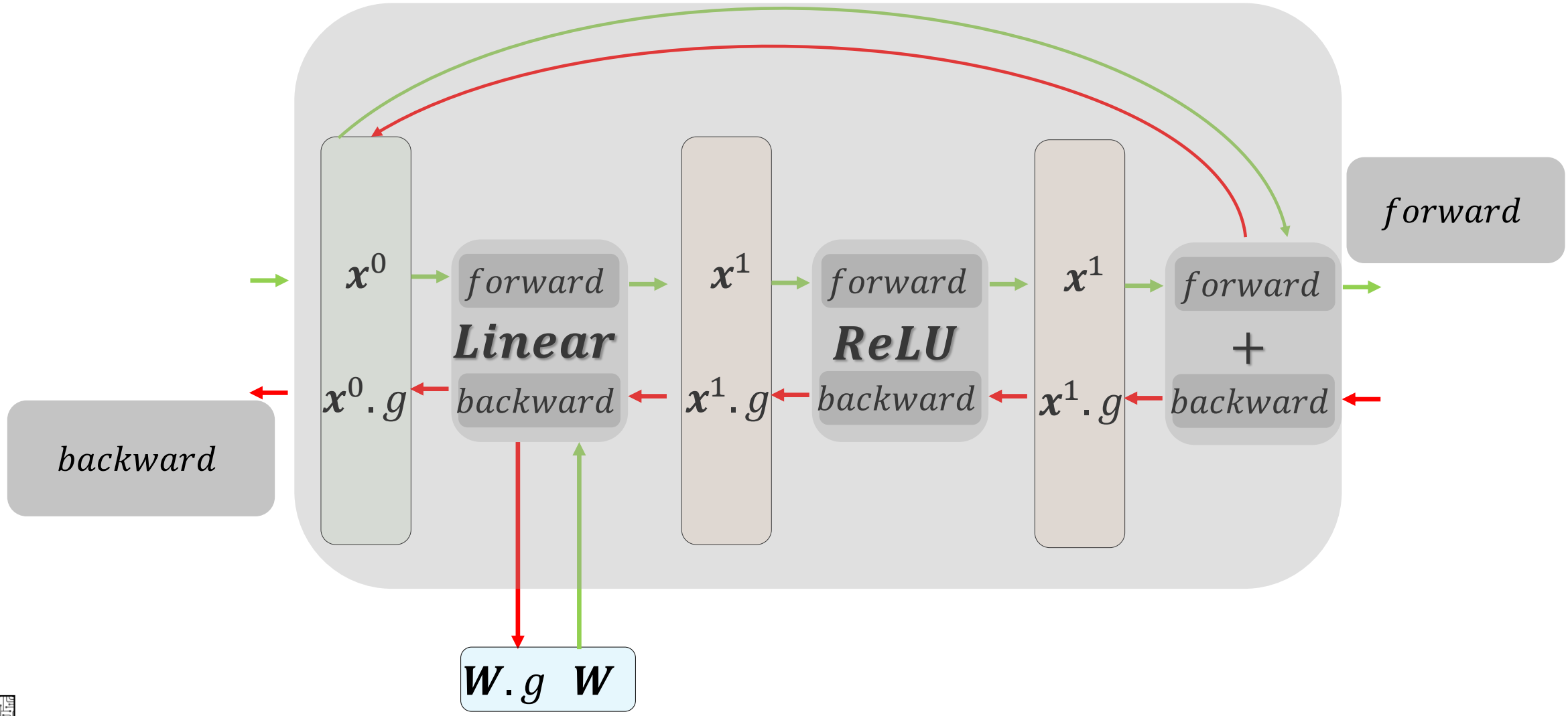


BTW : You can backprop any DAG!

BTW2: Layers (NN modules) can be nested!



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Yes you should understand backprop

Andrej Karpathy Dec 19, 2016 · 7 min read

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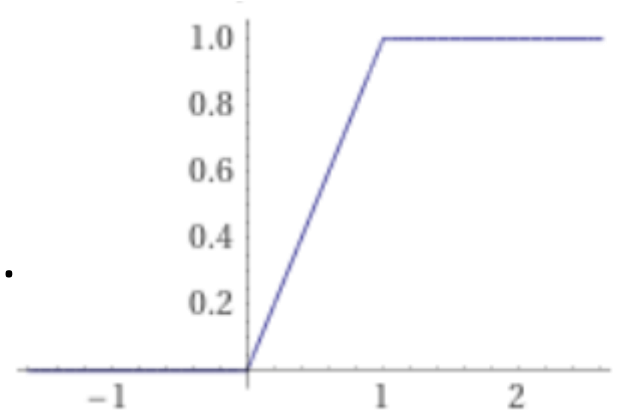

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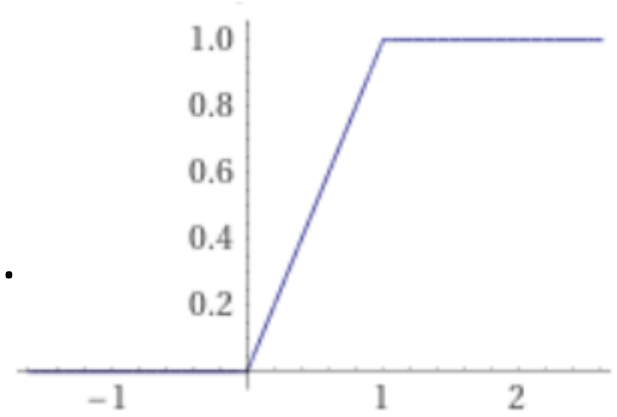

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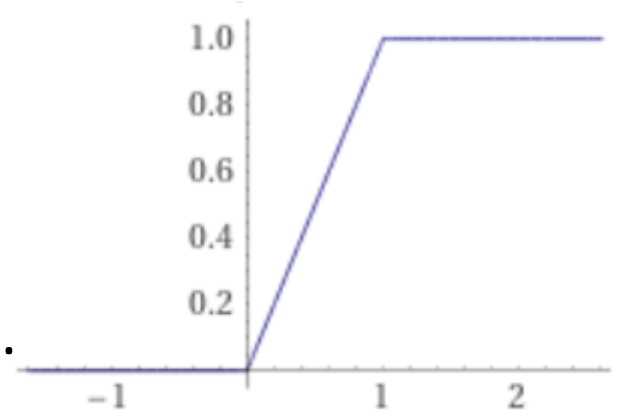

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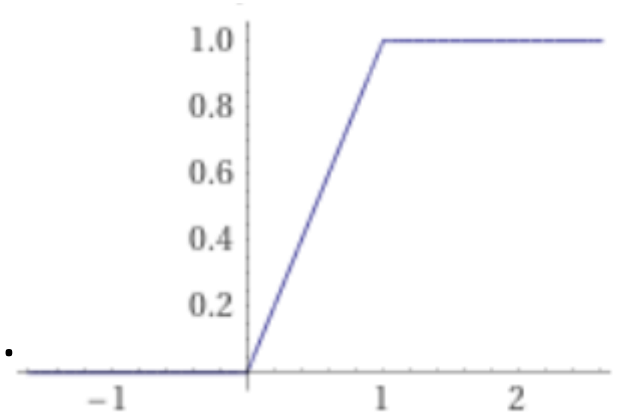

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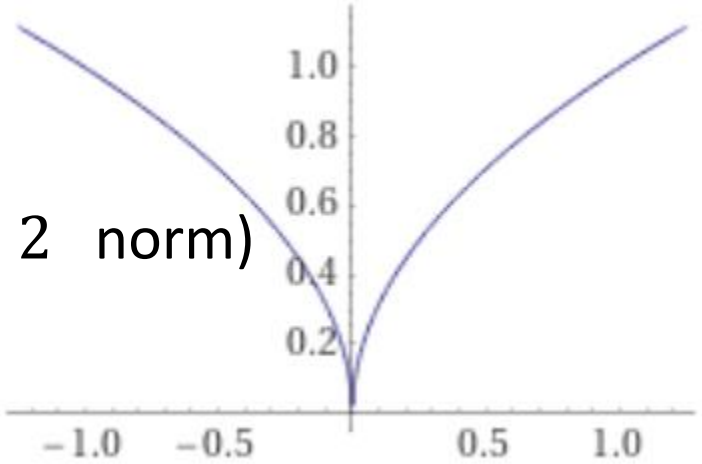

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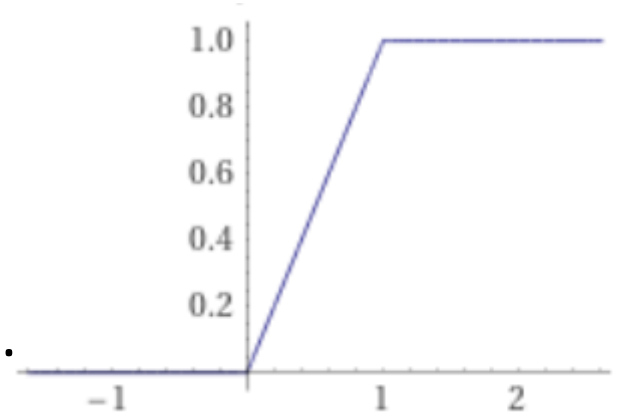

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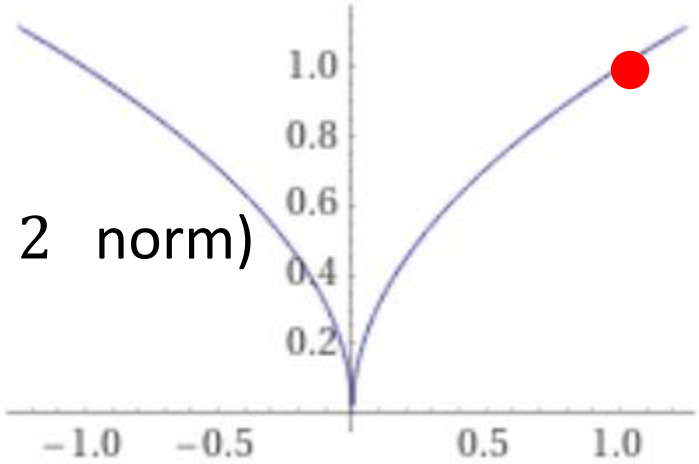

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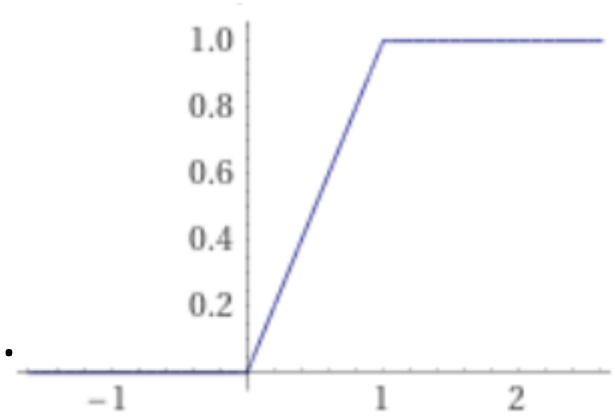

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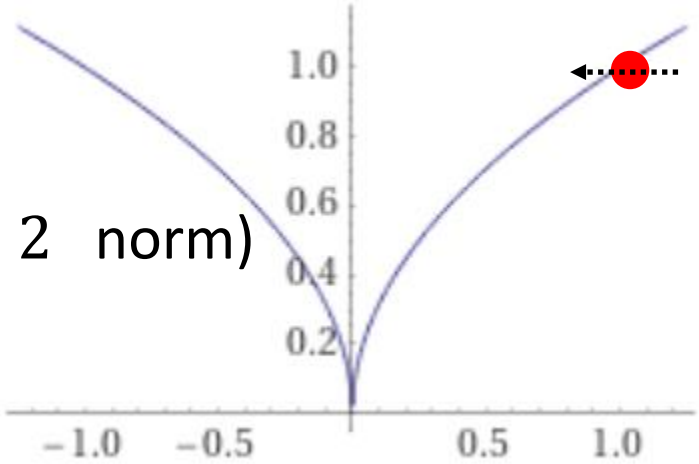
    - Ideas how to solve this?

  - Square root loss (L 1 2 1 2 1 2 norm)



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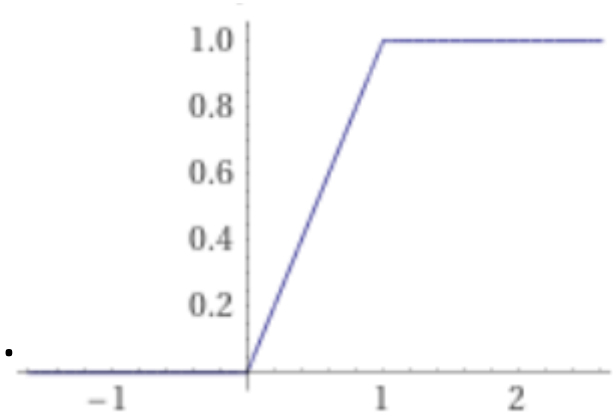

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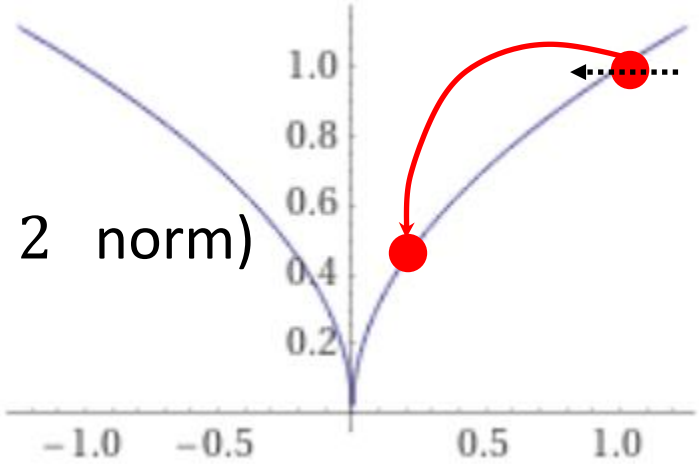

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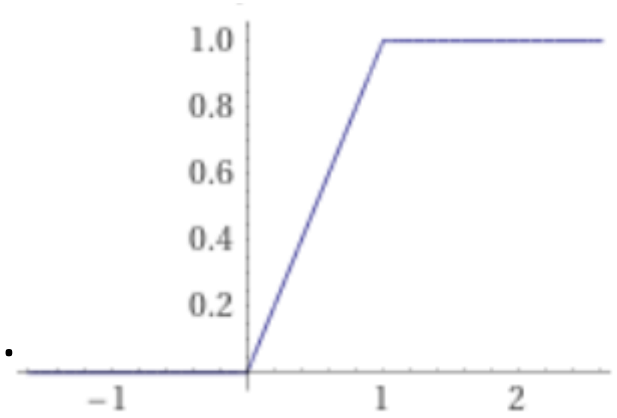

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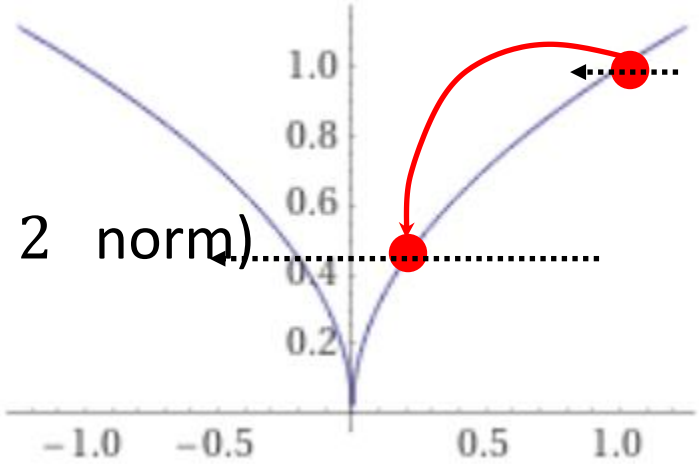

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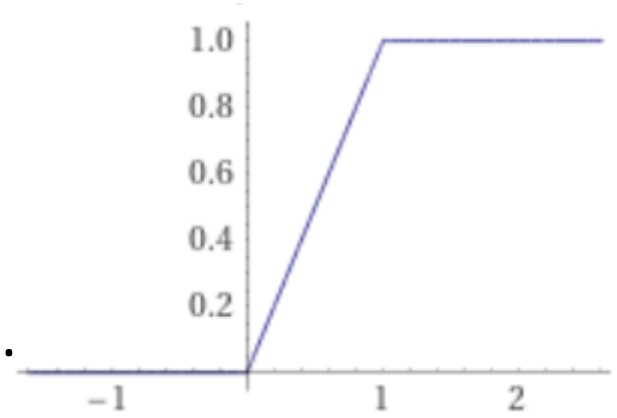

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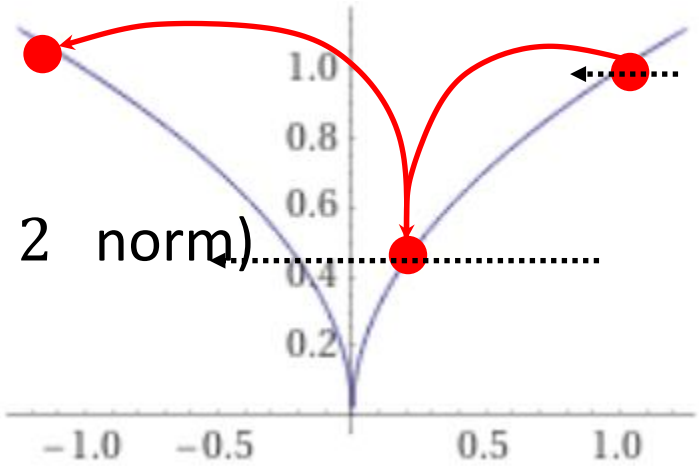

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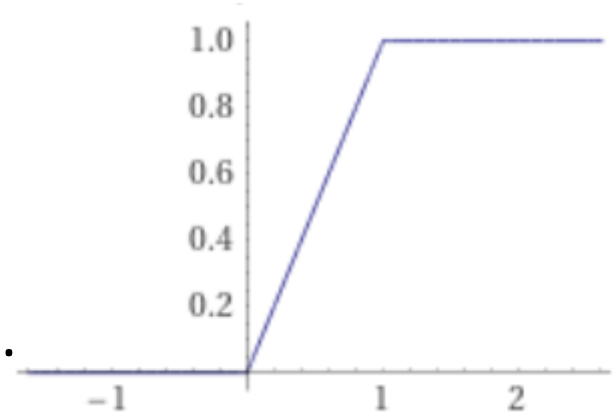

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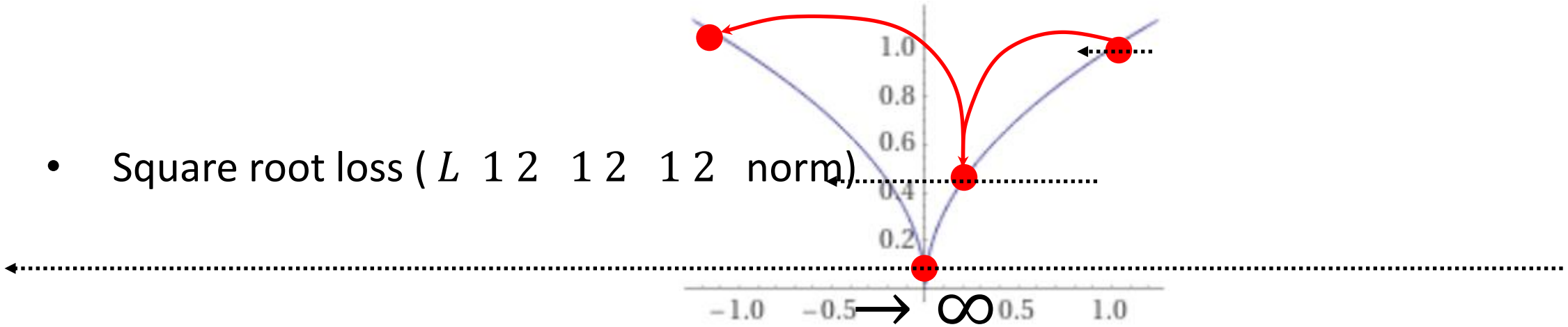

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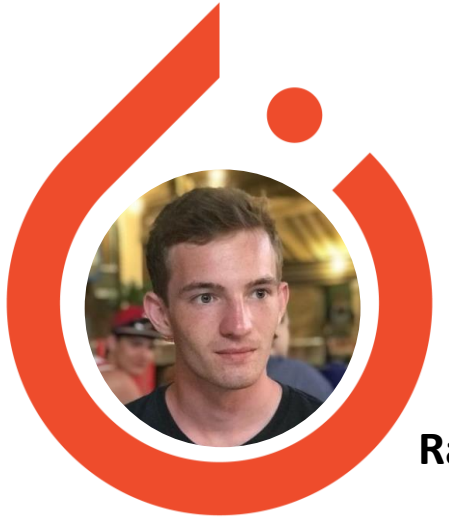
Be creative,
but always watch your back(prop)!

...

http://playground.tensorflow.org

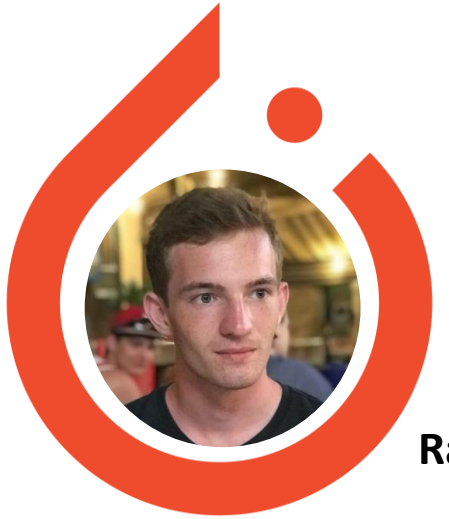
This week's tutorial:



Intro to PyTorch

Rafail Fridman

This week's tutorial:



Intro to PyTorch

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Next week's lecture:

(Me Again ☹)

Convolutional Neural Networks