Lecture 18 Implicit neural representations Neural rendering

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Last time

Deep image (implicit) prior for inverse problems

• Constructing an <u>implicit prior</u> by neural network

 $\min_{x} \|d(x) - \hat{x}\|$
s.t. x is an output of CNN



Last time computer graphics and rendering

The process of generating a photorealistic image from a 3D model





Last time Rendering equation (global illumination) James Kajiya, 1986 **Reflection equation (direct light)**

- Computing reflection equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing reflected radiance from surfaces
- So we have to compute another integral, we have exactly the same equation
- Rendering equation is recursive

DL4CV Weizmann

Slide by Lior Yariv

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i)f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$



Material covered today

 <u>Neural</u> rendering (Deep-based computer graphics)

Deep image or video generation approaches that enable explicit or implicit control of scene properties such as illumination, camera parameters, pose, geometry, appearance and semantic structure

State of the Art on Neural Rendering, A. Tewari et al., 2020



Material covered today

- Implicit neural scene representations
 A network can parametrize
 - Geometry
 - 3D volumes
 - Continuous functions

Why not utilizing geometry of a scene by an explicit representation? Why is it less beneficial to employ explicit representation in neural nets?



Based on

- 1. The ECCV 2022 Tutorial Neural Volumetric Rendering for Computer Vision
- 2. In particular, slides by Matt Tancik and Ben Mildenhall



Geometry

Scene representation

Explicit (discretization of the object geometry)

- triangle (polygon) mesh
- voxels
- point cloud





Geometry Mesh representation



Slides on explicit geometry by Matt Tancik







Target Geometry









Compute Gradients







Compute Gradients













Compute New Error



Target Geometry



















Gradient Based Optimization









Geometry Voxel representation







Initialized Grid







Initialized Grid











Target Geometry





Repeat







Reconstruction







Reconstruction





Geometry Representations

Implicit (continuous) representations



Mesh Representation

Small memory footprint Hard to optimize



Voxel Representation

Easy to optimize Large memory footprint



Geometry Implicit Scene representation

Implicit representations

• algebraic surfaces

How to represent a general shape with implicit functions?

complicated to tailor algebraic expressions, that will fit general shapes

More expressive implicit representations

- level set $f: \mathbb{R}^3 \to \mathbb{R}$, f(x, y, z) = 0
- signed distance function





Geometry Implicit shape representation



Surface represented implicitly $s = \{x \in \mathbb{R}^3 | f(x) = 0\}$



Geometry Implicit shape representation







Signed Distance Function (SDF)



Geometry Implicit shape representation

Eikonal equation $\|\nabla f(\mathbf{x})\| = 1, \mathbf{x} \in \Omega$ $f(\mathbf{x}) = 0, \mathbf{x} \in \partial \Omega$



Signed distance function (SDF)



Implicit representation Properties

- continuous representation
- can represent arbitrary topology at arbitrary resolution
- not limited by excessive memory requirements
- geometric quantities, e.g., normals
- blend well with deep learning techniques How?



Implicit <u>neural</u> representations

[Park et al. 2019, Chen & Zhang 2019, Mescheder et al. 2019, Atzmon et al. 2019]

Theorem (Universality).

Any watertight piecewise linear surface can be exactly represented as the neural level set *S* of MLP with ReLU activations.

$$S = \{x | f(x; \theta) = 0\}$$

After training, the obtained weights in the neural net actually represent the shape, in an implicit way.





How to learn implicit <u>neural</u> representations?

Surface represented implicitly

$$S_{\theta} = \{ \boldsymbol{x} | f(\boldsymbol{x}; \theta) = 0 \}$$

How to learn implicit neural representations?

- Full 3D supervision
- Raw data (weak supervision)





Learning implicit representation Full 3D supervision

- Representing the 3D geometry as the decision boundary of a classifier that *learns* to separate the object's inside from its outside
- After *training* the weights of the neural net represent the surface
- This yields a continuous implicit surface representation
- At inference, queries of 3D points, allows to construct watertight meshes, by using marching cubes algorithm




Occupancy Networks, Mescheder et al, 2019





Occupancy Networks, Mescheder et al, 2019



Occupancy network.

Learning non-linear function

$$f_{\theta} \colon \mathbb{R}^3 \to [0,1]$$

Input: $p \in \mathbb{R}^3$ Output: probability of occupancy

The decision boundary, $f_{\theta}(\mathbf{p}) = \tau$, $(\tau = 0.5)$, represents the surface of the reconstructed shape

Occupancy Networks, Mescheder et al, 2019



(Recap)

Occupancy networks, Mescheder et al., 2019



- Full 3D supervision of the occupancy function is needed
- After *training* the weights of the neural net represent the surface
- At inference, queries of 3D points, allows to construct watertight meshes, by using marching cubes algorithm
- Caveat. Full 3D supervision is complicated and expensive



By weak supervision, from the raw data

Point clouds



- given an input point cloud $\chi = \{x_i\}_{i \in I} \subset \mathbb{R}^3$
- our goal is to compute θ
- $f(x; \theta)$ is approximately the signed distance function to a plausible surface \mathcal{M} defined by χ
- without any additional supervised data preparation





By weak supervision



Eikonal PDE $\|\nabla f(\mathbf{x})\| = 1$ $f(\mathbf{x}) = 0, \mathbf{x} \in \Omega$

Implicit geometric regularization (IGR) by Gropp, Yariv, Haim, Atzmon and Lipman 2020



By weak supervision

Eikonal PDE

 $\|\nabla f(\boldsymbol{x})\| = 1$

 $f(\mathbf{x}) = 0, \mathbf{x} \in \Omega$



Signed distance function (SDF)



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By weak supervision



Implicit geometric regularization (IGR) by Gropp, Yariv, Haim, Atzmon and Lipman 2020





設定では Maic DL4CV Weizmann





DL4CV Weizmann

$$loss(\theta) = \sum_{i \in I} |f(x_i; \theta)|^2 + \lambda \mathbb{E}_x (||\nabla_x f(x; \theta)|| - 1)^2$$

vanish Eikonal



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Weak supervision

Implicit geometric regularization (IGR), Gropp et al., 2020





Weak supervision

Implicit geometric regularization (IGR), Gropp et al., 2020 Inductive bias

Theorem (Convergence and linear reproduction)

Gradient descent of the linear model with random initialization converges with probability 1 to the reproducing plane

$$\operatorname{loss}(\theta) = \sum_{i \in I} (w^T x_i)^2 + \lambda (||w||^2 - 1)^2$$





By weak supervision, from the raw data

Point clouds



Images



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Neural rendering

W/AI(



Slide by Lior Yariv

Neural rendering

• Learning from raw data (weak supervision)





• Building (implicit) neural representation of the scene





Slides on volume rendering formulation by Ben Mildenhall



computing color along rays through 3D space

What color is this pixel?



using a neural network as a scene representation, rather than a voxel grid of data





continuous, differentiable rendering model without concrete ray/surface intersections





Neural rendering



Want to know how ray interacts with scene



Neural rendering - surface vs. volume rendering



Surface rendering — loop over geometry, check for ray hits



Neural rendering - surface vs. volume rendering



Volume rendering — loop over ray points, query geometry



NeRF

Representing Scenes as **Ne**ural **R**adiance **F**ields for View Synthesis By Mildenhall, Srinivasan, Tancik, Barron, Ramamoorth and Ng, 2020



Representing Scenes as Neural Radiance Fields for View Synthesis By Mildenhall, Srinivasan, Tancik, Barron, Ramamoorth and Ng, 2020



A NeRf stores a volumetric <u>scene representation as the weights of an MLP</u>, trained on many images with known pose



Slides on NeRF are based on slides of Yoni Kasten and Dolev Ofri



The scene is represented by MLP <u>Input:</u> spatial location (x, y, z) and viewing direction (θ, ϕ) <u>Output:</u> volume density (opacity), radiance emitted at direction (θ, ϕ) at point (x, y, z)



<u>Inference:</u> render new photorealistic images from the learned scene



New views are rendered by integrating the density and color at regular intervals along each viewing ray (volume rendering)



Training

Objective: reconstruct all training views by volume rendering

Multiview Images of a single scene



Camera poses



Training

reconstruct all training views by differentiable volume rendering







Training Loss

Simulate the rendering of a learned neural scene representation in a differentiable way, and minimize:



WAIC



Neural volume rendering

Neural volume rendering refers to methods that generate images by tracing a ray into the scene and taking an integral over the length of the ray

A neural network (MLP) encodes a function from the <u>3D coordinates</u> on the ray to quantities like <u>density</u> and <u>color</u>, which are integrated to yield an image

Two key properties:

- Integration over the ray
- Coordinate-based scene representation



Scene representation





Slide credit: Jon Barron's talk



Scene representation



 σ (spatial location) c (spatial location, viewing direction)

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Scene representation



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Volume rendering



r(t) – camera ray r(t) = o + td σ – volume density



Volume rendering



r(t) – camera ray r(t) = o + td σ – volume density




Volume rendering



r(t) – camera ray r(t) = o + td σ – volume density



Volume rendering formulation



Scene is a cloud of tiny colored particles



Volume rendering formulation





Volume rendering formulation

What does it mean for a ray to "hit" the volume?



This notion is *probabilistic:* chance that ray hits a particle in a small interval around t is $\sigma(t) dt$.

 σ is called the "volume density"



Volume rendering formulation

Probabilistic interpretation



To determine if t is the *first* hit along the ray, need to know T(t): the probability that the ray makes it through the volume up to t. T(t) is called "transmittance"



Volume rendering formulation

Probabilistic interpretation







Volume rendering formulation

Calculating T given σ





Volume rendering formulation



Finally, we can write the probability that a ray terminates at t as a function of only the density σ

 $P[\text{first hit at } t] = P[\text{no hit before } t] \times P[\text{hit at } t]$ $= T(t)\sigma(t)dt$ $= \exp\left(-\int_{t_0}^t \sigma(s)ds\right)\sigma(t) dt$



Volume rendering formulation

Expected value of color along ray

This means the expected color returned by the ray will be

 $\int_{t_s}^{t_e} T(t)\sigma(t)c(t)dt$

Note the nested integral!



Volume rendering formulation

Approximating the integral



Approximate the nested integral,

splitting the ray up into *n* segments with endpoints $\{t_1, t_2, ..., t_{n+1}\}$ with lengths $\delta_i = t_{i+1} - t_i$



Volume rendering formulation

Approximating the integral

$$\int_{t_s}^{t_e} T(t)\sigma(t)c(t)dt \approx \sum_{i=1}^n \int_{t_i}^{t_{i+1}} T(t)\sigma_i c_i dt$$



We assume volume density and color are roughly constant within each interval



Volume rendering formulation

Approximating the integral

$$\int_{t_s}^{t_e} T(t)\sigma(t)c(t)dt \approx \sum_{i=1}^n \int_{t_i}^{t_{i+1}} T(t)\sigma_i c_i dt$$

<u>Caveat:</u> piecewise constant density and color **do not** imply constant transmittance T(t)!

Important to account for how early part of a segment blocks later part when σ_i is high



We assume volume density and color are roughly constant within each interval



Volume rendering formulation

Evaluating T for piecewise constant density σ

For
$$t \in [t_i, t_{i+1}]$$
, $T(t) = \exp\left(-\int_{t_1}^{t_i} \sigma_i ds\right) \exp\left(-\int_{t_i}^{t} \sigma_i ds\right)$

We need to evaluate at continuous *t* values that can lie *partway through* an interval





Volume rendering formulation

Evaluating T for piecewise constant density σ

For
$$t \in [t_i, t_{i+1}]$$
, $T(t) = \exp\left(-\int_{t_1}^{t_i} \sigma_i ds\right) \exp\left(-\int_{t_i}^{t} \sigma_i ds\right)$







Volume rendering formulation

Evaluating T for piecewise constant density σ

For
$$t \in [t_i, t_{i+1}]$$
, $T(t) = \exp\left(-\int_{t_1}^{t_i} \sigma_i ds\right) \exp\left(-\int_{t_i}^{t} \sigma_i ds\right)$

"How much light is blocked partway through the current segment?"







Volume rendering formulation

Approximating the integral

$$\int T(t)\sigma(t)c(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i c_i dt = \sum_{i=1}^{n} T_i \sigma_i c_i \int_{t_i}^{t_{i+1}} \exp\left(-\sigma_i (t-t_i)\right) dt$$

$$=\sum_{i=1}^{n}T_{i}c_{i}(1-\exp(-\sigma_{i}\delta_{i}))$$



Volume rendering formulation

Connection to material opacity

$$= \sum_{i=1}^{n} T_i c_i (1 - \exp(-\sigma_i \delta_i))$$

segment
opacity α_i

$$T_i = \exp\left(-\sum_{j=1}^{i-1} \sigma_i \delta_i\right) = \prod_{j=1}^{i-1} (1-\alpha_j)$$

$$\text{Color}_{\text{ray}} = \sum_{i=1}^{n} T_i \alpha_i c_i$$



Volume rendering formulation



How much light is transmitted earlier along ray:

$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$

How much light is contributed by ray segment *i*:

$$\alpha_i = 1 - \exp(-\sigma_i \delta_i)$$









Sampling along the ray

Sparse uniform sampling

 \rightarrow Low accuracy





Sampling along the ray

Dense uniform sampling

 \rightarrow Inefficient

Uniform sampling: free space and occluded regions that do not contribute to the rendered image are still sampled equally





Fine and coarse sampling along the ray



Non-uniform samples



$$(x, y, z, \theta, \phi) \rightarrow \square \rightarrow \widehat{C}_{f}, \sigma$$

$$F_{\Theta f}$$
Fine NeRF

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Nerf

Fine and coarse sampling along the ray

Train two networks

$$(x, y, z, \theta, \phi) \rightarrow \square \rightarrow \hat{C}_{c}, \sigma$$

$$F_{\Theta c}$$
Coarse NeRF

$$(x, y, z, \theta, \phi) \rightarrow \square \rightarrow \hat{C}_{f}, \sigma$$

$$F_{\Theta f}$$
Fine NeRF

$$Loss = \sum_{r \in \mathcal{R}} \left(\left\| \hat{C}_{c}(r) - C(r) \right\|_{2}^{2} + \left\| \hat{C}_{f}(r) - C(r) \right\|_{2}^{2} \right)$$





Ablation study



Ground Truth

Complete Model

No View Dependence No Positional Encoding



Challenge

Positional encoding





Basri et al., NeurIPS 2019

Spectral Bias

FC network fits the lower frequency component of the target function faster than the higher frequencies

How to get MLPs converged faster on high-frequency target functions?

Tancik et al., NeurIPS 2020





Implicit image representation





Positional encoding







Positional encoding

Introducing positional encoding



$$^{*}\gamma(x) = (\sin(2^{0}\pi x), \cos(2^{0}\pi x), \dots, \sin(2^{L-1}\pi x), \cos(2^{L-1}\pi x))$$











Target Image





Target Image







NeRF Synthetic scenes



NeRF Real scenes


Nerf





NeRF No positional encoding

NeRF With positional encoding

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Importance of positional encoding





With positional encoding

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WAI

NeRF

Summary

- Novel view synthesis by volume rendering (ray integration)
- Coordinate-base scene representation
- The viewing direction is taken into account
- Encoding the scene in the MLP weights



NeRF

Drawbacks / Future directions

- Trained per scene, not generalizable
- Limited by the dense cover of the scene
- Glossy/transparent surfaces are not modelled well
- The surface geometry is not characterized well by the density σ



Neural rendering

- Representing the surface itself, why?
- Volume rendering or estimation of volume density does not admit accurate surface reconstruction



Volume density thresholds of NeRF



UNISURF: Unifying Neural Implicit Surfaces and Radiance Fields for Multi-View Reconstruction, Oechsle et al., 2021

Next time, by Lior Yariv

Neural Surface Rendering and Reconstruction

