<미적분>

23. 정답 ③

$$\lim_{x\to 0}\frac{\ln(1+3x)}{\ln(1+5x)}=\frac{3}{5}$$

24. 정답 ②

$x=\ln(t^2+1)\,,\,y=\sin\pi t$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\frac{\pi\cos\pi t}{1}}{\frac{3t^2}{t^3+1}}=\frac{-\pi}{\frac{3}{2}}=-\frac{2\pi}{3}$$

25. 정답④

$g(f(x))=x\;,\;g'(f(x))=\dfrac{1}{f\,'(x)}$

$$\int_1^a\frac{1}{g'\,(f(x))f(x)}dx=\int_1^a\frac{f\,'(x)}{f(x)}dx=$$

$$=[\ln|f(x)|]_1^a=\ln f(a)-\ln f(1)=\ln\frac{f(a)}{8}=2\ln a+\ln(a+1)-\ln 2$$

$f(a)=4a^2(a+1)\quad,\;f(2)=48$

26. 정답 ③

$$\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}}(1-2x)\cos dx=[(1-2x)\sin x-2\cos x|]_{\frac{3\pi}{4}}^{\frac{5\pi}{4}}=2\sqrt{2}\pi-\sqrt{2}$$

27, 정답 ①

$f(t)=\dfrac{e^{-\alpha}+e^t}{\alpha}=-e^{-\alpha}$ ---(1)

$f(a)=-e\sqrt{e}=-e^{\frac{3}{2}}$ 이므로

$t=a,\,\alpha=-\dfrac{3}{2}$ 이다.

$f(t)=\dfrac{e^{\frac{3}{2}}+e^t}{-\frac{3}{2}}$ 에서 $f\,'(t)=-\dfrac{2}{3}e^t$, $f\,'(a)=-\dfrac{2}{3}e^a$

--(1)에서 $e^t=-(\alpha+1)e^{-\alpha}=\dfrac{1}{2}e^{\frac{3}{2}}$

$f'(t) = -\frac{2}{3}e^{t}$, $f'(a) = -\frac{2}{3}e^{a} = -\frac{1}{3}e^{\frac{3}{2}} = -\frac{1}{3}e\sqrt{e}$

28. 정답 ②

$x<0.\ f(x) = -4xe^{4x^2}$

$f'(x) = -4e^{4x^2} - 32x^2e^{4x^2} = -4(1+8x^2)e^{4x^2} < 0$

감소함수

$h(t) = k - 2g(t)$

평행이동으로

$\therefore f(g(t)) = f(h(t))$ $\quad g(t) = x$ 라두면

$f(x) = f(k-2x)$ $\quad k-2x = s > 0.$

$x = \frac{k-s}{2}$

$f\left(\frac{k-s}{2}\right) = f(s)$

$\therefore f(s) = -4\cdot\left(\frac{k-s}{2}\right)\cdot e^{(k-s)^2} = 2(s-k)\cdot e^{(s-k)^2}$

$\therefore f(x) = 2(x-k)e^{(x-k)^2}$

$\int_0^7 f(x)\,dx = \int_k^7 f(x)\,dx = \int_k^7 2(x-k)\cdot e^{(x-k)^2}dx$

$= \left[e^{(x-k)^2}\right]_k^7 = e^{(7-k)^2} - 1 = e^4 - 1$

$k=5$ $\quad \therefore f(x) = 2(x-5)e^{(x-5)^2}$ $\quad \therefore \frac{f(9)}{f(8)} = \frac{4}{3}e^7$

29. 정답 162

$a_n = ar^{n-1}$ $\quad -1<r<1$, $\quad b_n = bs^{n-1}$ $\quad -1<s<1$

$$\sum_{n=1}^{\infty} a_n b_n = \left(\sum_{n=1}^{\infty} a_n\right)\cdot\left(\sum_{n=1}^{\infty} b_n\right)$$

$$\frac{ab}{1-rs} = \frac{a}{1-r} \times \frac{b}{1-s}$$

$\therefore\ 1-rs = 1+rs-r-s \quad \therefore\ 2rs = r+s$ (1)

대입

$$3\times \sum_{n=1}^{\infty} |a_{2n}| = 7\times \sum_{n=1}^{\infty} |a_{3n}|$$

$$3(|a_2|+|a_4|+|a_6|+\cdots) = 7(|a_3|+|a_6|+|a_9|+\cdots)$$

i) $a<0,\ r>0$

$$3(-ar-ar^3-\cdots) = 7(-ar^2-ar^5-\cdots)$$

$$\frac{3(-ar)}{1-r^2} = 7\times\frac{-ar^2}{1-r^3} \Rightarrow 3(1+r+r^2) = 7r(1+r)$$

$4r^2+4r-3=0 \quad (2r-1)(2r+3)=0$

$r=\frac{1}{2}$

ii) $a>0,\ r<0$

$$3(-ar-ar^3-ar^5-\cdots) = 7(ar^2-ar^5+ar^8-\cdots)$$

$$\frac{3(-ar)}{1-r^2} = \frac{7ar^2}{1+r^3}$$

$\therefore\ 4r^2-4r-3=0$

$(2r+1)(2r-3)=0$

$r=-\frac{1}{2} \Rightarrow (1)\ s=\frac{1}{4}$

$$\sum_{n=1}^{\infty}\frac{b_{2n-1}+b_{3n+1}}{b_n}$$

$$=\sum_{n=1}^{\infty}\left(\left(\frac{1}{4}\right)^{n-1}+\left(\frac{1}{64}\right)\cdot\left(\frac{1}{16}\right)^{n-1}\right)$$

$$=\frac{1}{1-\frac{1}{4}}+\frac{1}{64}\times\frac{1}{1-\frac{1}{16}} = \frac{81}{60} = S$$

$\therefore\ 120S = 162$

30. 정답 125

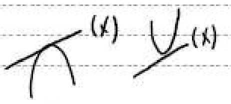
$$f'(x) = |\sin x|\cos x \begin{cases} \sin x\cos x & (\sin x \geq 0) \\ -\sin x\cos x & (\sin x < 0) \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{2}\sin^2 x + C_1 & (\sin x \geq 0) \\ -\frac{1}{2}\sin^2 x + C_1 & (\sin x < 0) \end{cases}$$

$$h(x) = \int_0^x (f(t) - g(t))\,dt$$

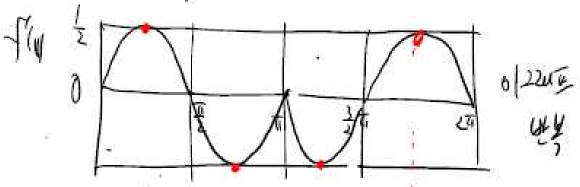
$h'(x) = f(x) - g(x) = 0$ 이고 부호 변화

접선

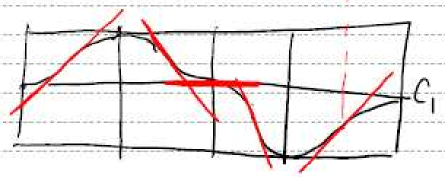


즉 변곡접선 ⇒ 변곡점 해석 문제

$$f'(x) = \begin{cases} \frac{1}{2}\sin 2x & (\sin x \geq 0) \\ -\frac{1}{2}\sin 2x & (\sin x < 0) \end{cases}$$



$$f(x) = \begin{cases} \frac{1}{2}\sin^2 x + C_1 & (\sin x \geq 0) \\ -\frac{1}{2}\sin^2 x + C_1 & (\sin x < 0) \end{cases}$$



$a_1 = \frac{\pi}{4}$ $a_2 = \frac{3}{4}\pi$ $a_3 = \pi$ $a_4 = \frac{5}{4}\pi$ $a_5 = \frac{7}{4}\pi$ $a_6 = 2\pi$

$\therefore (a_6 - a_2) \times \frac{100}{\pi}$

$\frac{5}{4}\pi \times \frac{100}{\pi} = 125.$