

Collected Homework Assignment – sequences and recursion – due Monday, December 5

Note: all terminology and notation used in this assignment is defined in the “sequences” handout on the class web page, as well as in recent class meetings.

1. List the first eight terms of the arithmetic sequence whose first two terms are 5, 9.

5, 9, __, __, __, __, __, __

2. List the first eight terms of the geometric sequence whose first two terms are 48, 24.

48, 24, __, __, __, __, __, __

3. Find the first eight terms of the sequence whose first two terms are 1, 3, which satisfies the following two-term recursion rule:

$$x_n = x_{n-1} + 2x_{n-2}.$$

(Hint: the third term is $3 + 2 \times 1 = 5$; the fourth term is $5 + 2 \times 3 = 11$.)

1, 3, 5, 11, __, __, __, __

4. Figure out the next three terms in each of the following sequences. Explain your reasoning for each.

a. 1, 3, 7, 15, 31, __, __, __

b. 1, 3, 6, 10, 15, __, __, __

c. 8, 12, 18, 27, _____, _____, _____

d. 2, 9, 11, 20, 31, __, __, __

5. Let t_n stand for the number of ways to write a rhythm whose length is n beats using only quarter notes (1 beat) and/or eighth notes ($1/2$ beat).

For example: the only 1-beat rhythms we can write under this rule would be either a single quarter note, or two eighth notes. Since there are two ways to write a rhythm whose length is 1 beat, the value of t_1 is 2. In other words, $t_1 = 2$.

a. Find the values of t_2 and t_3 by listing all acceptable 2-beat and 3-beat rhythms, respectively. (Hint: your answer for t_3 should be between 10 and 20.)

b, (Optional – challenge question) Based on your observations for t_1 , t_2 , and t_3 , see if you can find a recursion rule to predict how the sequence t_n will continue when n is greater than 3. See if your rule works when $n = 4$, by finding all acceptable 4-beat rhythms. (Hint: your answer for t_4 should be between 30 and 40.)