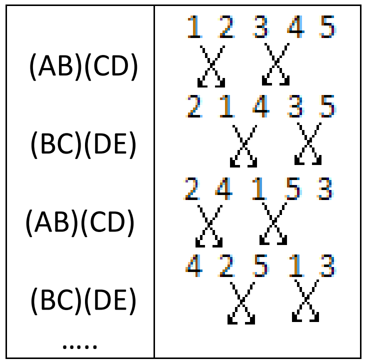
Math 105 Collected Homework Exercises: Due Thursday, April 13

Permutations & Change Ringing

1. For 5 “bells” – numbered 1,2,3,4,5 – alternate the permutations (ab)(cd) and (bc)(de) repeatedly. How many *different* rearrangements (including 1,2,3,4,5 itself) can you get using these two sequences of swaps? (Note: make sure not to count 1,2,3,4,5 twice!) List them.

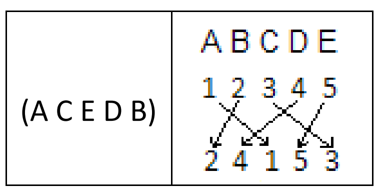
   To get you started, here are the first few rearrangements you from this process:



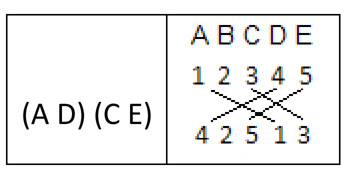
   …continue the list until you return to 1 2 3 4 5, at which point the list will begin to repeat. You can stop at this point!

2. For each rearrangement you ended up with in your list, find the permutation (written using cycle notation) which would have given you that rearrangement, starting from the first line, all in one step.

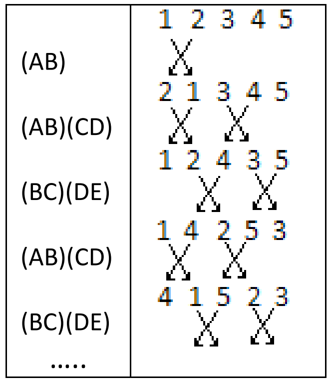
   For example: The third rearrangement in the list above is 2 4 1 5 3.
   To get from **1 2 3 4 5** to **2 4 1 5 3** all in one step, you would use the permutation (acedb):



   One more example: The fourth rearrangement in the list above is 4 2 5 1 3.
   To get from **1 2 3 4 5** to **4 2 5 1 3** all in one step, you would use the permutation (ad)(ce):

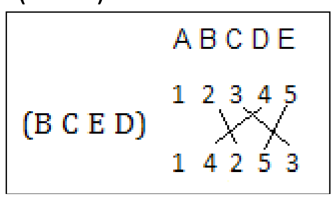


For the next problem, we'll repeat #1 and #2 – but, before starting, *first* apply the swap (ab). So, you'd start with…



3. a) List the different rearrangements you obtain in this way, starting with 2 1 3 4 5. How many are there? (Hint: your should get the same number of rearrangements here as you got in problem #1.)

   b) For each rearrangement you ended up with in your list for part (a), find the permutation (written using cycle notation) which would have given you that rearrangement, **starting from 1 2 3 4 5**, all in one step. For example: One of the arrangements in the list above is 1 4 2 5 3. To get from **1 2 3 4 5** to **1 4 2 5 3** all in one step, you would use the permutation (BCED):



Comments: The set of permutations you found in #1 is a *subgroup* of the full group of permutations on five bells. (As discussed in class – and at the beginning of Chapter 5 of the text – there are 5!=120 such permutations in all.) In #3(b), you found one *coset* of this subgroup.

4. Based on your answer to #1, how many cosets should this subgroup have in all? (You don't have to find them all – just predict how many there should be.) Explain your answer.