Math 155, Fall 2016
Test #3 Learning Objectives

Section 6-4: Sampling Distributions & Estimators
(material on means and proportions only; not variances)

- Know the meaning of "point estimator" (e.g., the sample mean is a point estimator for the population mean)
- Know the meaning of "sampling distribution"
    - understand the sample mean as a random variable, $\bar{x}$, from the distribution of all possible sample means of samples of the same size from the same population
    - understand the sample proportion as a random variable, $\hat{p}$, from the distribution of all sample proportions of samples with the same size from the same population
- Be able to find the sampling distribution of a sample proportion, or sample mean, for a very small sample (say n=2 or n=3) from a small population (so that all possible samples can be listed quickly)

Section 6-5: Central Limit Theorem

- Understand that for samples of size $n$ where $n$ is "large (meaning $n > 30$), the sampling distribution of $\bar{x}$ is approximately normal, with mean $\mu$ and standard deviation $\sigma/\sqrt{n}$ (where $\mu$ and $\sigma$ are the population mean and standard deviation, as usual)
- Be familiar with sampling distribution notation:
    - $\mu_{\bar{x}}$ : mean of the sampling distribution for $\bar{x}$
    - $\sigma_{\bar{x}}$ : standard deviation of the sampling distribution for $\bar{x}$
- If you are given the population mean, $\mu$, population standard deviation, $\sigma$, and sample size, $n$, you should be able to use the normal distribution (z-scores) to estimate the probability that the sample mean will fall in a given range

Section 7-2: Estimating a Population Proportion

- Understand the definition of, and correctly interpret, a confidence interval (CI) as an interval of real numbers with which we estimate the actual value of a population proportion.
- Understand and interpret the meaning of "confidence level," the fraction/percentage of the time that confidence intervals will contain the actual population proportion. The confidence level is often written as $1 - \alpha$ (meaning $\alpha$ may be interpreted as the fraction/percentage of the time that our confidence intervals will not contain the actual population proportion)
- Be able to use the standard normal distribution tables (which will be provided) to find critical values for z – in particular, be able to find $z_{\alpha/2}$, the z-value that cuts off a right "tail" of area $\alpha/2$ of the standard normal distribution
- Be familiar with the necessary requirements for constructing a confidence interval (as outlined on p. 330) for the population proportion
- Given a random sample that satisfies the necessary requirements, be able to find a confidence interval for the population proportion with a specified confidence level. The limits of the confidence interval will be $\hat{p} - E$ and $\hat{p} + E$, where $E$ denotes the "margin of error" of the confidence interval. (As per Formula 7-1, $E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$.)

Section 7-3: Estimating a Population Mean

- Understand the definition of, and correctly interpret, a confidence interval (CI) as an interval of real numbers with which we estimate the actual value of a population mean.
- Understand and interpret the meaning of "confidence level"
- Be able to use the Student t-table (which will be provided) to find critical values for t – in particular, be able to identify $t_{\alpha/2}$, the t-value that cuts off a "tail" of area $\alpha/2$ of the t-distribution
- Be familiar with the necessary requirements for constructing a confidence interval (as outlined on p. 344) for the population mean
- Given a random sample that satisfies the necessary requirements, be able to find a confidence interval for the population mean with the specified confidence level. The limits of the confidence interval will be $\bar{x} - E$ and $\bar{x} + E$, where $E$ denotes the "margin of error" of the confidence interval. (Note: $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$)

  *(Note: if the population standard deviation is somehow known, then we replace $t_{\frac{\alpha}{2}}$ with $z_{\frac{\alpha}{2}}$, and s with σ, in the above formula for E; however, this is very rare, and so I won't put any such questions on a test. This applies for Chapter 8 below as well.)*

HYPOTHESIS TESTING

For all material covered in Chapters 8, 9, and 13, the main focus will be on choosing the most appropriate hypothesis test method (based on which requirements are satisfied – e.g., study design population distribution, sample size(s)), correctly stating the null and alternative hypotheses, and correctly interpreting and communicating the results of the test. (For some, but not all, hypothesis test methods, you will need to be able to identify and calculate the test statistic – for example, you will need to be able to compute the test statistic $t$ for a hypothesis test for a single population mean, as in Section 8-4.)

**"Choosing a Hypothesis Test" Handout**: For any question regarding hypothesis tests, use the "Choosing a Hypothesis Test" handout to determine what type of test is appropriate. This handout WILL BE PROVIDED for you during the test (and also during the final exam), so you don't need to memorize it. You do, however, need to understand it, and you'll be expected to apply it correctly to any given hypothesis testing situation.

For ALL types of hypothesis tests, you will be expected to understand, and correctly state or apply, each of the following:

- Null hypothesis ($H_0$) and alternative hypothesis ($H_1$)
- The selected significance level, $\alpha$, for the hypothesis test
- The observed P-value, based on sample data
- The conclusion of the test: reject $H_0$ (if the P-value is less than or equal to $\alpha$), or fail to reject $H_0$ (if the P-value is greater than $\alpha$).

Sections 8-2 and 8-4: Hypothesis Tests; Testing Claims about a Population Mean

- In order to test a claim about a population mean, we must have either a "large" sample size (n>30) or a normally distributed population. In this case, a t-test for the population mean is appropriate. (If not, we use a "non-parametric" test, as indicated in the "Choosing a Hypothesis Test" handout. We have not yet covered these, so they will not show up on the third test, but they may show up on the final exam!)

*Each of the following assumes that we are using a t-test for the population mean.*

- Be able to identify, and correctly state, the null hypothesis, $H_0$, and alternative hypothesis, $H_1$, to test a claim about a population mean.
  Reminders:
  - In a hypothesis test for the population mean, the null hypothesis is always of the form $\mu =$ [hypothesized mean]. The purpose of any hypothesis test is to assess the strength of the evidence in favor of $H_1$, based on the (temporary) assumption that $H_0$ is true
  - The alternative hypothesis is always of one of the following forms:
    - $\mu <$ [hypothesized mean]
    - $\mu >$ [hypothesized mean]
    - $\mu \neq$ [hypothesized mean]
- Given a null hypothesis and an alternative hypothesis, be able to state the meaning of "Type I error" and "Type II error" in non-technical terms (that is, in the context of the original situation, in a way that's relevant to the claim being tested).
  Reminders:
  - A Type I error occurs when we decide to reject $H_0$ in favor of $H_1$, when in reality $H_0$ is true. (This is sometimes called a "false positive.")
  - A Type II error occurs when we decide not to reject $H_0$, when in reality $H_0$ is false. (This is sometimes called a "false negative.")
- Understand the meaning of the stated significance level, $\alpha$, of a hypothesis test. In particular, interpret $\alpha$ as the likelihood of making a Type I error. Understand also that $\alpha$ is always decided on independent of sample data – that is, do not compute $\alpha$. (In a test question, $\alpha$ will be given to you if you are expected to use it for anything.)
- Be able to find the "critical value(s)" of $t$ for your test, based on the stated value of $\alpha$ and on the whether the test is a one- or two-tailed test. (Whether it's a one- or two-tailed test depends on the alternative hypothesis.)
- *If* a sample mean and standard deviation are given, use this information to calculate the test statistic, $t = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}}$, and determine whether this value falls in the critical region bounded by the critical value(s) of t. If it does, we reject the null hypothesis; otherwise, we do not reject the null hypothesis.
- If, rather than sample statistics, an observed "P-value" is given (this is usually the case if we are using technology such as Minitab to handle computations), correctly decide whether or not to reject the null hypothesis by comparing the P-value to $\alpha$.
  Reminder:
  - If the P-value is less than or equal to $\alpha$, we reject the null hypothesis
  - If the P-value is greater than $\alpha$, we do not reject the null hypothesis

- Be able to interpret the P-value as a probability – specifically, it is the probability of observing a result that is at least as extreme (in favor of the alternative hypothesis) as the result that was actually observed, assuming the null hypothesis is true.
- Be able to write the conclusion of the test in non-technical terms – that is, not just "reject the null hypothesis" or "fail to reject the null hypothesis," but say what that means in the context of the original claim.

Sections 9-3 and 13-4: Two independent samples

In both of these sections, we are comparing two populations by selecting a sample from each, in such a way that the samples are "independent." (After Thanksgiving, we will earn about "paired" samples – no such examples will show up on the third test, but they will be on the final.)

For these two sections, you will NOT be asked to compute any test statistics or find critical values! Instead, we'll assume that Minitab is used to perform all relevant calculations. (You won't actually use Minitab during the test; instead, Minitab output will be given to you, and your job will be to interpret it.) Your job for these problems will be the following:

- Identify the correct hypothesis test to use, depending on which requirements are satisfied (according to the "Choosing a Hypothesis Test" handout)
- Correctly state the null and alternative hypotheses
- Interpret the result of the test (P-value from Minitab), and determine the correct conclusion of the test by comparing the P-value to the selected significance level, $\alpha$
- Write the conclusion of the test in non-technical terms

COMMENTS: There is a lot of material on this test, and some of the types of question types covered can be time-consuming. Also, a lot of the material covered here is more about critical thinking (e.g. interpretation of concepts, correct decision-making) than about calculation. For these reasons, I plan to include a few multiple-choice questions on this test. For multiple-choice questions, I may or may not require you to explain your choice – please read the instructions carefully to be sure.

After Thanksgiving break, we'll cover several more hypothesis test methods as well that cover other situations not yet considered. (You can get a "preview" of these by reading the rest of the "Choosing a Hypothesis Test" handout.) For most of these (the "sign test" is a possible exception), we will leave the test statistic computations entirely to Minitab, and your job will be to correctly choose the right test to use, set up hypotheses, and then interpret and communicate the results of the test, as outlined above.