MATH 210, TEST #1
SEPTEMBER 25, 2015

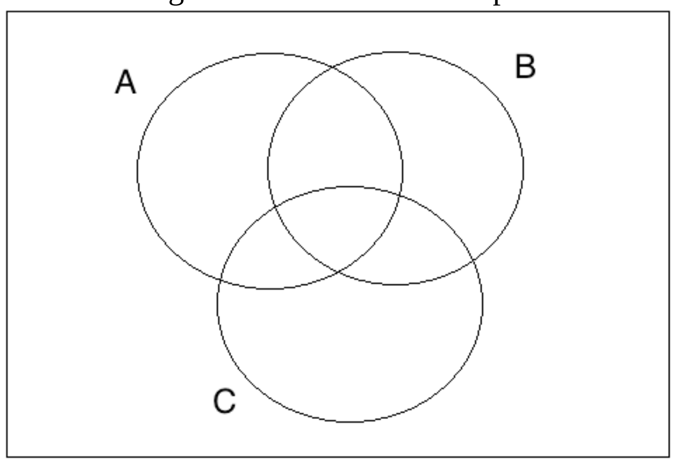
Read all instructions carefully. Use the space provided on each page to write your work and answers. If you need more space than what is provided, please use the back of the page rather than another sheet of paper.

For each question, clearly state your answer, and – unless instructed otherwise – explain (through words, diagrams or both) how you arrived at your answer. Answers without sufficient justification, even if correct, will not receive full credit. (If you're not sure you're showing enough work and/or explaining sufficiently, you may *ask* about this during the test.)

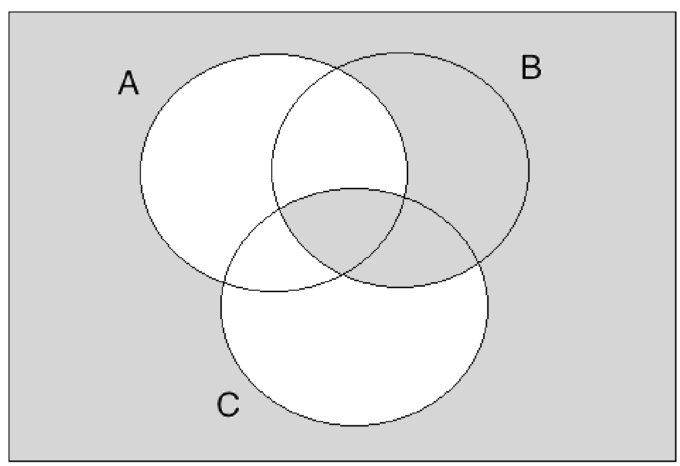
1. Assume that A, B and C are sets. Solve each of the following problems. No additional work or explanation is required for the problems on this page.

a) Describe, in words, the set $(A \cap B') \cup (B \cap C)$. Your written description may not include any of the words "union," "intersection," or "complement."

b) Shade in the region of the Venn diagram below which corresponds to the set $(B \cap C)'$.



c) Use set operations (union, intersection, complement) to describe the set that corresponds to the shaded region in the Venn diagram below.



2. Use a truth table to determine whether the proposition

$$\big((\boldsymbol{p} \to \boldsymbol{q}) \land (\boldsymbol{p} \lor \boldsymbol{q})\big) \leftrightarrow \boldsymbol{q}$$

is a theorem. Explain your conclusion (write at least one sentence of explanation).

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3. Use an *element chasing* argument to prove: $(\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{B}' \cup \boldsymbol{C})' \subseteq \boldsymbol{B}$.

Hint: Notice that this is a *subset* proof, *not* a set equality proof – this is intended to save you some time!

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4. Do **ONE** of the following; pick the **one** you feel more comfortable with. If you try both, clearly indicate which **one** you want to have graded. (You won't get extra credit for doing both!)

a. Use appropriate Boolean identity laws to prove the theorem:

$$\boldsymbol{\overline{x}y + (\overline{z + x}) = \overline{x}(y + \overline{z})}.$$

Show all necessary steps of your proof. (Hint: You should be able to complete this proof using only those laws you were instructed to memorize – commutative, associative, distributive, and/or De Morgan)

…OR…

b. Use appropriate set identity laws to prove the identity:

$$(\boldsymbol{A}' \cap \boldsymbol{B}) \cup (\boldsymbol{C} \cup \boldsymbol{A})' = \boldsymbol{A}' \cap (\boldsymbol{B} \cup \boldsymbol{C}')$$

Show all necessary steps of your proof. (Hint: You should be able to complete this proof using only those laws you were instructed to memorize – commutative, associative, distributive, and/or De Morgan)

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5. Find the maxterm and minterm expansions for the Boolean function, f, which is defined by the following table:

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

6. Consider the following argument:

PREMISES
Everyone who can drive has a driver’s license.
No toddler has a driver’s license.
Ruby cannot drive.

CONCLUSION
Ruby is a toddler.

a) Use propositional calculus to write the premises and the conclusion of this argument symbolically. (Clearly define each of your “atoms” – don’t make me guess at what your notation means.)

b) Determine whether the argument is valid. Show all necessary work, and explain your conclusion.