Math 201, Test #1B
September 25, 2015

This test covers sections 2.1-2.7 of the textbook.

You may NOT use a calculator or any reference materials during this test.

For each problem, *show your work*, and/or otherwise explain how you got your answer. Correct answers with insufficient justification may not receive full credit, and partial credit for incorrect answers can only be given based on work shown and/or written explanation.

Please write all work and answers on this test, rather than using any separate sheets of paper. If you find that you need more space than what is provided, write any additional work on the back of the page.

1. For this problem,
$$f(x) = \frac{2x^2 - 7x + 3}{x^2 - 9}.$$

Find each limit, if it exists. If a limit does not exist, write DNE.

(a) $\lim_{x \to 3} f(x)$

(b) $\lim_{x \to -3} f(x)$

(c) $\lim_{x \to \infty} f(x)$

(d) $\lim_{x \to \infty} 2^{f(x)}$

2. If an object is dropped from the top of a 75-meter-tall building, its height after t seconds is given by the function $f(t) = 75 - 5t^2$ (meters).

(a) Find the object's average velocity for the time period from $t = 0$ to $t = 3$. (Include the correct units of measurement.)

(b) Find the object's instantaneous velocity at time $t = 3$. (Include the correct units of measurement.)

3. The following diagram shows the graph of $y = f(x)$. The dashed line at $x = 1$ is a vertical asymptote. Each point represented by a dot has integer coordinates.
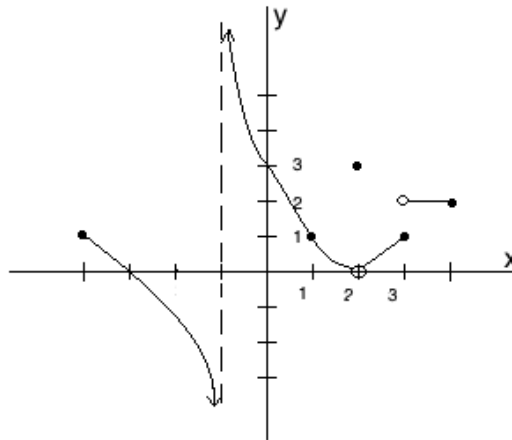
Note: For part (a), no explanations are required. For (b) and (c), write your answers in the space provided; use the back of this page if you need more room.

(a) Find each of the following limits. If an answer does not exist, write "DNE."

$$\lim_{x \to -3} f(x) = \qquad \lim_{x \to -1} f(x) = \qquad \lim_{x \to 0} f(x) =$$

$$\lim_{x \to 1} f(x) = \qquad \lim_{x \to 2} f(x) = \qquad \lim_{x \to 3} f(x) =$$

(b) List all of the values of x at which $f(x)$ is not continuous. Provide a brief (one sentence) explanation for each of your answers.

(c) Find the interval(s) on which $f'(x)$ is positive, the interval(s) on which $f'(x)$ is negative, and the interval(s) on which $f'(x)$ is zero. Briefly (one sentence) explain each of your answers.
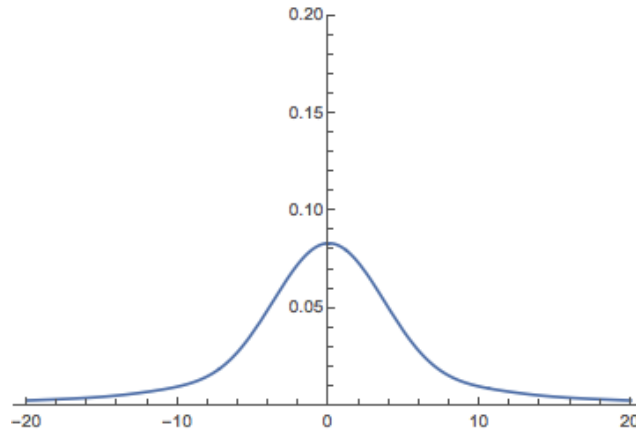
4. Find an equation for the line tangent to the parabola $y = x^2 - 6x + 10$ at the point (6, 10).

5. Use the precise definition of the limit to prove that $\lim_{x \to 1}(7 - 4x) = 3$.

   (Hint: Recall that $\lim_{x \to a} f(x) = L$ if, for any positive number ε, we can find a positive number δ such that $|f(x) - L| < \epsilon$ whenever $|x - a| < \delta$.)

6. The diagram below is a Mathematica graph of the function

$$f(x) = \frac{2\cos(x) + x^2 - 2}{x^4}.$$

(a) From the graph, estimate $\lim\limits_{x \to 0} f(x)$, correct to the nearest hundredth.

(b) Is f continuous at $x = 0$? Why, or why not? Write at least one sentence of explanation.

(c) (Extra credit – optional): Based on your answer to part (a), evaluate

$$\lim_{x \to 0} \frac{2\cos(x) - 2}{x^2}.$$

Explain your answer. (No credit will be given without a valid explanation!)