4.3

2. $L = \{a^n b^k c^n : k \geq n \geq 0\}$

* $\boxed{\text{Assume}}$ L is regular.

By P. Lemma, $\exists\, m \in \mathbb{N}$, $|w| \geq m \rightarrow \exists\, x,y,z$ such that $w = xyz$, $|xy| \leq m$, $|y| \geq 1$, and $xy^iz \in L$ $\forall i \geq 0$.

Let $w = a^m b^m c^m$. Then $w \in L$.

Since a^m is a prefix of w, $|xy| \leq m \rightarrow x, y$ consist entirely of a's.

So, we have $x = a^k$, $y = a^j$, $z = a^{m-(k+j)} b^m c^m$ for some j, k, with $j \geq 1$ (and $k+j \leq m$).

~~and follow~~ and $xy^iz \in L$ for all $i \geq 0$.

So, for example, $xz \in L$ (selecting $i=0$)

$$\rightarrow \underbrace{a^k a^{m-k-j}} b^m c^m \in L$$

$$\underline{a^{m-j} b^m c^m \in L}.$$

must match

But $a^{m-j} b^m c^m$ is not of the form $a^n b^k c^n$,

(fewer a's than c's!) so $\underline{a^{m-j} b^m c^m \notin L}$

Thus, $a^{m-j} b^m c^m$ both is and isn't in L - a $\boxed{\text{contradiction.}}$ *

$\therefore$ Our initial assumption must be false!

$\therefore$ L is not regular. $\square$

#4. Similar to Example 4.7. (Use $w = a^m b^m$, then "pump out" initial string of a's)

#5(e). Recall: L is regular $\rightarrow \overline{L}$ is regular (closure properties)

$\{w : n_a(w) \neq n_b(w)\}$ is the complement of $\{w : n_a(w) = n_b(w)\}$

from #4... So, if the language in 5(e) is regular, then the language in #4 is regular - which it's not, as shown earlier.

$\therefore$ The language in #5(e) cannot be regular

(Note: More generally, a language L is regular iff $\overline{L}$ is regular!)

4.2

#5: $L_1 \subseteq L_2$ iff $L_1 - L_2$ is empty.

$L_1 - L_2$ is regular (see Example 4.1),
an algorithm exists to determine if $L_1 - L_2$ is empty
by Theorem 4.6. Thus, we can use this algorithm
to determine whether $L_1 \subseteq L_2$.

#12 (Discussed in class.)
Since Tail(L) is regular, we can use Theorem 4.7
to determine whether a given regular language, L, is equal to tail(L).

4.1

#10. $S_1 \ominus S_2 = (S_1 \cup S_2) - (S_1 \cap S_2)$.

(or, $(S_1 \cup S_2) \cap (\overline{S_1 \cap S_2})$
or, $(S_1 \cup S_2) \cap (\overline{S_1} \cup \overline{S_2})$)

Since the family of regular languages is closed
under union, intersection, complementation, and difference,
this language is regular if S_1 and S_2 are regular.

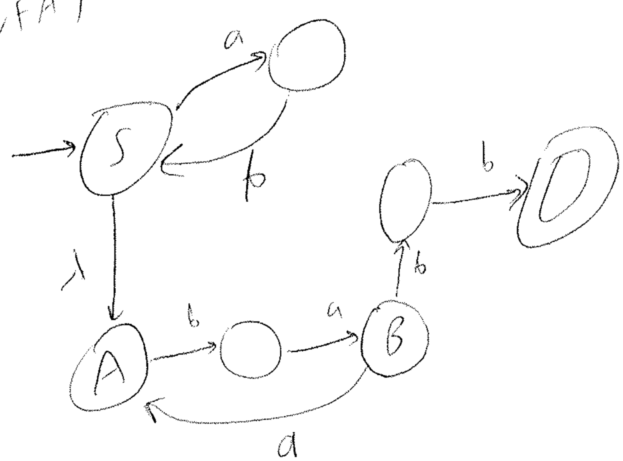
4.1 #12 - Similar to #10

$$cor(L_1, L_2) = \bar{L_1} \cup \bar{L_2} \quad (\text{or } (\overline{L_1 \cap L_2})).$$

#21 - discussed in class.

3.3

#2. (NFA)



#6. $S \to aaaA \mid \lambda$

$A \to bA \mid B$

$B \to ab \mid abS$

$L((aaab^*ab)^*)$