

Math 210, Test #3  
November 20, 2015

For each problem, show your work and/or explain your answer as appropriate, unless the instructions for that problem indicate otherwise. Partial credit may be given for incorrect answers based on work shown and/or written explanations; an incorrect answer with no supporting work or explanation will receive no partial credit.

You may not use your book, notes, or any other reference materials during this test.

The use of a calculator on this test is **not** permitted.

Due to the size of some of the numbers involved, you may leave answers in factored form:

- **If your answer is an integer**, you may write it as in factored form – that is, as a product of two or more integers. For example, if your answer to a question is  $37 \times 6 \times 35$ , you can just leave it in that form, rather than taking the time to simplify it to 7,770.

You may use factorial notation – e.g.,  $5!$  rather than  $5 \times 4 \times 3 \times 2 \times 1$ .

You may use exponent notation – e.g.,  $13^5$  rather than  $13 \times 13 \times 13 \times 13 \times 13$ .

- **If your answer is a rational number**, (when calculating a probability, for example), you may write the numerator and the denominator as products of integers, then cancel any common factors to reduce the fraction to lowest terms. You will not receive full credit for a fractional answer unless it is simplified as much as possible by canceling common factors.

Write all work and answers in the space provided. If you need more space than what is provided, please use the back of the page. (If that's still not enough space, I will provide extra paper on request.)

1. (30 pts) Complete *one* of the following inductive proofs. (Your choice!) If you attempt both, make it clear which *one* you want to have graded. You will *not* get extra credit for trying both.

OPTION #1:

Use induction to prove: for all  $n \geq 1$ ,

$$\sum_{i=1}^n \frac{2}{3^i} = 1 - \frac{1}{3^n}.$$

(In other words, prove that for all  $n \geq 1$ ,  $\frac{2}{3} + \frac{2}{9} + \dots + \frac{2}{3^n} = 1 - \frac{1}{3^n}$ .)

OPTION #2:

Use induction to prove that every power of 6 (6, 36, 216, etc.) has a units digit of 6.

(In other words, prove that for all  $n \geq 1$ ,  $6^n - 6$  is divisible by 10.)

2. (20 pts) Recall: a standard deck contains exactly 4 cards of each “rank,” and exactly 13 cards of each “suit.”

In the game “three-card poker,” a “hand” consists of an unordered selection of three cards from a standard fifty-two card deck.

- (a) (4 pts) How many different ways are there to select a three-card poker hand?

For (b) and (c), assume a three-card poker hand is selected at random. Find the number of different ways in which each event can occur, and find the probability of each event.

- (b) (8 pts) All three cards are of the same suit.

- (c) (8 pts) Exactly one card is a face card. (Note: a “face card” is a Jack, Queen, or King.)

3. (15 pts) Find the number of different rearrangements of each of the following nine-letter words.

(a) (5 pts) BALTIMORE

(b) (10 pts) TENNESSEE

4. (10 pts)

- (a) In the space below, write out the first eight rows of Pascal's Triangle. (Recall that the first row consists of two 1's.)
- (b) Identify (circle) the entry in the triangle that corresponds to  $C(6, 3)$ . Use the combinations formula to verify that this entry in the triangle has the correct value.

5. (10 pts) Use the Binomial Theorem to find the coefficient of  $x^3$  in the expansion of  $(2x + 5)^6$ .

6. (15 pts) There are 110 students in this year's incoming freshman class at Arrakis University. Every one of these incoming freshmen is taking at least one of the introductory courses in architecture, celestial mechanics, or ecology.

There are 60 freshmen taking architecture, 75 taking celestial mechanics, and 45 taking ecology. Many students are taking more than one of these courses – in particular, 30 freshmen are taking both architecture and celestial mechanics, 25 are taking both architecture and ecology, and 20 are taking both celestial mechanics and ecology.

(a) How many freshmen are taking *all three* introductory classes at the same time?

(b) How many freshmen are taking architecture *or* ecology?

(c) How many freshmen are taking *only* celestial mechanics?