Pythagorean Tuning – Practice Exercises

Recall that a Pythagorean scale consists of some base frequency, as well as the following frequencies which are found by raising/lowering by fifths: R1, R2… R6, and L1, L2… L5.

For the practice exercises below, find the frequencies of the tones in a Pythagorean scale with a base frequency of 288 Hz. (That is, find frequencies that lie in the octave between 288 Hz and 576 Hz.) A few of the needed frequencies are found below to get you started; the answers appear on the last page of this handout.

R1: To find R1, we raise 288 Hz by a perfect fifth: $288 \times \frac{3}{2} = 432$ Hz.
So, R1 = 432 Hz for this example.

R2: To find R2, raise R1 by a perfect fifth: $432 \times \frac{3}{2} = 648$ Hz. But, since $648 > 576$, we will lower 648 Hz by an octave: $648 \div 2 = 324$ Hz. So, R2 = 324 Hz for this example.

Practice Exercise #1: Continue the Pythagorean tuning process to find R3, R4, R5 and R6. The correct answers appear on the next page.

We'll continue by finding the frequencies obtained through lowering by fifths (starting back at the base frequency of 288 Hz)…

L1: To lower by a perfect fifth, we divide by 3/2; this is the same as multiplying by 2/3. So, our result is $288 \div \frac{3}{2} = 288 \times \frac{2}{3} = 192$ Hz. However, this result is lower than our base frequency of 288 Hz, so we will raise it by an octave: $192 \times 2 = 384$ Hz. So, L1 = 384 Hz for this example.

L2: To find L2, lower L1 by a perfect fifth: $384 \div \frac{3}{2} = 384 \times \frac{2}{3} = 256$ Hz. Again, our result is less than the base frequency of 288 Hz, so raise it by an octave: $256 \times 2 = 512$ Hz.
So, L2 = 512 Hz for this example.

Practice Exercise #2: Find L3, L4, and L5. The correct answers appear on the following page.

Practice Exercises #3: Starting from a base frequency of 500 Hz, use the Pythagorean tuning method to find the frequencies you would include in one octave (between 500 Hz and 1000 Hz) of a pentatonic scale. The correct answers appear on the following page.

(Hint: recall that the pentatonic scale includes, in addition to the base frequency, the frequencies that correspond to R1, R2, R3 and R4.)

Practice Exercise #1 Answers:

R3: 486 Hz R4: 364.5 Hz R5: 546.75 Hz R6: 410.0625 Hz

Answers to Practice Exercise #2:

L3: $\frac{1024}{3} \approx 341.33$ Hz L4: $\frac{4096}{9} \approx 455.11$ Hz L5: $\frac{8192}{27} \approx 303.41$ Hz

Note: From the above answers, we find that the twelve-tone Pythagorean scale with a base frequency of 288 Hz would consist of tones with the following frequencies (in order, lowest to highest, each measured in Hz and rounded – when rounding is necessary – to the nearest hundredth):

288, 303.41, 324, 341.33, 364.5, 384, 410.06, 432, 455.11, 486, 512, 546.75, 576

Answer to Practice Exercise #3:

R1: 750 Hz

R2: $\frac{1125}{2} = 562.5$ Hz

R3: $\frac{3375}{4} = 843.75$ Hz

R4: $\frac{10125}{16} \approx 632.81$ Hz

So, the pentatonic scale, with a base frequency of 500 Hz, would consist of tones with the following frequencies (in order, lowest to highest):

500 Hz, 562.6 Hz, 632.81 Hz, 750 Hz, 843.75 Hz, 1000 Hz

Note that these notes fall in the following order: Base, R2, R4, R1, R3, Octave.