Practice problems – Sequences & Recursion

1. Find the next few terms of each of the following sequences. For each, describe the recursive rule for the sequence in words. Then, let x_n denote the nth term of the sequence, and write an equation describing the recursive rule for the sequence.

a. 3, 7, 11, 15, 19, 23, 27, …
b. 2, 5, 11, 23, 47, 95, …
c. 1, 3, 4, 7, 11, 18, 29, …
d. 1, 3, 6, 10, 15, 21, …

2. Write the first ten terms of each sequence, given the recursive rule for the sequence. (Recall: when describing recursive rules in this way, we write "x_n" to stand for the nth term of the sequence.)

a. $x_n = x_{n-1} + 2x_{n-2}$, with $x_1 = 1$ and $x_2 = 1$.
(Hint: this sequence starts with 1, 1, 3, 5, 11, …)
b. $x_n = 3x_{n-1} - 1$, with $x_1 = 1$
(Hint: this sequence starts with 1, 2, 5, 14, …)
c. $x_n = 3x_{n-1} - 1$, with $x_1 = 3$
(Hint: this sequence starts with 3, 8, 23, …
d. $x_n = x_{n-1} + x_{n-2} + x_{n-3}$, with x_1, x_2, and x_3 all equal to 1.
(Hint: this sequence starts with 1, 1, 1, 3, 5, …)

In class, we worked out (or will work out) an example in which we find the number of n-beat rhythms that consist entirely of quarter notes and half notes. The following problems are similar to that example.

3. Let b_n stand for the number of n-beat rhythms we can write under the rule that every note is either a quarter note (one beat) or a dotted half note (3 beats).

a. Write out all possible rhythms for n = 1, 2, 3, 4, 5 and 6. Use your results to find the values of b_1, b_2, b_3, b_4, b_5 and b_6. (Hint: none of these should be a very large number.)
b. See if you can figure out, and explain, a recursive rule for the sequence of numbers b_n. (Hint: the reasoning for this one will be *similar* to the reasoning for the example from class in which we restricted ourselves to only quarter notes and half notes.)

4. Let a_n stand for the number of n-beat melodies we can write with the following rules:

- every note is a quarter note (one beat) or a half note (two beats), and
- every note is a C or a D.

a. Find all possible 1-beat, 2-beat and 3-beat melodies under these rules. (This will give you the values of $a_1, a_2,$ and a_3.)

b. Based on your results from part (a), figure out the recursive rule for the sequence of terms a_n – that is, a rule which will let you predict the values of a_4, a_5, etc. without actually having to find and count all of the possible 4-beat and 5-beat melodies.

c. Once you've found a recursive rule in part (b), explain *why* it works.