Math 105: Music & Mathematics
February 24, 2017
Test #1

For each question, show your work and/or explain your answer. Always write *something* to justify your answer, unless the instructions say otherwise; you will not receive full credit for an answer with no supporting work or explanation, even if it is correct. Also, keep in mind that partial credit (for an incorrect answer) can be given only if your supporting work or explanation is shown.

If you need more space for your work on a problem, please use the back of the page on which the problem appears rather than a separate sheet of paper.

1. For each of the following, circle the correct choice. No further work or explanation is required for this multiple-choice problem. Be sure to read all possible answers before making your selection!

a. This interval has a width of four semitones.

Major Third Perfect Fourth Perfect Fifth Major Sixth

b. This is the ideal (just intonation) frequency ratio of a perfect fifth.

2/1 3/2 5/3 5/4

c. This interval's ideal (just intonation) frequency ratio is 4/3.

Major third Perfect Fourth Perfect Fifth Major Sixth

d. This tuning system has consistent semitones.

Pythagorean Just Intonation Equal Temperament ALL of these

e. In this tuning system, the frequency ratio of an octave is always exactly 2/1.

Pythagorean Just Intonation Equal Temperament ALL of these

f. This describes a standard, modern piano keyboard.

i) A total of 36 keys, including three octaves and five perfect fifths

ii) A total of 64 keys, including five octaves and nine perfect fifths

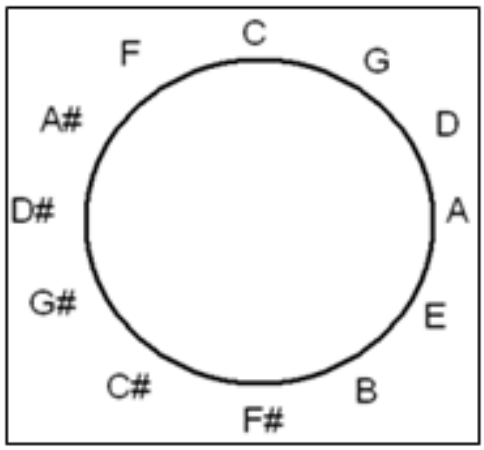
iii) A total of 88 keys, including seven octaves and twelve perfect fifths

iv) A total of 373 keys, including thirty-one octaves and fifth-three perfect fifths

2. Suppose an instrument is tuned using Pythagorean tuning, with a base note D:540 Hz. (Hint: you may wish to make use of the Circle of Fifths diagram at the bottom of this page.)

a. Find the correct frequency for the next higher E. (Your answer should be between 540 Hz and 1080 Hz.)

b. What note would be "L3" in this Pythagorean tuning system? Give the name of the note (A, B, C, etc.) and its frequency in Hertz (a number between 540 and 1080).



CIRCLE OF FIFTHS

3. In a just intonation tuning system with base note C:300 Hz, find each of the following.

a. The correct frequency for the next higher E (between 300 Hz and 600 Hz).

b. The correct frequency for the next *lower* A (between 150 Hz and 300 Hz).

c. The note name (A, B, C, etc.) of the note whose frequency is 1000 Hz.

4. The standard piano tuning uses 12-TET, with base note A:440 Hz. Based on this, find each of the following:

a. The correct frequency for the next higher D (between 440 Hz and 880 Hz).

b. The correct frequency for the next lower G (between 220 Hz and 440 Hz).

c. The note name (A, B, C, etc.) for the note whose frequency is (approximately) 2489 Hz.

5. In just intonation, a “minor sixth” interval has a frequency ratio of 8/5.

a. Find the width of a minor sixth in cents measurement. Round your answer to the nearest whole number of cents.

b. Based on your result for (a), what is the width of a “minor sixth” in semitones? (Round your answer to the nearest whole number of semitones.) Briefly explain your answer.

6. An “augmented fourth” interval has a width of 427 cents.

a. Find the frequency ratio for an “augmented fourth.”

b. If the lower note of an “augmented fourth” has frequency 640 Hz, find the frequency of the higher note.

7. Use continued fractions to find a rational approximation for $\sqrt{7}$ (approx. 2.645751311). Use a list of five whole numbers to find your answer. After you've found your answer, *check* to make sure that it is a good approximation for $\sqrt{7}$.