Math 105: Music & Mathematics
May 5, 2017
Test #3

For each question, show your work and/or explain your answer as appropriate (unless the instructions for that question indicate otherwise). <u>You will not receive full credit for an answer with insufficient supporting work or explanation, even if it is correct</u>. Also, keep in mind that partial credit (for an incorrect answer) can be given only if your supporting work or explanation is shown.

If you need more space for work or explanation than is provided, please use the back of the page rather than a separate sheet of paper.

1. Starting from the ordered list 1 2 3 4 5 6 7…

a. Find the effect of applying the permutation (ACG)(BF)(ED) to this list.

b. Find the effect of applying (ACG)(BF)(ED) again to your result from part (a).

c. Use cycle notation to describe the permutation that rearranges 1 2 3 4 5 6 7 into your answer from part (b) all in one step.

2. A change ringing group produces changes of six bells by repeating a sequence of three permutations. The first several changes they generate using their system are shown in the diagram to the right.

1	2	3	4	5	6
2	1	4	3	6	5
2	4	1	3	5	6
4	2	1	3	6	5
2	4	3	1	5	6
2	3	4	1	6	5
3	2	4	1	5	6

a) What are the three permutations that are being used to generate these changes? Write your answers using cycle notation.

b) Continue the list shown in the diagram by filling in the next three changes that will be obtained if the same pattern is continued.

c) What permutation would rearrange the list in the first row, 1 2 3 4 5 6, into the list in the fourth row, 4 2 1 3 6 5, all in one step? Write your answer using cycle notation.

d) If this same repeating pattern is continued long enough, how many different changes will be generated before repetition occurs? Explain your answer. (Note: do not try to answer this question by actually listing all of the changes; that's not the idea here. Find another way to make this prediction.)

3. For each of the following, all tones are to be selected from the set {**C, D, E, G, A, B**}. (There are six notes in this set). Show your work; make it clear how you are getting your answers. Remember that a <u>melody</u> is an <u>ordered selection</u> of notes.

a) How many ways are there to write a <u>three note melody</u> if no repetition of notes is allowed?

b) How many ways are there to write a <u>four note melody</u> if there are no restrictions on repeating notes?

c) How many ways are there to write a <u>four note melody</u> if repetition is allowed, with the restriction that you can't use the same note twice in a row? (For example, if the first note of the melody is an **A**, then the second note can be anything <u>except</u> an **A**.)

4. DaNizza Pizza (at 1400 South, near campus) has a menu with sixteen different toppings. They allow you to choose three toppings for your pizza.

a. Is the selection of pizza toppings a combination, a permutation, or something else? Briefly explain your answer.

b. How many different ways are there to choose three toppings (from the sixteen available toppings) for your pizza?

c. Two of the sixteen available toppings are pepperoni and spinach. Suppose you decide that you must have pepperoni, but you absolutely don't want spinach. Subject to these restrictions, how many ways are there to choose three toppings for your pizza?

5. Find the number of distinct rearrangements of each of the following words.

a. PERMUTE

b. REARRANGE

6. For this problem, start by writing out the first eight rows of Pascal's triangle (Remember, the first row consists of two 1's.) Then answer the questions that appear below.

a. Write out the first eight rows of Pascal's triangle (the first two rows are provided for you):

$$\begin{array}{ccccc} & 1 & & 1 & \\ 1 & & 2 & & 1 \end{array}$$

b. Circle the number in the above triangle that corresponds to $C(7,3)$. Briefly explain your choice.

c. What is the value of the 4th number in the 25th row of Pascal's triangle? Briefly explain how you came up with your answer. (Note: please do not try to write out the first 25 rows of Pascal's triangle! You don't have time for that, and it shouldn't be necessary.)

7. Find the next three terms of each of the following sequences. For each, please show your work *and/or* briefly explain how you are getting your results.

a. The arithmetic sequence 3, 11, 19, ____________________

b. The geometric sequence 2, 6, 18, ________________________

c. The two-term recursion starting with 1, 2, and with the recursive rule $x_n = 2x_{n-1} + x_{n-2}$

1, 2, 5, 12, _____________________