Math 210, Fall 2015
Collected Homework Assignment #4 – **Due Friday, October 9**

Let A = {1,2,3,4,5}, B={a,b,c,d}, C ={red, orange, purple}

1. For each of the following, draw arrow diagrams for functions f and g that satisfy the given conditions, **if possible**. Or, if you believe no such example exists, explain why. (I'm not looking for any formal proofs – not yet, at least. For each part, just provide a suitable example, *or* write a sentence or two to explain what goes wrong when you try to find a suitable example and are unable to do so.)

a) Functions $f: A \to B$ and $g: B \to C$ such that f is onto, g is *not* onto, and $g \circ f$ is onto.
b) Functions $f: C \to B$ and $g: B \to A$ such that f is one-to-one, g is *not* one-to-one, and $g \circ f$ is one-to-one.
c) Functions $f: A \to B$ and $g: B \to C$ such that g is onto, f is *not* onto, and $g \circ f$ is onto.
c) Functions $f: C \to B$ and $g: B \to A$ such that g is one-to-one, f is *not* one-to-one, and $g \circ f$ is one-to-one.

2. Based on your results from #1, decide which of the following are *probably* theorems, and which ones are definitely *not* theorems. For each one that is not theorem, give a counterexample (this should come from one of your answers to problem #1).

For functions f and g such that the image of f is contained in the domain of g (that is, for which $g \circ f$ is defined)…

a) If $g \circ f$ is onto, then g is onto
b) If $g \circ f$ is onto, then f is onto
c) If $g \circ f$ is 1-1, then g is one-to-one
d) If $g \circ f$ is 1-1, then f is one-to-one

3. Optional: In #2, you should have found that some of the given propositions appear to be theorems. Now, see if you can PROVE them!