Pythagorean Tuning – Practice Exercises

Recall that a Pythagorean scale consists of some base frequency, as well as the following frequencies which are found by raising/lowering by fifths: R1, R2… R6, and L1, L2… L5.

For the practice exercises below, find the frequencies of the tones in a Pythagorean scale with a base frequency of 288 Hz. (That is, find frequencies that lie in the octave between 288 Hz and 576 Hz.) A few of the needed frequencies are found below to get you started; the answers appear on the last page of this handout.

R1: To find R1, we raise 288 Hz by a perfect fifth: $288×\frac{3}{2}=432$ Hz.

So, R1 = 432 Hz for this example.

R2: To find R2, raise R1 by a perfect fifth: $432×\frac{3}{2}=648$ Hz. But, since $648>576$, we will lower 648 Hz by an octave: $648÷2=324$ Hz. So, R2 = 324 Hz for this example.

**Practice Exercise #1:** **Continue the Pythagorean tuning process to find R3, R4, R5 and R6.** The correct answers appear on the next page.

We’ll continue by finding the frequencies obtained through lowering by fifths (starting back at the base frequency of 288 Hz)…

L1: To lower by a perfect fifth, we divide by 3/2; this is the same as multiplying by 2/3. So, our result is $288÷\frac{3}{2}=288×\frac{2}{3}=192 $ Hz. However, this result is lower than our base frequency of 288 Hz, so we will raise it by an octave: $192×2=384 $Hz. So, L1 = 384 Hz for this example.

L2: To find L2, lower L1 by a perfect fifth: $384÷\frac{3}{2}=384×\frac{2}{3}=256$ Hz. Again, our result is less than the base frequency of 288 Hz, so raise it by an octave: $256×2=512$ Hz.
So, L2 = 512 Hz for this example.

**Practice Exercise #2:** **Find L3, L4, and L5.** The correct answers appear on the following page.

**Practice Exercise #3:** **Starting from a base frequency of 500 Hz, use the Pythagorean tuning method to find the frequencies you would include in one octave (between 500 Hz and 1000 Hz) of a pentatonic scale.** The correct answers appear on the following page.

(Hint: recall that the pentatonic scale includes, in addition to the base frequency, the frequencies that correspond to R1, R2, R3 and R4.)

Practice Exercise #1 Answers:

R3: 486 Hz R4: 364.5 Hz R5: 546.75 Hz R6: 410.0625 Hz

Answers to Practice Exercise #2:

L3: $\frac{1024}{3}≈341.33 $ Hz L4: $\frac{4096}{9}≈455.11$ Hz L5: $\frac{8192}{27}≈303.41$ Hz

Note: From the above answers, we find that the twelve-tone Pythagorean scale with a base frequency of 288 Hz would consist of tones with the following frequencies (in order, lowest to highest, each measured in Hz and rounded – when rounding is necessary – to the nearest hundredth):

288, 303.41, 324, 341.33, 364.5, 384, 410.06, 432, 455.11, 486, 512, 546.75, 576

Answer to Practice Exercise #3:

R1: 750 Hz

R2: $\frac{1125}{2}=562.5$ Hz

R3: $\frac{3375}{4}=843.75$ Hz

R4:$\frac{10125}{16}≈632.81 $ Hz

So, the pentatonic scale, with a base frequency of 500 Hz, would consist of tones with the following frequencies (in order, lowest to highest):

500 Hz, 562.6 Hz, 632.81 Hz, 750 Hz, 843.75 Hz, 1000 Hz

Note that these notes fall in the following order: Base, R2, R4, R1, R3, Octave.