Math 160, Test #3B
October 26, 2015

You will need a graphing calculator for this test. (No cell-phone calculators, nor any other devices capable of taking photos or accessing the internet, are allowed.) No other reference materials are allowed.

Read all instructions for each individual problem carefully. For each problem, *show your work*, and/or otherwise explain how you got your answer. Correct answers with insufficient justification may not receive full credit, and partial credit for incorrect answers can only be given based on work shown and/or written explanation.

Please write all work and answers on this test, rather than using any separate sheets of paper. If you find that you need more space than what is provided, write any additional work on the back of the page.

1. Find the derivative (with respect to x) of each of the following functions.

   (a) $\dfrac{1}{(6x+3)^4}$

   (b) $\sqrt{6x^3 - 3x^4}$

2. For this problem, show all necessary work; do *not* use your graphing calculator.

   Let $f(x) = 6x^3 - 3x^4$.

   (a) Find the critical numbers of f.

   (b) Find the x-values of the relative maxima and minima (if any) of f. (If an answer does not exist, write "DNE.")

   Relative maxima:

   Relative minima:

3. For this problem, show all necessary work; do *not* use your graphing calculator.

Let $g(x) = x^2 - \dfrac{6}{x}$.

(a) Find the inflection point of the graph of $y = g(x)$. Find the *exact* x- and y-coordinates.

(b) For what values of x is the graph of $g(x)$ concave up? (Write your answer using interval notation.)

(c) For what values of x is the graph of $g(x)$ concave down? (Write your answer using interval notation.)

4. Recall that elasticity of demand is defined as

$$E(p) = \frac{-pf'(p)}{f(p)},$$

where $x = f(p)$ is the demand function.

For the demand function

$$x = 18 - \frac{p^2}{50},$$

compute the elasticity of demand, and use your result to answer each of the following questions.

(a) Is demand elastic or inelastic when $p = 20$?

(b) For what value of p is demand unitary? Give an exact answer, and also a decimal answer rounded to the nearest cent.

5. Suppose the weekly demand for burritos is given by $p = 7.8 - 0.06x$, where p denotes price per burrito (in dollars) and x is the number of burritos demanded per week at price p.

   (a) Find the revenue function, R. (Hint: recall that revenue is price per unit times number of units sold.)

   (b) Find the marginal revenue function, R'.

   (c) Find the value of $R'(70)$, and interpret the result. (Write at least one sentence, demonstrating that you understand what "marginal revenue" measures.)

6. For this problem, sketch a graph of the function:

$$f(x) = 600x^3 - 300x^4$$

Your graph must clearly show the exact coordinates of all of the following points:

- The y-intercept
- The x-intercepts
- Relative maxima and relative minima
- Inflection point(s)

In addition, the following attributes should be clear from a quick look at your sketch:

- Intervals on which f is increasing or decreasing
- Intervals on which f is concave up/concave down
- The general behavior of f (up or down) as $x \to -\infty$ and as $x \to \infty$

You may use your calculator to help you with the shape of this graph – in particular, you may use the calculator graph to determine the intervals of increase/decrease, the intervals where f is concave up/concave down, and how f behaves as $x \to -\infty$ and as $x \to \infty$, which means it is not necessary to show work for these. However, you *must* show your work for finding the exact locations of the intercepts, extrema, and inflection point(s).

Note: please use plenty of space, both for your work and for your sketch. If you need more space than what is provided below, you may use the next page of this exam (which has been left blank for this purpose) to finish showing your work and/or to draw your sketch.