Math 105: Music & Mathematics

October 25, 2016

Test #2A

You may not use a calculator for this test. (None of the test problems should require a great deal of computation; if you find yourself needing a calculator, you’re probably on the wrong track.)

For each question, show your work and/or explain your answer. Always write *something* to justify your answer, unless specifically instructed otherwise; you will not receive full credit for an answer with insufficient supporting work or explanation, even if it is correct. Also, keep in mind that partial credit (for an incorrect answer) can be given only if your supporting work or explanation is shown.

If you need more space for your work on a problem, please use the back of the page on which the problem appears rather than a separate sheet of paper.

1. For this problem, consider the melody consisting of the notes: **C, E, G, A#, F**

For each variation listed below, find the result of applying that variation to the above melody. Just write each answer as a list of notes, as the melody is written above; you do not need to write the music on a staff.

a) $T\_{2}R$

b) $T\_{2}I$

c) $I\_{G}$ (an inversion centered at G)

2. Simplify each of the following combinations of variations. Write each answer in one of the following forms: $T\_{n}, T\_{n}R, T\_{n}I, or T\_{n}IR,$ with $n$ between 0 and 11. Show your work.

a) $R T\_{6} R T\_{10} R$

b) $T\_{9} I T\_{4} I T\_{2}$

c) $I R T\_{10} I R T\_{5}$

3. Starting with the melody **C, E, D, A** (shown to the right), determine which variation we could use to end up with each of the following results. Write each answer in one of the following forms: $T\_{n}, T\_{n}R, or T\_{n}I.$ Briefly explain each of your answers.



a) F, A, G, D



b) D, G, A, F



c) E, C, D, G

4. Determine whether each of the following sets, with the given operation, is a group. That is, either show it’s a group, or give a specific example showing why it’s not a group.

a) {0, 3, 6} under addition (mod 9)

b) {1, 4, 8} under multiplication (mod 12)

 c) $\{T\_{0}, T\_{3}R, T\_{6}R, T\_{9}R\} $under the usual rules for combining variations

5. The set {1, 2, 4, 7, 8, 11, 13, 14} turns out to be a group under multiplication (mod 15).
(Note: you do NOT need to show that this is a group. Just assume it’s a group, and follow the instructions based on that assumption.)

a) Find $\left〈2\right〉$ - that is, find the cyclic subgroup (of the above group) that is generated by 2.

b) Find all coset(s) of the subgroup from part (a). Explain how you know you’ve found them all.

6. Find each of the following “opposites.” Briefly explain each answer – make it clear that you know what “opposite” actually means in each case.

a) Find the opposite of variation $T\_{8}$

b) Find the opposite of 5 under mod 30 addition
(Your answer should be between 0 and 29)

c) Find the opposite of 5 under mod 12 multiplication
(Your answer should be between 0 and 11)

“Musical Clock”

