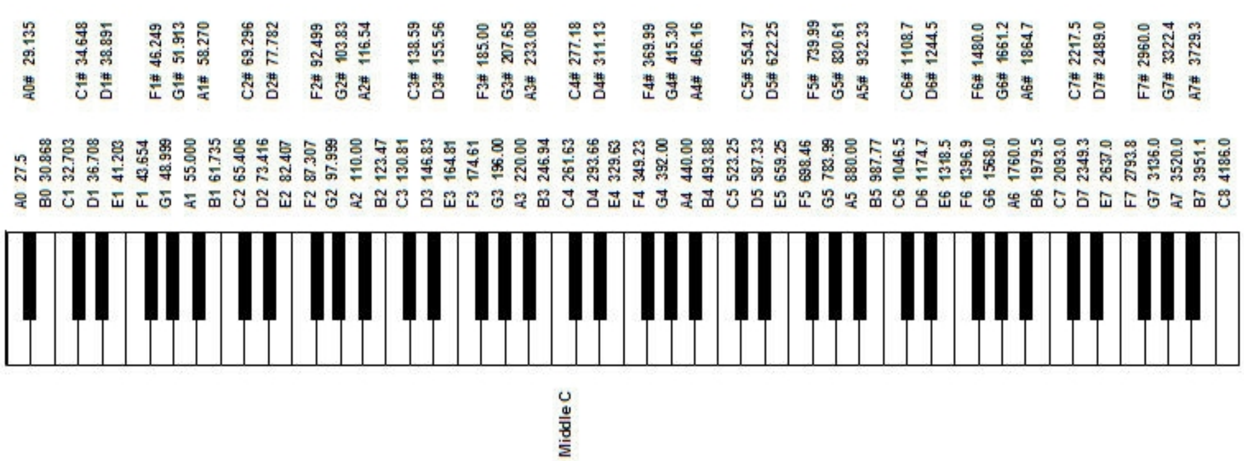
CHAPTER 1 – TUNING SYSTEMS

Section 1.0 – INTRODUCTION

What we perceive as "sound" is actually our sensory interpretation of vibrations in the air around us. Much as our eyes detect light and process this as vision, our ears detect changes in air pressure and process these as sound. In most cases, what we hear is our nervous system's interpretation of oscillations (regular up-and-down alternations) in air pressure. Generally, the magnitude of these oscillations determines the "volume" (loudness) of the sound, and their frequency determines the "pitch" (highness or lowness) of the sound. These frequencies are typically measured in "Hertz" (Hz); one "hertz" is equal to one vibration, or oscillation, per second.

For example, a loud, deep, booming bass – one of those sounds you sometimes feel as much as hear – is caused by an air vibration with relatively high magnitude and low frequency. On the other hand, a very high-pitched sound, such as a piccolo or a bird-call, is caused by a vibration with a much higher frequency. The typical range of human hearing runs from a low of 20 Hz to a high of 20,000 Hz. (This range varies from person to person and tends to narrow with age.) By contrast, a dog's hearing range typically runs from about 65 Hz to 45,000 Hz. This is how "dog whistles" work – they emit a vibration whose frequency is far above the human range, but still well within a dog's range. (Curious about the hearing ranges of other animals? Visit http://soundphysics.ius.edu/?page_id=2657.)

A "tone" is a steady sound with a distinct pitch. More precisely, it is an air vibration of a constant frequency. For example, the "tuning A," which is the tone an oboe plays before a concert to help tune the rest of the orchestra, is tuned to a frequency of exactly 440 Hz. As another example, "middle C," the C near the middle of the piano keyboard, typically produces a tone with frequency 261.63 Hz. As seen in the figure below, listing standard piano key frequencies, the tones emitted by a standard piano range from 27.5 Hz to 4186.0 Hz. Not surprisingly, these frequencies are all well within the typical range of human perception.



An "interval" is made up of two tones that are sounded simultaneously. Some intervals sound better (or nicer, or more concordant) than others, though this is often a subjective judgment.

As noted earlier, a tone is a sound with a constant frequency. Thus, an interval consists of two distinct frequencies. The character of an interval is determined by its "frequency ratio" – that is, by the ratio of the higher frequency to the lower frequency in an interval. In other words, the frequency ratio of an interval is the fraction (or decimal) obtained when the frequency of the higher tone is divided by the frequency of the lower tone.

Certain intervals are generally considered more desirable than others. Listed below are a few of the most fundamental intervals – these are intervals that musicians often want to use when creating music, and thus they are intervals we would typically like to make available when we create a tuning system.

Octave: An interval whose frequency ratio is exactly 2
Perfect Fifth: An interval whose frequency ratio is exactly 3/2
Perfect Fourth: An interval whose frequency ratio is exactly 4/3
Major Third: An interval whose frequency ratio is exactly 5/4
Major Sixth: An interval whose frequency ratio is exactly 5/3

A “tuning system” is a set of frequencies that may be used to produce music. Any tuning system has a tone that’s called the “base tone” or “base frequency;” other tones to be included in the tuning system follow, in some way, from the selection of the “base tone.” (Sometimes this tone is called the “root” rather than the “base” of the tuning system; this vocabulary may vary from one source to the next.)

There are many different ways to create a tuning system; we will explore just a few of these. A primary objective in the development of a tuning system is to “preserve” as many different intervals as possible – that is, to have frequencies that produce intervals with “nice” frequency ratios, such as the fundamental intervals listed above. In particular, every tuning system that we’ll look at will preserve octaves – this means that for every frequency used in a tuning system, the frequency an octave higher (found by doubling the original frequency) and an octave lower (found by dividing the original frequency by 2) will generally be included in that tuning system as well.

EXAMPLE: Suppose we set out to create a tuning system whose “base” tone has a frequency of 450 Hz. This system would also include the tone an octave higher, whose frequency is 900 Hz, since $2\times450 = 900$. In addition, we would keep “doubling” to obtain tones with frequencies of 1800 Hz, 3600 Hz, and so on.
Similarly, this system would include the tone an octave lower than 450 Hz; this is found by dividing, rather than multiplying, by 2, so the frequency of this tone would be 225 Hz. If we keep lowering by octaves (dividing frequencies by 2), we would also include tones with frequencies of 112.5 Hz, 56.25 Hz, and so on.

This process of multiplying or dividing by 2 to add more tones to the tuning system may be continued indefinitely, in theory. In practice, it is continued only as far as practically feasible, depending on the logistics of the instrument being used or built, or ultimately on the range of human hearing. For example, if we’re just building a stringed instrument with a two-octave range, then we might only include tones ranging from 225 Hz to 900 Hz. On the other hand, if we are designing a much larger instrument without range limitations, then we may elect to include one key (or string, or whatever our hypothetical instrument uses to create sounds) for each of the following frequencies (all in Hz): 28.125, 56.25, 112.5, 225, 450, 900, 1800, 3600, 7200, 14400. (Make sure you understand where these numbers are coming from, and also why we elected to stop the process where we did. Hint: recall that the human ear can only detect frequencies within a limited range!)

EXERCISE: An instrument is being designed based on a tuning system with the following specifications.

- The tuning system will include tones with frequencies of 300 Hz, 400 Hz, 500 Hz, and 600 Hz.
- The overall range of this instrument will be from 150 Hz to 2000 Hz. That is, its highest tone will have frequency 2000 Hz, and its lowest tone will have frequency 150 Hz.
- Within the overall range of the instrument (150-2000 Hz), we want to include all tones that can be obtained by raising or lowering an existing tone by an octave. (For example, since 400 Hz is included, we will also include the tone with frequency 800 Hz, since 800 Hz is one octave above 400 Hz.)

Based on these specifications, find the frequencies of ALL tones that MUST be included in this instrument's tuning system. (Hint: you should come up with a set of 12 distinct tones.)

Solutions (with an explanation) appear on the following page. But, try to figure it out yourself first before looking at the solution. (Note – that's good advice for any exercise you see in these notes – try it yourself before reading the answer!)

Since there's still lots of extra space to fill on this page, please enjoy this photo of cats on a piano.

[Image courtesy of pixabay.com]

SOLUTION: The included frequencies (each measured in Hertz) will be as follows:

150	200	250
300	400	500
600	800	1000
1200	1600	2000

Explanation: To see why these frequencies are included, we'll focus on the starting frequencies – 300, 400, 500, and 600 Hz – as our starting points. For each, any new frequency that can be obtained by raising or lowering by octaves (that is, multiplying or dividing the frequency by 2) is to be included in our system. So here's what happens:

- Starting from 300 Hz.
    - Lowering by an octave (dividing by 2) gives us 150 Hz, so we include this in our system. (Lowering 150 Hz by an octave gives us 75 Hz, which is below our minimum limit.)
    - Raising by octaves (multiplying by 2) gives us, successively, 600 Hz, then 1200 Hz, then 2400 Hz, etc. Since our maximum limit is 2000 Hz, we will "keep" the frequencies of 600 and 1200 Hz, but not the higher ones.

      Summary: Our tuning system must include the frequencies 150, 300, 600, and 1200 Hz.

- Starting from 400 Hz.
    - Lowering by octaves (dividing by 2) gives us 200 Hz, 100 Hz, 50 Hz etc.
    - Raising by octaves (multiplying by 2) gives us 800 Hz, 1600 Hz, 3200 Hz, etc.
    - Summary: Our tuning system must include 200, 400, 800, and 1600 Hz. (Again, note that we are only keeping frequencies between 150 Hz and 2000 Hz.)

- Starting from 500 Hz.
    - Lowering by octaves gives us 250 Hz, 125 Hz, 62.5 Hz, etc.
    - Raising by octaves gives us 1000 Hz, 2000 Hz, 4000 Hz, etc.
    - Summary: Our tuning system must include 250, 500, 1000, and 2000 Hz.

- Starting from 600 Hz.
    - Lowering by octaves gives us 300 Hz, 150 Hz, 75 Hz, etc.
    - Raising by octaves gives us 1200 Hz, 2400 Hz, 4800 Hz, etc.
    - Summary: Our tuning system must include 150, 300, 600, and 1200 Hz.
    - Say, that looks familiar! We had already included these frequencies (see 300 Hz above). Why does this duplication occur?

Overall summary: putting all of our results together, we must include the following frequencies (all in Hz): 150, 300, 600, 1200, 200, 400, 800, 1600, 250, 500, 1000, and 2000, which are the frequencies listed at the top of this page.