

MATH 105: COLLECTED HOMEWORK #3

Use the method of “continued fractions” to find rational approximations for each of the following numbers:

1. $\log(50)$

2. $\sqrt{5}$ (that is, $5^{1/2}$)

3. $\log(1.25)/\log(2)$

Use “whole number” lists as shown in the examples. For each of these problems, find three different rational approximations – first use 3 whole numbers (in your “whole number list”), then 4 whole numbers, then 5 whole numbers; this should give you three different answers for each of the numbers to be approximated. Make sure to show your work – that is, show the steps outlined in the above examples, and also check (using your calculator) that each of your answers is “close” to the number you are trying to approximate!

As a hint – to help you make sure you’re on the right track – here are a few answers you should get...

- $\log(50)$ – you should get $5/3$, then $17/10$, then $158/93$ for your answers.
- $\sqrt{5}$ -- you should get $38/17$ for your first answer (using three whole numbers)
- $\log(1.25) / \log(2)$ -- one of your answers should be $19/59$

One more hint: here’s what I mean by “use 3 whole numbers” to find a rational approximation for $\log(50)$:

- Using the method described in this handout, you should get the following “whole number parts” for $\log(50)$: 1, then 1, then 2. So, for your first answer (using three whole numbers), your list of “whole number parts” would be $[1, 1, 2]$.
(Note: you’d just continue the same process to find the fourth whole number, then the fifth...)
- So, running through the second part of the process, we start with 2.
The reciprocal of 2 is $1/2$. Move to the left to find a 1. $1 + 1/2 = 3/2$.
The reciprocal of $3/2$ is $2/3$. Move to the left to find another 1. $1 + 2/3 = 5/3$.
Since we’re at the left end of the list, we’re done at this point. Answer: $5/3$.
Check: $5/3 = 1.66666\dots$, and $\log(50) = 1.69897\dots$, so $5/3$ is a pretty close estimate. The “error” would be $\log(50) - 5/3$, which is about $0.03230\dots$

Or, in “continued fraction” notation...

$$\log(50) = 1 + \frac{1}{1 + \frac{1}{2.321\dots}} \approx 1 + \frac{1}{1 + \frac{1}{2}} = 1 + \frac{1}{3/2} = 1 + \frac{2}{3} = \frac{5}{3}$$