Code : S-02

Register Number							
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I Semester Diploma Examination, November 2007

SCIENCE BOARD

APPLIED MATHEMATICS – I

Time : 3 Hours] **[Max. Marks : 100**

Note : (1) Part – A is *compulsory*.
(2) Answer any ***six*** questions from Part – B.
(3) Answer any ***thirteen*** questions from Part – C.
(4) Answer any ***three*** questions from Part – D.

PART – A

(Compulsory)

$5 \times 1 = 5$

I. Fill in the blanks :

(1) If $\begin{vmatrix} -3 & 2 \\ x & 4 \end{vmatrix} = 0$, then $x =$ _____.

(2) If $\vec{a}$ is unit vector, then $|\vec{a}|$ = _____.

(3) The number of terms in the expansion of $(x^2 + 2)^{14}$ is _____.

(4) If sin θ is positive and cos θ is negative, then θ lies in _____ quadrant.

(5) If $\theta = 45°$ then cos 2θ = _____.

PART – B

II. Answer any **six** questions from the following. Each question carries **2** marks $6 \times 2 = 12$

(1) Evaluate $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

(2) If $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ find A + B

(3) If $A = \begin{bmatrix} -2 & 3 \\ -5 & 7 \end{bmatrix}$ find adj A

(4) Prove that the vectors $2i - 5j + k$ and $3i + j - k$ are perpendicular to each other.

(5) If $\vec{a} = 2i + 3j + k$ and $\vec{b} = i - 3j + 2k$ find $\vec{a} - \vec{b}$.

(6) Find the last term in the expansion of $\left(ax - \frac{b}{x}\right)^6$

(7) Express 1.75 radian in degree.

(8) Without using tables/calculator find the value of $\sin^2 45° + \cos^2 45°$.

(9) If $\cos A = \frac{3}{5}$ and A is in fourth quadrant, find sin A.

PART – C

III. Answer any **thirteen** questions from the following. Each question carries **5** marks.

$13 \times 5 = 65$

(1) Show that $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix} = 0$

(2) If $A = \begin{bmatrix} 5 & 6 \\ 8 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

Show that $(AB)^T = B^T A^T$

(3) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, show that $A \cdot \text{adj } A = |A| \cdot I$

(4) Solve for x and y by detrminant method :

$3x - 2y = 4$

$4x + 5y = 13$

(5) Find the projection of $\vec{a} = 3i - 4j - 5k$ on $\vec{b} = 2i - 3j + k$.

(6) Find the cosine of the angle between the vectors $2i + j + 3k$ and $i - 2j + 2k$

(7) Find the 7^{th} term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{10}$

(8) Evaluate $\sqrt{102}$ upto 4 decimal places using binomial expansion.

(9) Simplify $(2+\sqrt{3})^4 + (2-\sqrt{3})^4$

(10) Find the angle subtended by an arc of length 5.5 cm at the centre of a circle of radius 3.5 cm in degree.

(11) Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2 \text{ cosec } A$

(12) Without using tables/calculator show that

$$\sqrt{\frac{1-\cos 780^\circ}{1+\cos 780^\circ}} = \tan 30^\circ$$

(13) Without using tables / calculator find the value of

$$\sin^3 60^\circ \cdot \cot 30^\circ - 2\,\text{cosec}^2 45^\circ + 3\cos 60^\circ \cdot \tan^2 60^\circ$$

(14) Simplify

$$\frac{\cos(90^\circ - A) \cdot \sec(-A) \cdot \tan(180^\circ - A)}{\sec(360^\circ + A) \cdot \sin(180^\circ + A) \cdot \cot(90^\circ - A)}$$

(15) Prove that $\cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B$

(16) Prove that $\frac{1 - \cos 2A + \sin 2A}{1 + \cos 2A + \sin 2A} = \tan A$

(17) Without using tables or calculator prove that

$$\frac{\sin 68^\circ + \sin 52^\circ}{\cos 68^\circ + \cos 52^\circ} = \sqrt{3}$$

(18) Prove that

$$\tan 7A - \tan 5A - \tan 2A = \tan 7A \times \tan 5A \times \tan 2A.$$

(19) Find the greatest angle of the triangle whose sides are 3, 4 and 5.

(20) Show that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

PART – D

IV. Answer any **three** questions from the following. Each question carries **6** marks. $6 \times 3 = 18$

(1) Solve for x, y, z using determinants only

$$x + y = 3$$
$$y + 2z = 2$$
$$x + z = 1$$

(2) Verify Cayley-Hamilton Theorem for the matrix $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$

(3) Show that the vectors $i + j + 4k$, $4i - 2j + 4k$ and $3i - 3j$ form an isosceles right-angled triangle.

(4) Prove geometrically that

$$\sin(A + B) = \sin A \cos B + \cos A \cdot \sin B$$

(5) A man observes the elevation of a balloon to be 30°. He then walks 500 metres towards the balloon which is stationary and finds that the elevation is 60°. Find the height of the balloon.