

Variations Practice Exercises III

For each of the following, combine the given sequence of variations into a single variation. Write your answer in the form T_n , T_nR , T_nI , or T_nIR , where $0 \leq n \leq 11$.

1. $RT_4RT_4RT_8$
2. T_3IRIT_8
3. $T_5IRT_7IRT_8$

Solutions appear on the following page...

Solutions

1. $RT_4RT_4RT_8$

Remember that transpositions and retrogrades commute – that is, you can interchange them without affecting the result. So, using the rules presented in class (and also in the text), we can proceed as follows:

$$\underbrace{RT_4}_{T_4R} RT_4 \underbrace{RT_8}_{T_8R} = T_4 \underbrace{RR}_{T_0} \underbrace{T_4T_8}_{T_0} R = \mathbf{T_4R}$$

Note that we can “cancel” the two consecutive retrogrades (RR); similarly, T_4 and T_8 are opposites, so they “cancel” as well.

2. IT_3IRIT_8

For #2 and #3, there are inversions, so we have to be a bit more careful about simplifying expressions. Don't forget the rule for interchanging inversions and transpositions: $IT_n = T_{12-n}I...$

$$\underbrace{IT_3}_{T_9I} IR \underbrace{IT_8}_{T_4I} = T_9 \underbrace{II}_{T_0} \underbrace{RT_4}_{T_4R} I = \underbrace{T_9T_4}_{T_1} \underbrace{RI}_{IR} = \mathbf{T_1IR}$$

Note that at the last step above, T_9T_4 combine to give us T_{13} ; we then apply our “mod 12” rule to replace T_{13} with T_1 .

3. $T_5I \underbrace{RT_7}_{T_7R} I \underbrace{RT_8}_{T_8R} = T_5 \underbrace{IT_7}_{T_5I} R \underbrace{IT_8}_{T_4I} R = T_5 T_5 I \underbrace{RT_4}_{T_4R} \underbrace{IR}_{RI} = \underbrace{T_5T_5}_{T_{10}} \underbrace{T_4}_{T_8I} \underbrace{RR}_{T_0} I = \underbrace{T_{10}T_8}_{T_6} \underbrace{II}_{T_0} = \mathbf{T_6}$