

Math 210, Fall 2015

Collected Homework #5 – due Tuesday, 10/13

1. Define relation  $\vdash$  on  $\mathbb{N}$  as follows:

$$x \vdash y \text{ iff } xy \equiv 0 \text{ or } 1 \pmod{3}$$

For example:  $4 \vdash 6$ , since  $4 \times 6 = 24 \equiv 0 \pmod{3}$ .

Also,  $4 \vdash 7$ , since  $4 \cdot 7 = 28 \equiv 1 \pmod{3}$ .

However,  $4 \not\vdash 8$ , since  $4 \cdot 8 = 32 \equiv 2 \pmod{3}$ .

For each of the five relation properties discussed in Section 1.4, determine which properties are possessed by  $\vdash$  on the set of natural numbers. Justify each of your answers with a proof or a counterexample.

2. Reconsider relation  $\vdash$ , but this time defined on the set,  $S$ , of natural numbers that are not divisible by 3. That is,  $S = \{1, 2, 4, 5, 7, 8, 10, 11, \dots\}$

For each of the five relation properties discussed in Section 1.4, determine which properties are possessed by  $\vdash$  on the set  $S$ . Justify each of your answers with a proof or a counterexample; you may refer back to your proof or example from #1, if it is still valid on set  $S$ .

(Note: your results for  $\vdash$  as defined on set  $S$  should *not* be entirely identical to your results for  $\vdash$  on  $\mathbb{N}$ . At least one of your results should be different!)