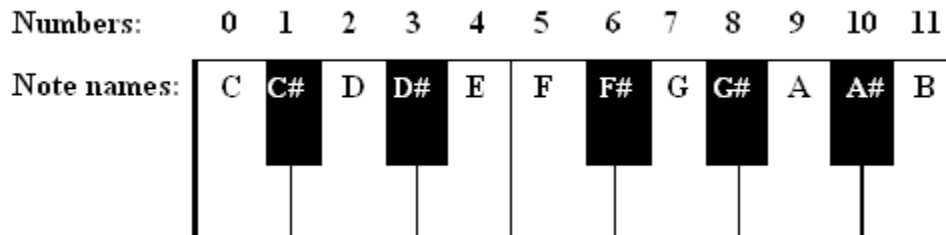


Math 105: Musical Variations “By The Numbers”

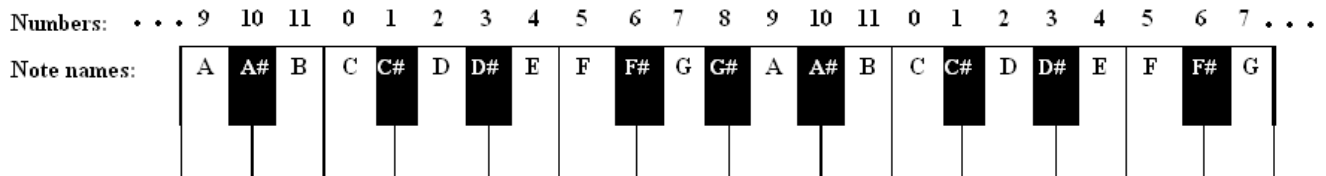
Each of the variations described in our text – transpositions, inversions, and retrogrades – can be thought of entirely in terms of numbers, rather than notes on a music staff.

Numbers for notes:

Since we are used to starting from C as our “base” pitch, we will start counting from C. So, we will index the note C as note number zero. Starting from C (or 0), we label each note according to its location in the 12-tone scale starting from C:

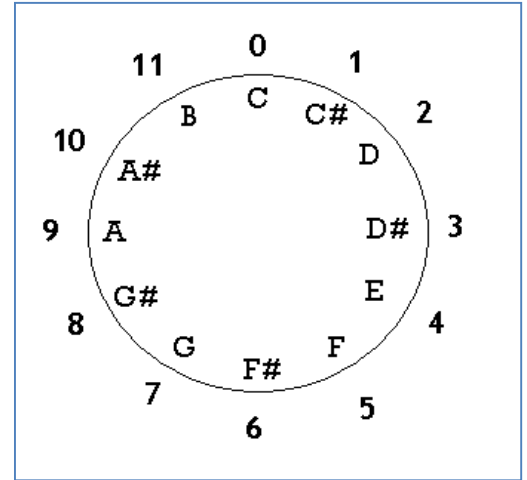


Now, if we need to count beyond 11 – for example, if a transposition raises us to a pitch above B – then we *start over* from 0. This corresponds to our convention of associating notes separated by an octave as “equivalent” notes. So, the rest of the keyboard (not just the octave shown above) would “cycle” through the numbers 0-11, just as the note names “cycle” through twelve names (C, C#, D, etc... back to A#, B, C):

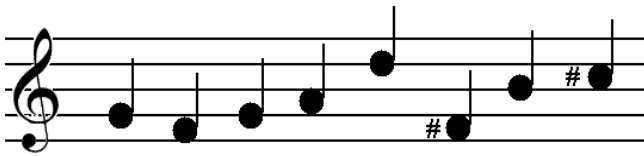


Musical Clock

Another way to visualize this cyclic correspondence between tones and numbers is the “musical clock,” shown below to the right. Using the “clock,” transposition of a given tone can be thought of by counting around the “clock” in the clockwise direction (or counterclockwise in the case of negative numbers). For example, if the first note in a melody is an A (9), and we transpose the melody using T_7 , then the new starting tone would be found by counting seven places in the clockwise direction starting from A. (What note do you end up with?)



For example, consider the melody:



The notes in this melody are, in order: G, F, G, A, D, F#, B, C#.

“By the numbers,” we’d write this melody as the sequence: 7, 5, 7, 9, 2, 6, 11, 1.

Let’s see what happens if we apply the transposition T_5 to this melody.

T_5 : just add 5 to each number (subtracting 12 as necessary), then convert back to note names:

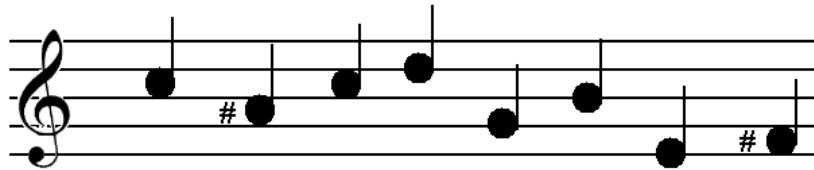
Original melody: G, F, G, A, D, F#, B, C#

“By the numbers:” 7, 5, 7, 9, 2, 6, 11, 1.

After we apply T_5 : 0, 10, 0, 2, 7, 11, 4, 6.

Transposed melody: C, A#, C, D, G, B, E, F#

On the music staff, the transposed melody would look like this:



Also try to get the above result using the “musical clock” visualization from the previous page. You should get the same results.

Here are a few examples for you to try. We'll go over these in class, if necessary.

For each, I'll just give you a "melody" as an ordered list of notes. Your job is to apply the given transposition to each one, to find the new variation (or "transposed melody"). Also, try to write the original melody and the transposed melody on a musical staff. (You should especially give this a try if you're not familiar with reading/writing music on a staff. It's something you'll need to learn to do as we progress through the semester.) You can find some sheet music on the "notes" section of the class web page (click the "free sheet music" link).

1. G, F#, A, G, G, C – use the transposition T_4
2. D, C, B, A, A, G, B – use the transposition T_6
3. D, D, C#, A, B, C#, D – use T_{-5}
4. E, A, D, G, C, D, E, F, D, G – use T_8