Math 105: Collected Homework #6 – Solutions & Comments

1. How would we define "inversions" in a 30-tone system? (Think by analogy – take what we do with our usual 12-tone system, and extend the same reasoning to a 30-tone system.) It may help to "name" the notes numerically, according to their positions on a 30-tone "musical clock."

Answer: In this system, an inversion centered at [0] would replace each note with its mod 30 opposite. For example, the opposite of [1] would be [29], the opposite of [2] would be [28], etc. In general, the mod 30 opposite of any number between 1 and 29 would be 30 minus that number – in other "words," the opposite of note [n] would be note [30-n] for any value of n between 1 and 29.

(Note: similarly to the "musical clock" diagram for 12-TET, one could sketch a circle with the numbers from 0 through 29 (rather than 0 through 11) spaced evenly around the perimeter, with 0 at the top and 15 at the bottom. This would be difficult to draw precisely though (since spacing 30 numbers evenly around a circle is harder than evenly spacing 12 numbers), so it's probably better just to do this numerically.)

2. Suppose a melody (in 30-TET) consists of the following tones, in order, where each tone is named as suggested in #1 above:

[0] , [20] , [12] , [25] , [15] , [10]

Find each of the following variations of this melody.

a. T_{10} b. $\mathcal{T}_{20}$ c. IT_{10} d. $T_{10}I$ e. $T_{20}I$

Answers:

a. Add 10 to each number (mod 30) to get [10], [0], [22], [5], [25], [20]

b. Add 20 to each number (mod 30) to get [20], [10], [2], [15], [5], [0]

c. First we use mod 30 opposites to apply the inversion, I: [0], [10], [18], [5], [15], [20]
Next, we add 10 (mod 30) to find the result of IT_{10}: [10], [20], [28], [15], [25], [0]

d. We already found the result of T_{10} in part (a), so we just need to find the mod 30 opposite of each number in that list. This gives us the result [20], [0], [8], [25], [5], [10]

e. We already found the result of T_{20} in part (b), so we just need to find the mod 30 opposite of each number in that list. This gives us the result [10], [20], [28], [15], [25], [0] COMMENT: Note that the answers to 2(c) and 2(e) are the same. This is because, for a 30-tone scale, the variations $I\mathcal{T}_{10}$ and $\mathcal{T}_{20}I$ are equivalent! This is the 30-tone version of the transposition/inversion rule for variations in the 12-tone scale: $IT_n = T_{-n}I$. For any number of notes in a scale, this rule would still apply – not just for 12 notes or 30 notes.

3. Find the *cyclic subgroup* of variations *generated* by each of the following. Show your work.

a. $\mathcal{T}_6$ b. $\mathcal{T}_{10}$ c. $\mathcal{T}_{18}$ d. $\mathcal{T}_{20}$ e. $\mathcal{T}_{25}$ f. $\mathcal{T}_{20}I$

(Hint: in 30-TET, transpositions should combine according to a "mod 30" rule, rather than "mod 12." So, for example, $\mathcal{T}_{20}\mathcal{T}_{20} = \mathcal{T}_{40} = \mathcal{T}_{10}$, since in 30-TET there are 30 tones to the octave.)

Hint: as an example, the cyclic subgroup generated by $\mathcal{T}_5$ *would be* $\{\mathcal{T}_5, \mathcal{T}_{10}, \mathcal{T}_{15}, \mathcal{T}_{20}, \mathcal{T}_{25}, \mathcal{T}_0\}$.

Answers: Listed in the order in which the transpositions would be generated in each case. (Recall that the order in which the elements of a set are listed is not relevant; what matters is which elements are, or are not, in the set at all.)

a. $\langle\mathcal{T}_6\rangle = \{\mathcal{T}_6, \mathcal{T}_{12}, \mathcal{T}_{18}, \mathcal{T}_{24}, \mathcal{T}_0\}$
b. $\langle\mathcal{T}_{10}\rangle = \{\mathcal{T}_{10}, \mathcal{T}_{20}, \mathcal{T}_0\}$
c. $\langle\mathcal{T}_{18}\rangle = \{\mathcal{T}_{18}, \mathcal{T}_6, \mathcal{T}_{24}, \mathcal{T}_{12}, \mathcal{T}_0\}$
d. $\langle\mathcal{T}_{20}\rangle = \{\mathcal{T}_{20}, \mathcal{T}_{10}, \mathcal{T}_0\}$
e. $\langle\mathcal{T}_{25}\rangle = \{\mathcal{T}_{25}, \mathcal{T}_{20}, \mathcal{T}_{15}, \mathcal{T}_{10}, \mathcal{T}_5, \mathcal{T}_0\}$
f. $\langle\mathcal{T}_{20}I\rangle = \{\mathcal{T}_{20}I, \mathcal{T}_0\}$

Comments: Notice that several of these results are the same (keeping in mind that the order in which elements of a set are listed is not relevant).

Pairs of identical cyclic subgroups:

$\langle\mathcal{T}_{10}\rangle = \{\mathcal{T}_{10}, \mathcal{T}_{20}, \mathcal{T}_0\}$
$\langle\mathcal{T}_{20}\rangle = \{\mathcal{T}_{20}, \mathcal{T}_{10}, \mathcal{T}_0\}$

$\langle\mathcal{T}_6\rangle = \{\mathcal{T}_6, \mathcal{T}_{12}, \mathcal{T}_{18}, \mathcal{T}_{24}, \mathcal{T}_0\}$
$\langle\mathcal{T}_{18}\rangle = \{\mathcal{T}_{18}, \mathcal{T}_6, \mathcal{T}_{24}, \mathcal{T}_{12}, \mathcal{T}_0\}$

$\langle\mathcal{T}_{25}\rangle = \{\mathcal{T}_{25}, \mathcal{T}_{20}, \mathcal{T}_{15}, \mathcal{T}_{10}, \mathcal{T}_5, \mathcal{T}_0\}$
$\langle\mathcal{T}_5\rangle = \{\mathcal{T}_5, \mathcal{T}_{10}, \mathcal{T}_{15}, \mathcal{T}_{20}, \mathcal{T}_{25}, \mathcal{T}_0\}$

Note that $\mathcal{T}_{20}I$ is its own opposite, so its cyclic subgroup contains only two elements – itself and the identity. There is a similar rule for all inversions T_nI for the 12-tone scale. We can expect that a similar rule would hold for any number of notes to a scale.

(Note: To see why $\mathcal{T}_{20}I$, you could try a few examples: choose a note in the 30-tone scale, apply $\mathcal{T}_{20}I$ twice, and you will always end up back with the same note you started with. Or, you could do all the examples at once algebraically, as shown in class.)