MATH 105: Counting Problems

Note:
$P(n,k)$ denotes the number of ways to select a permutation of k elements from a set of n elements.
$C(n,k)$ denotes the number of ways to select a combination of k elements from a set of n elements.

<u>Practice Exercises</u>

1. Evaluate each of the following: $P(8,4)$, $P(9,5)$, $P(12,4)$

2. How many different 5-note melodies are possible, if each note can be any of the 12 notes from the standard 12-tone scale, and no note may be repeated?

3. How does the answer to #2 change if notes may be repeated?

4. How many ways are there to rearrange the letters of the word "MUSICAL"?

5. Evaluate each of the following: $C(8,4)$, $C(9,5)$, $C(12,4)$

6. How many 5-note chords are possible, if we're selecting from the 12-tone scale?

7. There are fourteen students in a class. In how many ways can we select four of these students under each of the following scenarios?

   a) We are selecting a committee of four students
   b) We are selecting "officers" – a president, vice president, secretary and treasurer (assuming that no student may hold multiple offices)
   c) We are selecting a committee consisting of two males and two females – for this one, assume that there are nine female students and five male students in the class

8. Selecting notes from the 12-tone scale: how many different nine-note melodies include exactly four D's, exactly two F#'s, and no other repeated notes?

9. How many 7-note melodies are there with three C's, one D, one E, one F and one G?

   (e.g., CCCDEFG, FGDECCC, CGDFCEC, FCCDECG, etc… there are a lot more!)

10. Find the number of different rearrangements of each of the following words:
    MELODY   RHYTHM   AARDVARK   TWITTER   MISSISSIPPI

11. Write out the first 12 rows of Pascal's Triangle. Then, use Pascal's Triangle to find the values of $C(5,3)$, $C(5,4)$, $C(6,4)$, $C(10,3)$, and $C(10,7)$. Check your answers using the combinations formula.

Card problems:

The following few problems refer to the standard 52-card deck, as described in class. Recall that a “hand” is an unordered selection of cards, without replacement (i.e. a combination).

Hands discussed in class include: four-of-a-kind, full house, flush, ...

12. How many different ways are there to select a 5-card poker hand?

13. How many ways are there to select a “suits full house,” where three cards are all of one suit, and the other two cards share some other suit. (e.g., three clubs and two hearts would be a “suits full house.”)

14. How many ways are there to select a three-card hand which consists entirely of face cards (jack, queen, king)?

15. How many ways are there to select a four-card hand which consists of only numbered cards (2, 3, 4, ..., 10)?

16. How many ways are there to select “three of a kind” – that is, three cards all of one rank, and two other cards, each of different ranks? (e.g., Three kings, a jack, and a ten would count; three jacks and two tens would not count, since that’s a “full house,” as described in class)