Math 105, Spring 2017

Practice Exercises on Sequences and Recursion

Note: all terminology and notation used in this assignment is defined in the "sequences" handout on the class web page, as well as in recent class meetings.

1. List the first eight terms of the arithmetic sequence whose first two terms are 5, 9.

5 , 9 , ___ , ___ , ___ , ___ , ___ , ___

2. List the first eight terms of the geometric sequence whose first two terms are 48, 24.

48 , 24 , ___ , ___ , ___ , ___ , ___ , ___

3. Find the first eight terms of the sequence whose first two terms are 1, 3, which satisfies the following two-term recursion rule:

$$x_n = x_{n-1} + 2x_{n-2}.$$

(Hint: the third term is $3 + 2 \times 1 = 5$; the fourth term is $5 + 2 \times 3 = 11$.)

1 , 3 , 5 , 11 , ___ , ___ , ___ , ___

4. Figure out the next three terms in each of the following sequences. Explain your reasoning for each.

a. 1, 3, 7, 15, 31, ___ , ___ , ___

b. 1, 3, 6, 10, 15, ___ , ___ , ___

c. 8, 12, 18, 27, _____ , _____ , _____

d. 2, 9, 11, 20, 31, ___ , ___ , ___

5. Find the next few terms of each of the following sequences. For each, describe the recursive rule for the sequence in words. Then, let x_n denote the nth term of the sequence, and write an equation describing the recursive rule for the sequence.

a. 3, 7, 11, 15, 19, 23, 27, …
b. 2, 5, 11, 23, 47, 95, …
c. 1, 3, 4, 7, 11, 18, 29, …
d. 1, 3, 6, 10, 15, 21, …

6. Write the first ten terms of each sequence, given the recursive rule for the sequence.

a. $x_n = x_{n-1} + 2x_{n-2}$, with $x_1 = 1$ and $x_2 = 1$.
(Hint: this sequence starts with 1, 1, 3, 5, 11, …)
b. $x_n = 3x_{n-1} - 1$, with $x_1 = 1$
(Hint: this sequence starts with 1, 2, 5, 14, …)
c. $x_n = 3x_{n-1} - 1$, with $x_1 = 3$
(Hint: this sequence starts with 3, 8, 23, …
d. $x_n = x_{n-1} + x_{n-2} + x_{n-3}$, with x_1, x_2, and x_3 all equal to 1.
(Hint: this sequence starts with 1, 1, 1, 3, 5, …)

7. Let t_n stand for the number of ways to write a rhythm whose length is n beats using only quarter notes (1 beat) and/or eighth notes (1/2 beat).

For example: the only 1-beat rhythms we can write under this rule would be either a single quarter note, or two eighth notes. Since there are two ways to write a rhythm whose length is 1 beat, the value of $t_1$is 2. In other words, $t_1 = 2$.

a. Find the values of t_2 and t_3 by listing all acceptable 2-beat and 3-beat rhythms, respectively. (Hint: your answer for t_3 should be between 10 and 20.)

b, (Optional – challenge question) Based on your observations for t_1, t_2, and t_3, see if you can find a recursion rule to predict how the sequence t_n will continue when n is greater than 3. See if your rule works when $n = 4$, by finding all acceptable 4-beat rhythms. (Hint: your answer for t_4 should be between 30 and 40.)

8. Let b_n stand for the number of n-beat rhythms we can write under the rule that every note is either a quarter note (one beat) or a dotted half note (3 beats).

a. Write out all possible rhythms for n = 1, 2, 3, 4, 5 and 6. Use your results to find the values of b_1, b_2, b_3, b_4, b_5 and b_6. (Hint: none of these should be a very large number.)
b. See if you can figure out, and explain, a recursive rule for the sequence of numbers b_n. (Hint: the reasoning for this one will be *similar* to the reasoning for the example from class in which we restricted ourselves to only quarter notes and half notes.)