A+ Winner

March 3, 2013

Arithmetic Sequence

1. The arithmetic sequence "3, 7, 11, ... ,407" is given.

   (a) How many terms are in it ?
   (b) What is the sum of the first 10 terms.
   (c) Which is the 20th term when counted backward from the last term ?

2. In an arithmetic sequence the fourth term is thrice the first term. Its seventh term is 1 more than twice the third term. Then

   (a) Which is the first term ?
   (b) What is the common difference ?
   (c) Which is the 10^{th}term?

3. The sum of three consecutive terms of an arithmetic sequence is -3 and their product is 8. Then

   (a) What is the common difference ?
   (b) Which is the first term ?
   (c) What is the difference between the first term and the fifth term ?

4. In an arithmetic sequence the sum of the first 7 terms is 10, and the sum ot the next 7 terms is 17.

   (a) Which is the first term ?
   (b) What is the common difference ?
   (c) Is 1000 a term of this sequence ?

5. Syam took a loan of Rs. 3250/- from a bank. He, being a newspaper boy having only a meager income, decided to pay back Rs.20/- the first month and to increase the monthly payment by an amount of Rs.15/- every month.

   (a) Do the monthly payments form an arithmetic sequence ? Why ?

(b) What amont will have been repaid by the end of the eighth month ?

(c) By how many months will he pay back the amount.

6. To find the sum of the first 200 integers, Appu made use of the following method. $1+2+3+ .... +200 = 100 \times 201 = 20100$

(a) Is Appu's answer correct ? Explain.

(b) Find the answer to "5+10+15+....+100" in this method.

7. Ammu got 2n+3 as the sum of n terms of an arithmetic progression. It is said that Anu got $2n^2 + 3n$ as the answer to the same problem. Whose answer is probably correct ? Why ? Explain.

8. Angles of a quadrilateral are in arithmetic sequence. Its common difference is $10°$. Then

(a) What are the measures of the four angles.

(b) Is it a cyclic quadrilateral ? Why ?

9. If 10 times the tenth term of an arithmetic sequence is equal to 15 times the fifteenth term, find the 25th term.

10. The ninth term of an arithmetic sequence is zero. Ravi says that the $29^{th} term$ of this sequence is twice the 19th term. Do you agree with his opinion ? Why ?

Second Degree Equations

1. Consider the second degree equation $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$

(a) Find the discriminant.

(b) Is the discriminnt a second degree equation ? Why ?

(c) For what value of k will this equation be a perfect square ?

2. $(b - c)x^2 + (c - a)x + (a - b) = 0$ is a second degree equation.

(a) Find the discriminant.

(b) Under what condition will its solutions be equal ?

(c) If the solutions are equal, what is the relationship between a,b and c ?

3. The product of the digits of a two digit number is 14. When 45 is added to the number the place values of its digits get reversed. If x denotes the digit in the tenth place,

(a) What is the digit in its unit's place ?

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(b) Form an equation using the informatiom given in the question.

(c) Find the two digit number.

4. A train travels a distance of 300 km. If its speed is increased by 5 km/hr it will take 2 hours less to complete the journey. What is the original speed of the train?

5. Some students of a school decided to go for an excursion. They kept apart Rs.500/- for food. But five students could not participate in the excursion. So they had to increase the amount for food by Rs.5 from each student. How many students participated in it ?

6. A takes 6 days less than B to complete a work. If they together complete the work in 4 days, how many days will B take to complete the work alone ?

7. The product of the ages of Ramu and his father is 45. Five years ago the product of their ages was 124. How old are they now ?

8. Two pipes together fill a tank in $3\frac{1}{13}$ minutes. If the first pipe alone fills the tank 3 minutes faster than the second, how long will each pipe take to fill the tank separately.

9. Each boy in a group of boys gives a gift to every other boy in the same group. If the total number of gifts is 132, how many boys are there in the group ?

10. The speed of a boat is 15 km/hr. To travel 30 km down stream and the same distance upstream it takes 4hrs and 3 minutes. If the speed of the river is x km/hr:-

(a) What is the speed of the boat against the stream ? Does it travel at the same speed in the direction of the river ?

(b) What is the speed of the river ?

Probability

1. What is the probability of having 53 Sundays in a leap year ? Is there any change in a normal year.

2. The cards in a box are numbered from 1 to 25. If a card is drawn from it .

(a) What is the probabilty of getting a prime number ?

(b) What is the probabilty of getting an even prime number ?

(c) What is the probability of getting a perfect square less than 17 ?

3. There are 12 balls in a box. x of them are white. If a ball is taken from it,

   (a) What is the probabilty of it being white ?
   (b) When a ball is taken after adding 6 more balls in the box, the probability doubles that of part (a).Find x.

4. There are 4 prizes in the 600 tickets sold. Manoj has bought a ticket. What is the probability that he would get a prize.

5. In a bag there are 20 beads in three different colours green, red and blue. A bead is taken from it. If the probability of getting a green bead is $\frac{1}{4}$ and that of a red bead is $\frac{2}{5}$,

   (a) What is the probability of taking a blue bead ?
   (b) How many red beads are there ?
   (c) How many green beads are there ?

6. In a bag there are 5 red balls and 4 white balls. In another bag 6 red balls and 2 white balls are there. A ball is taken after choosing a bag at random. What is the probability of it being white ?
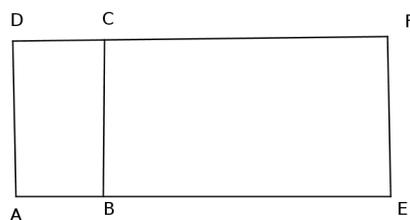
7.

   Some points are made in the figure at random. The probability of getting a dot in the rectangle ABCD is calculated to be $\frac{1}{4}$. Then

   (a) If AE =10, EF=4, what is the area of the rectangle ABCD.
   (b) What is the probabilty that the point falls in the rectangle BEFC ?

8. In a box there are some black beads and white beads. They are 15 in number. If the probability of taking a black bead is $\frac{1}{3}$,

   (a) How many black beads are there ?
   (b) What is the probability of getting a black bead if a bead is taken after removing a black bead from the box
   (c) Two white beads are put in the box after removing a black bead. If a bead is taken from it, What is the probability of it being white.

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9. Before the battle of Mahabharata, the Pandavas played die with Sakuni. Sakuni threw two dies upward in his own usual way. What are the probabilities of getting the following results ?

    (a) The sum of the numbers getting on both dies is even.
    (b) The sum of the numbers getting on both dies is a prime.
    (c) The sum of the numbers getting on both dies is at least 10.

Geometry and algebra

1. A line passes through the points A(1,-3), B(3,3).

    (a) What is the slope of the line ?
    (b) What is the slope of a line perpendicular to the line ?
    (c) Show that its equation is $\frac{x}{2} - \frac{y}{6} = 1$.
    (d) Does the line AB intersect the x-axis and the y- axis ? If so where ?

2. A line passes through A(0,2), and B(3,0).

    (a) What is its slope ?
    (b) What is its equation ?
    (c) If "O" is the origin what is the area of triangle OAB ?

3. A line passes through A(-1,3) and B(2,-9).

    (a) What is its slope ?
    (b) What is the equation of the line perpendicular to the line and passing through C(-3,4) ?
    (c) Which point is common to the line AB and to its perpendicular line ?

4. The x-axis, the y-axis and a line AB make a right triangle with hypotenuse 5 units and area 6 sq: units.

    (a) What are the points where the line AB meets the x and y axes ?
    (b) What is the slope of AB ?
    (c) What is the equation of AB ?

5. A line passes through both the point (3,5) and the point common to the lines 4x+y-1=0, and 7x-3y-35=0.

    (a) What is its slope ?
    (b) What is its equation ?
    (c) What is the slope of the line perpendicular to this line ?

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6. A line passes through (2,3) and is perpendicular to the line 3x-4y+5=0.

   (a) What is its slope ?
   (b) What is its equation ?
   (c) What is the slope of a line perpendicular to the line ?

7. If a, b, c are in arithmetic sequence, which is the point common to ax+2y+1=0, bx+3y+1=0 and cx+4y+1=0 ?

8. Find the centre of the circle passing through A(5,-8), B(2,-9) and C(2,1).

9. Show that A(1,1), B(-1,-1), C(-$\sqrt{3}$,$\sqrt{3}$) are the vertices of an equilateral triangle. What is the slope of the line drawn perpendicular to the side AB from the vertex C ?

10. Find the circumcentre and circumradius of the triangle with vertices A(8,6)., B(8,-2), C(2,-2).

11. The lines x-2y+8=0 and 2x+y+1=0 are given.

   (a) Which is their common point ?
   (b) In which of these lines does (0,4) lie ?
   (c) What is the equation of the line perpendicular to x-2y+8=0 and passing through (0,4)

Solids

1. Find the area of colour paper required to wrap a square pyramid with base edge 18 cm and slant height 41 cm.

2. What is the height of the square pyramid formed by joining four isosceles triangles with base 8 cm and height 5 cm?

3. What are the radius and central angle of the sector used to make a cone of radius 9 cm and height 12 cm ?

4. A can in the shape of a cone with height 8 cm and radius 5 cm is filled with water. If one fourth of water overflows when some spheres with the same radius of 0.5 cm are put into it, find the number of spheres immersed in the water.

5. A circular sheet of radius 10 cm was taken. A sector having an area of 40% of the sheet was removed from it. If a cone was made from the remaining part of the sheet, find the ratio between its radius and height.

6. Water flows through a pipe of diameter 5 mm at the speed of 10 m/minute. How long will it take to fill a can in the shape of a cone of base diameter 40 cm and depth 24 cm ?

7. A tank in the shape of a hemisphere with radius $1\frac{3}{4}$ cm is full of water. A pipe at the bottom of the tank can take out water from it at the rate of 7 litre per second. How long will it take to empty a full tank using the pipe ?

8. A cone has 30 cm height. A small cone with base parallel to the base of the original cone is removed from its top. If the volume of the small cone is $\frac{1}{27}$ part of the large one, at what height from the base of the large cone is the small cone removed ?

9. A vessel in the shape of a cylinder has a base diameter 15 cm and height 12 cm. It is full of icecream. A small can is in the shape of a cone of diameter 6 cm and height 12 cm with a hemisphere of the same radius attached to its base. How many of such small cans are required to keep the icecream ?

10. A tank in the shape of a cylinder, with cones attached to both ends, is full of petrol. The base diameter of the cylinder is 21 cm and height 18 cm. The height of the cone is 9 cm. How much litre of petrol can the tank contain ?

Trigonometry

1. If in triangle ABC a=2, b=3, and Sin A $=\frac{2}{3}$, what is the measurement of $\angle B$? What is the circumradius of the triangle ?

2. The angles of $\triangle ABC$ are in arithmetic sequence. If b:c $=\sqrt{3}:\sqrt{2}$, what is the measurement of $\angle A$?

3. If sin A $= \frac{3}{5}$, find the value of $\frac{1+tanA}{cosA}$.

4. Two sides of a triangular card are 20 cm and 16 cm; the angle between them is 135°.

   (a) Mark the measurements in a rough sketch.
   (b) What is the perpendicular distance from the opposite vertex to the side of 20 cm ?
   (c) What is the area of the triangle ?
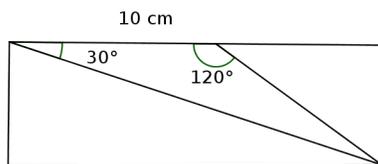
5. Find the area of the rectangle given in the figure.

6. In the figure 'O' is the centre of the circle, $\angle AOB = 90°, AB = 6\sqrt{2}, \angle BOC =$
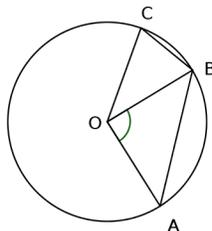
   30°.

   (a) Find the area of $\triangle ABC$.
   (b) What is the area of the quadrilateral OABC ?

7. A man sitting on a tree sees a car 30 m away from the tree at an angle of depression of $30°$. At what height is he from the ground ?

8. What is the value of $3cos^2 30 + sec^2 30 + 3sin30 - tan^2 60$?

9. An equilateral triangle is drawn with its vertices on a circle of radius 6 cm .Find the length of a side of the triangle.

10. The angles of elevations from two positions on the ground to the top of a building are complementary. The first and second positions are 'a' metre and 'b' metre away from the building. Drawing a suitable figure show that the height of the building is $\sqrt{ab}$ metre.

11. A boy standing on the top of a light house sees a ship in the sea at an angle of depression of $20°$. A man, standing at a distance of 100 m away from the light house, sees the boy at an angle of elevation of $45°$.

   (a) Draw a rough skecth.
   (b) What is the height of the light house ?
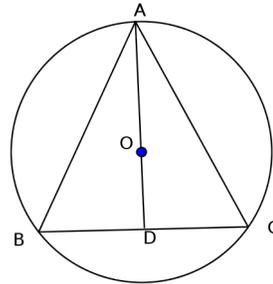   (c) How far is the ship from the shore ? (tan 20=0.3420)

Circles, Tangents

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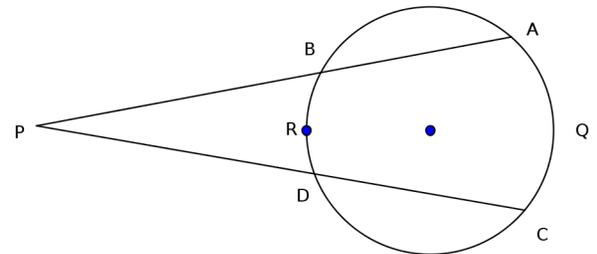1. In the figure AB is a diameter. BC =13 cm, QC=15 cm, PQ=9 cm. Find
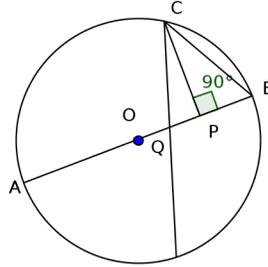
   PC, PB, OQ.

2. In the figure, the angle bisector AD of $\angle BAC$ passes through the circum-

   centre of $\triangle ABC$. Prove that AB=AC.

3. Chords AB and CD of a circle meet at P. Prove that $AP \times PB = PC \times PD$
   .

4. From the figure show that measurement of $\angle P$ is half the difference be-


   tween the central angles of arcs AQC and BRD.

5. Prove that an isosceles trapezium is cyclic.

6. If a parallelogram is cyclic, prove that it is a rectangle.

7. If the diagonals of a trapezium are equal, prove that it is a cyclic quadrilateral.

8. The tangents drawn through the points A and B meet at a point P outside the cirle with centre 'O'. Then

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(a) Show that the line OP bisects $\angle APB$.

(b) Also show that OP bisects AB.

9. In the figure, $\triangle ACD$ is an equilateral triangle. 'O' is the centre of the circle. PB is a tangent. If the central angle of the arc BC is 50°,what is
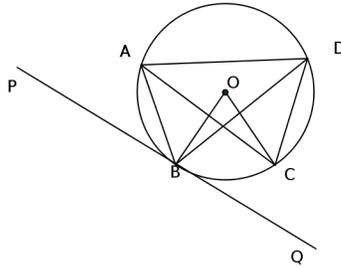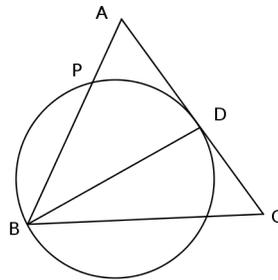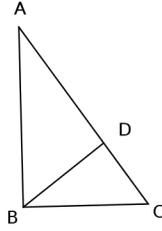

$\angle CAB$? Find $\angle ABP$.
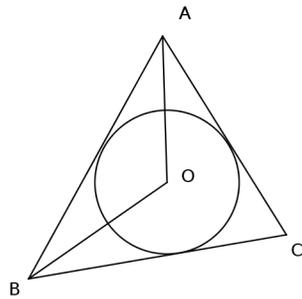
10. 'O' is the incentre of $\triangle ABC$. If $\angle C = 70°$, find $\angle AOB$?

11. In $\triangle ABC$, AB=AC. A circle is drawn with AC as a tangent. D is the mid-

dle point of AC. Then prove that 4AP=AB.

12. In the figure $\triangle ABC$ is a right triangle. BD is drawn perpendicular to the

hypotenuse AC. Prove that

(a) $AC \times AD = AB^2$

(b) $AC \times CD = BC^2$

13. $\triangle ABC$ is an isosceles triangle. (AB=AC). Prove that the tangent to the circumcircle drawn through A is parallel to BC.