

Math 105, Practice Exercises: Cosets

Note: A “coset” of a subgroup was defined in class. Refer to that definition to solve each of the following problems.

1. The set of whole numbers $\{0, 1, 2, \dots, 12, 13, 14\}$ under addition (mod 15) turns out to be a group.

- a) Show that the set $\{0, 5, 10\}$ is a subgroup of this group.
- b) Find the coset generated by 1.
- c) Find the coset generated by 2.
- d) Does this subgroup have any other cosets? If so, find them. If not, how do you know?

2. The set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ under multiplication (mod 11) turns out to be a group.

- a) Show that the set $\{1, 3, 4, 5, 9\}$ is a subgroup of this group.
- b) Find the coset generated by 2.
- c) Does this subgroup have any other cosets? If so, find them. If not, how do you know?

3. Consider the set of variations (as defined in Ch. 4 of the text) $\{T_0, T_3R, T_6, T_9R\}$.

- a) Verify that this is a subgroup of the full group of 48 musical variations.
- b) Find the coset generated by T_2R .
- c) Find the coset generated by IR .
- d) How many cosets does this subgroup have? (Hint: recall that there are a total of 48 variations in all.)
Note: You don't need to find them all to answer this question.