Math 105, Music & Mathematics – Spring, 2017

PRACTICE EXERCISES covering Change Ringing and Permutations

1. Find the result of applying each of the following permutations to the ordered list 1 2 3 4 5 6.

a. (ab)(cd)(ef)	b. (ab)(de)	c. (ad)(be)
d. (acd)(be)	e. (afce)(bd)	f. (afc)(ebd)

2. Find the permutation that is used in each of the following rearrangements. Write your answer using cycle notation, with disjoint (non-overlapping) cycles.

a.	1 2 3 4 5 6	to	1 4 3 5 2 6
b.	1 3 5 7 9 X Y	to	7 1 X 3 5 Y 9
c.	S T U V W X Y Z	to	V W U Z Y S T X
d.	0 1 2 3 4 5 6 7 8 9	to	2 1 4 8 6 7 5 3 0 9

3. Describe each of the following permutations of the ordered list 7 6 5 4 3 2 1 using cycle notation. Write each answer as a combination of disjoint cycles.

a. 6 7 5 4 3 2 1	b. 1 7 6 5 4 3 2	c. 5 6 7 4 1 2 3	d. 1 2 7 6 5 4 3

4. Find the effect of applying each permutation to the ordered list 1 2 3 4 5 6 7.

a. (acg)(bdfe)	b. (bdfe)(acg)	c. (cafe)(bad)	d. (bad)(cafe)

5. Rewrite each of the following permutations as a sequence of disjoint cycles. (To do this – apply the permutation to some ordered list to see what rearrangement results; this will allow you to trace each of the cycles, as shown in class, so that your result consists of disjoint cycles.)

a. (acd)(adc)	b. (ab)(bc)(cd)(da)	c. (cafe)(bad)	d. (bad)(cafe)

6. Show that the set { ( ), (acd) , (adc) } is a group.

7. Show that the set { ( ), (abc), (acb), (bcd) , (bdc) } is *not* a group.

In #8-11, we are considering subgroups of the group of permutations of five objects.
Note that since there are $5! = 120$ ways to rearrange five objects, there are, correspondingly, 120 permutations in this group.

8. Find the subgroup generated by the 5-cycle (adbec).

9. Find the subgroup generated by the permutation (adb)(ce).

10. Find each of the rearrangements that are obtained by applying each of the permutations from the subgroup generated in #9 (above) to the ordered list 1 2 3 4 5.

11. Consider the set of permutations: {( ), (abc), (acb), (de), (abc)(de), (acb)(de)}

    a) Verify that this set is a subgroup of the full set of permutations of five objects.
    b) Find the coset of this subgroup that is generated by the permutation (ab).
    c) Find the coset of this subgroup that is generated by the permutation (bcd).
    d) How many other cosets does this subgroup have?
    e) It turns out that this subgroup is actually a cyclic subgroup, generated by one of the permutations in the set. Figure out which permutation generates this subgroup.

For #12 below, recall that "adjacent swaps" are 2-cycles that swap adjacent positions in a list – for example, (ab), (bc), (cd), etc. are "adjacent swaps." It turns out that every permutation can eventually be carried out using some sequence of adjacent swaps.

For example: the permutation $(adb)(ce)$ rearranges the list 1 2 3 4 5 into 2 4 5 1 3. This permutation can also be achieved by the following sequence of adjacent swaps:

$$(ab), (cd), (bc), (de), (cd).$$

(Note that this is not the only way to use adjacent swaps to carry out $(adb)(ce)$ – can you find others?)

12. For each of the following permutations, find a way to use a sequence of adjacent swaps in such a way that the combined effect is the same as that of the given permutation. Keep count – how many swaps does it take? Try to find the most efficient way (that is, the lowest number of swaps possible) to write out this permutation using only adjacent swaps.

    i. (ae) ii. $(ae)(bd)$ iii. $(aebd)$ iv. $(ace)(bd)$ v. $(acebd)$