

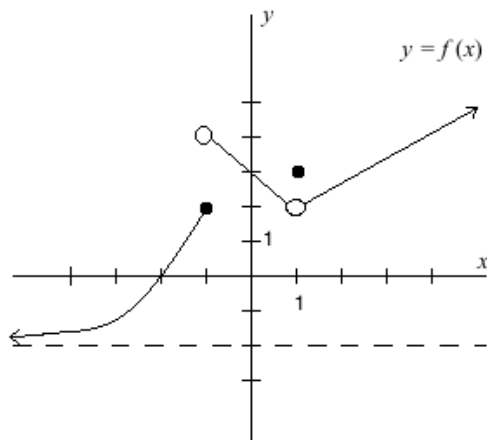
Math 160, Test #2A  
October 26, 2015

You may use a calculator on this test. (No cell-phone calculators, nor any other devices capable of taking photos or accessing the internet, are allowed.) No other reference materials are allowed.

Read all instructions for each individual problem carefully. For each problem, *show your work*, and/or otherwise explain how you got your answer. Correct answers with insufficient justification may not receive full credit, and partial credit for incorrect answers can only be given based on work shown and/or written explanation.

Please write all work and answers on this test, rather than using any separate sheets of paper. If you find that you need more space than what is provided, write any additional work on the back of the page.

1. The following diagram shows the graph of  $y = f(x)$ . The domain of  $f$  is  $(-\infty, \infty)$ . The dashed line indicates a horizontal asymptote, which the graph approaches as it goes off to the left (but not to the right).



Note: For part (a), no explanations are required. For parts (b) and (c), write your answers in the space provided; use the back of this page if you need more room.

- (a) Based on the diagram, write the apparent value of each of the following. If an answer does not exist, write "DNE" rather than a number. (Note: each of these should be clear from a quick look at the diagram; none should take much time to figure out. Don't spend more than a couple of minutes on this part!)

$$f(-1) =$$

$$f(0) =$$

$$f(1) =$$

$$\lim_{x \rightarrow -1^-} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow -1^+} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

$$\lim_{x \rightarrow -1} f(x) =$$

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow 1} f(x) =$$

- (b) Write the interval(s) on which  $f'(x)$  appears to be positive, and the interval(s) on which  $f'(x)$  appears to be negative. Briefly (one sentence) explain each of your answers.

2. Evaluate each of the following according to the instructions. Show all of your work. You should not use your calculator for either part of this problem.

(a) Use algebra to find the exact value of  $\lim_{x \rightarrow 2} \frac{2x^2 - 10x + 12}{3x - 6}$ .

(b) Use the *definition of the derivative* to find  $f'(x)$  if  $f(x) = 2x^2 + 3x$ .

(Note: “Use the definition of the derivative” means you may *not* use any of the differentiation formulas from Chapter 3 for this problem.)

3. An object is moving along a straight line in such a way that its position at time  $t$  (in seconds) is given by  $f(t) = 2t^3 + 4t$  (in feet). For each of the following, make sure to include the correct units of measurement with your answer.

(a) Find the object's *average* velocity over the time interval  $3 \leq t \leq 5$ .

(b) Find the object's *instantaneous* velocity at time  $t = 4$ .

4. Let  $f(x) = \frac{2x+4}{3-x^2}$ .

(Note that, if you do not work out part (a) successfully, you may still make use of its result in order to complete parts (b) and (c).)

(a) Use the quotient rule to show that  $f'(x) = \frac{2x^2 + 8x + 6}{(3 - x^2)^2}$ . (Show your work!)

(b) Find an equation for the line tangent to the graph of  $f$  at the point (2,-8).

(c) Find the points on the graph of  $f$  where the tangent line is horizontal. For each of these points, find the  $x$  and  $y$  coordinates.

5. Consider the function  $f(x) = \frac{4^x - 64}{x - 3}$ . (That's 4 to the  $x$  power, not 4 times  $x$ !)

(a) Use your calculator to evaluate  $f(3.01)$ ,  $f(3.0001)$ , and  $f(3.000001)$ . (Note: count those zeros carefully!) Round each answer to the nearest thousandth.

- $f(3.01) \approx$

- $f(3.0001) \approx$

- $f(3.000001) \approx$

(b) Based on your results in part (a), estimate the value of  $\lim_{x \rightarrow 3^+} f(x)$ . Round your answer (if rounding is necessary) to the nearest hundredth. Briefly (one or two sentences) explain your reasoning.

6. Suppose  $P(t) = 3000 - 40t + 6t\sqrt{t}$  is the population of the kingdom of Arandelle after  $t$  months, where  $t = 0$  corresponds to January, 2010.

Interpret the following values of  $P(t)$  or  $P'(t)$  in terms of what they mean about the population of Arandelle. For each, write at least one complete sentence of explanation; include correct dates and units of measurement, where appropriate, for full credit.

(a) Find and interpret the value of  $P(9)$ .

(b) Find and interpret the value of  $P'(9)$ .