Universal Gravity*

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Abstract

This paper studies the theoretical properties and counterfactual predictions of a large class of general equilibrium trade and economic geography models. We begin by presenting a framework that combines aggregate factor supply and demand functions with market clearing conditions. We prove that existence, uniqueness and – given observed trade flows – the counterfactual predictions of any model within this framework depend only on the demand and supply elasticities (the "gravity constants"). We propose a new strategy to estimate these gravity constants using an instrumental variables approach that relies on the general equilibrium structure of the model. Finally, we use these estimates to compute the impact of a trade war between US and China.

1 Introduction

Over the past fifteen years, there has been a quantitative revolution in spatial economics. The proliferation of general equilibrium gravity models incorporating flexible linkages across many locations now gives researchers the ability to conduct a rich set of real world analyses. However, the complex general equilibrium interactions and the variegated assumptions underpinning different models has resulted in our understanding of the models' properties to lag behind. As a result, many important questions remain either partially or fully unresolved, including: When does an equilibrium exists and when is it unique? Do different models have different counterfactual implications?

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In this paper, we characterize the theoretical and empirical properties common to a large class of gravity models spanning the fields of international trade and economic geography. We first provide a "universal gravity" framework combining aggregate demand and supply equations with standard market clearing conditions that incorporates many workhorse trade and economic geography models.[1] We show that existence and uniqueness of the equilibria of all models under the auspices of our framework can be characterized solely based on their aggregate demand and supply elasticities (the "gravity constants"). Moreover, the counterfactual predictions for trade flows, incomes, and prices of these models can be expressed solely as a function of the gravity constants and observed data. Hence, the key theoretical properties and positive counterfactual predictions of all gravity models depend ultimately on the value of two parameters – the elasticities of supply and demand. We show how these gravity constants can be estimated using an instrumental variables approach that relies on the general equilibrium structure of the model. Finally, we use these estimates to compute the impact of a trade war between US and China.

To construct our framework, we consider a representative economy in which an aggregate good is traded across locations subject to the following six economic conditions: 1) "iceberg" type bilateral trade frictions; 2) a constant elasticity of substitution (CES) aggregate demand function; 3) a CES aggregate supply function; 4) market clearing; 5) exogenous trade deficit; and 6) a choice of the numeraire. Any model in which the equilibrium can be represented in a way that satisfies these conditions is said to be contained within the universal gravity framework. Moreover, these conditions impose sufficient structure to completely characterize all general equilibrium interactions of trade flows, incomes, and prices. It turns out that the aggregate demand elasticity from condition 2 and the aggregate supply elasticity from condition 3 play a particularly important role in this characterization.

We first provide sufficient conditions for the existence, uniqueness, and interiority of the equilibrium of the model that depend solely on the gravity constants. Existence occurs everywhere except for a knife-edge constellation of parameters (corresponding e.g. to Leontief preferences in an Armington trade model or when agglomeration forces are just strong enough to create a "black hole" equilibrium in an economic geography model). An equilibrium is unique as long as the demand elasticity is (weakly) negative and the supply elasticity is (weakly) positive (or vice versa and both elasticities are greater than one in

[1] Examples of gravity trade models included in our framework are perfect competition models such as Anderson (1979), Anderson and Van Wincoop (2003), Eaton and Kortum (2002),Dekle, Eaton, and Kortum (2008), Caliendo and Parro (2010) monopolistic competition models such as Krugman (1980), Melitz (2003) as specified by Chaney (2008), Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008), Di Giovanni and Levchenko (2008), , and the Bertrand competition model of Bernard, Eaton, Jensen, and Kortum (2003). Economic geography models incorporated in our framework include Allen and Arkolakis (2014) and Redding (2016). See Table 1 for the mapping from work-horse trade and economic geography models into the universal gravity framework.

magnitude); moreover, if the inequalities are strict, an iterative algorithm is guaranteed to converge to the the unique equilibrium from any interior starting point. Multiplicity may occur if demand and supply elasticities are both negative (for example, in an economic geography model if agglomeration forces are sufficiently strong) or if demand and supply elasticities are both positive (for example, in a trade model if goods are complementary). We also show that these sufficient conditions can be extended further if trade frictions are "quasi" symmetric – a common assumption in the literature and provide conditions under which an equilibrium exists and an iterative algorithm is guaranteed to converge to the equilibrium.

We then examine how a shock to bilateral trade frictions affects equilibrium trade flows, incomes, and prices. To do so, we derive an analytical expression for the counterfactual elasticities of these endogenous variables to changes in all bilateral trade frictions that elucidates the networks effects of trade. In particular, we show how can this expression be written as series of terms expressing how a shock propagates through the trading network, e.g. the direct effect of a shock, the effect of the shock on all locations' trading partners, the effect on all locations' trading partners' trading partners, etc. Importantly, we show that this expression depends only on observed trade flows and the gravity constants, demonstrating that conditional on these two model parameters, the positive macro-economic implications for all gravity models are the same.[2] Moreover, we analytically prove that when trade frictions are "quasi" symmetric, the impact of a trade friction shock on the real output prices and real expenditure in directly-affected locations will always exceed the impact on other indirectly-affected locations.

We proceed by estimating the gravity constants using a novel procedure that can be applied to any model contained within the universal gravity framework. We show that the supply and demand elasticities can be estimated by regressing a location's fixed effect (recovered from a gravity equation) on its own expenditure share (the coefficient of which is the supply elasticity) and its income (the coefficient of which is the demand elasticity). Identifying the elasticities requires a set of instruments that are correlated with own expenditure share and income, but uncorrelated with unobserved supply shifters (such as productivity) in the residual. We construct such instruments using the general equilibrium structure of the model by calculating the equilibrium own expenditure shares and incomes of a hypothetical world where no such unobserved supply shifters exist and bilateral trade frictions are only a function of distance. Using this procedure, we estimate a demand elasticity in line with previous estimates from the trade literature (e.g. Simonovska and Waugh (2014)) and a supply elasticity that is larger than is typically calibrated to in trade

[2] While the implications for real output prices are the same for all gravity models, the mapping from real output prices to welfare will in general depend on the particular model. As a result, the normative (welfare) implications will vary across different models, as we discuss in detail below.

models but appears reasonable given estimates from the economic geography literature.

Finally, we use the estimated gravity constants along with the expression for comparative statics to evaluate the effect of a trade war between the U.S. and China on the real expenditure of all countries in the world. Given our large estimated supply elasticity, we find modest declines in (real) prices but large declines in (real) expenditure. Third country effects are also substantial, with important trading partners of China (e.g. Vietnam and Japan) and the U.S. (e.g. Canada and Mexico) being especially adversely affected.

This paper is related to a number of strands of literature in the fields of international trade, economic geography, and general equilibrium theory. There is a small but growing literature examining the structure of general equilibrium models of trade and economic geography. In particular, Arkolakis, Costinot, and Rodríguez-Clare (2012) provide conditions under which a model yields a closed form expression for changes in welfare as a function of changes in openness, while in a recent paper Adao, Costinot, and Donaldson (2017) show how to conduct counterfactual predictions in neo-classical trade models without imposing gravity. In contrast, our paper incorporates models with elastic aggregate supply curves, thereby allowing analysis of both economic geography models and trade models with intermediate "round-about" production. A key characteristic of the class of models we study is that the "gravity constants" are the same across all locations; while strong, this assumption imposes sufficient structure to completely characterize all general equilibrium interactions while retaining tractability even in the presence of a large number of locations.[3]

In terms of the theoretical properties of the equilibrium, Alvarez and Lucas (2007) use the gross substitutes property to establish sufficient conditions for uniqueness for gravity trade models. We instead generalize results from the study of nonlinear integral equations (see e.g. Karlin and Nirenberg (1967); Zabreyko, Koshelev, Krasnosel'skii, Mikhlin, Rakovshchik, and Stetsenko (1975); Polyanin and Manzhirov (2008)) to systems of nonlinear integral equations. As a result, the sufficient conditions we provide are strictly weaker than those derived by Alvarez and Lucas (2007). In particular, our conditions allows the supply elasticity to be larger in magnitude than the demand elasticity (in which case gross substitutes may not hold), which is what we find when we estimate the elasticities. In previous work, Allen and Arkolakis (2014) provide sufficient conditions for existence and uniqueness for economic geography models. Unlike those results, our conditions do not require symmetric trade frictions nor do we require finite trade frictions between all locations. Unlike both Alvarez and Lucas (2007) and Allen and Arkolakis (2014), our

[3]In contrast, the literature on Computable General Equilibrium models typically focuses on models with a large number of elasticities (e.g. location or region specific) but only a small number of regions; for a review of these models see Menezes, Fortuna, Silva, and Vieira (2006). Although outside the purview of this paper, it would be perhaps be interesting future work to determine whether some of the tools developed below could be applied to those models.

theoretical results cover both trade and economic geography models simultaneously.

Our analytical characterization of the counterfactual predictions is related to the "exact hat algebra" methodology pioneered by Dekle, Eaton, and Kortum (2008) and extended in Costinot and Rodriguez-Clare (2013) (and many others). Unlike that approach, we characterize the elasticity of endogenous variables to trade shocks (i.e. we examine local shocks instead of global shocks). There are several advantages of our local approach: first, all possible counterfactuals can be calculated simultaneously through a single matrix inversion. Second, our analytical characterization holds for local shocks around the observed equilibria even if there are other possible equilibria (in which case we are unaware of a procedure that ensures the solution to the "exact hat" approach that corresponds to the observed equilibria). Third, the local analytical expression admits a simple economic interpretation as a shock propagating through the trading network. In this regard, our paper is related to the recent working paper by Bosker and Westbrock (2016) which examines how shocks propagate through global production networks. Fourth, our analytical derivation allows us to characterize the relative size of the elasticity of real output prices and real output in different locations from a trade friction shock, providing (to our knowledge) one of the first analytical results about the relative size of the direct and indirect impacts of a trade friction shock in a model with many locations and arbitrary bilateral frictions.[4]

Our estimation strategy uses equilibrium income and own expenditure shares from a hypothetical economy as instruments to identify the demand and supply elasticities. Following Eaton and Kortum (2002), we use the fixed effects of a gravity equation as the dependent variable in an instrumental variables regression (although we use the regression to estimate the supply elasticity along with the demand elasticity). One advantage of our approach is the simplicity of calculating our instruments using bilateral distances and observed geographic variables; in this regard, we owe credit to Frankel and Romer (1999) who instrument for trade with geography (albeit not in a general equilibrium context).

The idea of using the general equilibrium structure of the gravity model to recover key parameters is originally due to Anderson and Van Wincoop (2003). Following this, several papers have sought to improve the typical gravity equation estimation by accounting for equilibrium conditions. For example, Anderson and Yotov (2010) pursues an estimation strategy imposing that the equilibrium "adding up constraints" of the multilateral resistance terms are satisfied, whereas Fally (2015) proposes the use of a Poisson Pseudo-Maximum-Likelihood estimator whose fixed effects ensure that such constraints are satisfied, and Egger and Nigai (2015) develops a two-step model consistent approach that overcomes bias arising from general equilibrium forces and unobserved trade frictions. Unlike these papers, here our focus is on recovering the demand and supply elasticities rather

[4] Mossay and Tabuchi (2015) prove a similar result in a three country world.

than estimating trade friction coefficients in a model consistent manner.

Recent work by Anderson, Larch, and Yotov (2016) explores the relationship between trade and growth examined by Frankel and Romer (1999) in a structural context. They recover the demand (trade) elasticity from a regression of income on a multilateral resistance term, where endogeneity concerns are addressed by calculating multilateral resistance based on international linkages only. Our estimation strategy, in contrast, recovers both the demand and supply elasticities from a gravity regression and overcomes endogeneity concerns using an instrumental variables approach based on the general equilibrium structure of the model.

Finally, we should note that the brief literature review above is by no means complete and refer the interested reader to the excellent review articles by Baldwin and Taglioni (2006), Head and Mayer (2013), Costinot and Rodriguez-Clare (2013) and Redding and Rossi-Hansberg (2017), where the latter two focus especially on quantitative spatial models.

The remainder of the paper is organized as follows. In the next section, we present the universal framework and discuss how it nests existing general equilibrium gravity models. In Section 3, we present the theoretical results for existence and uniqueness. In Section 4, we present the results concerning the counterfactual predictions of the model. In Section 5, we estimate the gravity constants. In Section 6 we calculate the effects of a U.S. - China trade war. Section 7 concludes.

2 A universal gravity framework

Before turning to the universal gravity framework, we present two variants of the simple Armington gravity model to provide a concrete example of the type of models that fall within our framework. Suppose there are N locations each producing a a differentiated good and in what follows we define the set $S \equiv \{1, ..., N\}$. The only factor of production is labor, where we denote the allocation of labor in location $i \in S$ as L_i and assume the total world labor endowment is $\sum_{i \in S} L_i = \bar{L}$. Shipping the good from $i \in S$ to final destination j incurs an iceberg *trade friction*, where $\tau_{ij} \geq 1$ units must be shipped in order for one unit to arrive. Consumers have CES preferences with elasticity of substitution $\sigma \geq 0$.

In the first variant, which we call the "trade" model, suppose that the labor endowed to a location is exogenous and perfectly inelastic, as in Anderson (1979) and Anderson and Van Wincoop (2003). Suppose too that there is roundabout production, as in Eaton and Kortum (2002), that combines labor and an intermediate input in a Cobb-Douglas fashion. Thus, the quantity of output produced in location i is $Q_i = (A_i L_i)^{\zeta} I_i^{1-\zeta}$, with $\zeta \in (0, 1]$ the labor share, A_i is the labor productivity in location $i \in S$ and I_i is an intermediate input equal to a CES aggregate of the differentiated varieties in all locations with the same elasticity of substitution σ as final demand. In this case, the output price in location i is $p_i = (w_i/A_i)^{\zeta} P_i^{1-\zeta}$, where w_i is the wage and $P_j \equiv \left(\sum_{k\in S} (p_j\tau_{kj})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is both the CES price index for the consumer and the price per unit of intermediate input.

In the second variant, the "economic geography" model, we suppose instead that the labor supplied to a location is perfectly elastic so that welfare is equalized across locations, as in Allen and Arkolakis (2014).[5] Welfare in this model is the product of the real expenditure of labor and the amenity value of living in a location, denoted by u_i , and welfare equalization implies $\frac{w_i}{P_i}u_i = \frac{w_j}{P_j}u_j$ for all $i, j \in S$. We further assume that productivities and amenities are subject to spillovers: $A_i = \bar{A}_i L_i^a$ and $u_i = \bar{u}_i L_i^b$. In this variant of the model, the quantity of output produced in location i is $Q_i = \bar{A}_i L_i^{1+a}$ and the output price is $p_i = w_i/\left(\bar{A}_i L_i^a\right)$.[6]

In both variants of the model, CES consumer preferences for the goods from each location yields a gravity equation that characterizes the aggregate demand in location j for the differentiated variety from location i:

$$X_{ij} = \frac{(p_i\tau_{ij})^{1-\sigma}}{\sum_{k\in S} (p_j\tau_{kj})^{1-\sigma}} E_j, \quad \text{for all } j, \tag{1}$$

where $E_j = \sum_{j\in S} X_{ji}$ is the expenditure in location j.

More subtly, both variants of the model also feature an aggregate supply for the quantity of output produced in each location. In the trade variant of the model – despite the labor supply being perfectly inelastic – we can use the fact that a constant share of revenue is paid to both workers and intermediates to write the output of location i as:

$$Q_i = A_i L_i \left(\frac{p_i}{P_i}\right)^{\frac{1-\zeta}{\zeta}}. \tag{2}$$

Similarly, in the economic geography variant of the model we can use the welfare equalization condition to write:

$$Q_i = \kappa \bar{A}_i^{\frac{b-1}{a+b}} \bar{u}_i^{-\frac{1+a}{a+b}} \left(\frac{p_i}{P_i}\right)^{-\frac{1+a}{a+b}}, \tag{3}$$

where $\kappa \equiv \left(\bar{L}/\left(\sum_{i\in S} (\bar{A}_i\bar{u}_i)^{-\frac{1}{a+b}} \left(\frac{p_i}{P_i}\right)^{-\frac{1}{a+b}}\right)\right)^{1+a}$ is an (endogenous) scalar that depends on the aggregate labor endowment $\bar{L}$ and we refer to $\frac{p_i}{P_i}$ as the *real output price* in location $i \in S$.[7] Finally, in both variants, we close the model by requiring that the value of total

[5] In addition, this formulation incorporates many prominent economic geography models, e.g. Helpman (1998); Donaldson and Hornbeck (2012); Bartelme (2014); Redding (2016).

[6] It is straightforward to add round-about production into the economic geography variant of the model (see Table 1); we omit to do so here to keep our illustrative examples as simple as possible.

[7] In these two examples – as in most of the analysis that follows – we focus on interior equilibria where production is positive in all locations. In the Online Appendix B.2 we generalize our setup to allow for the possibility of non-interior solutions where production is zero in some locations, which allows e.g. for the

output equals total sales (market clearing), i.e.

$$Y_i \equiv p_i Q_i = \sum_{j \in S} X_{ij}, \tag{4}$$

and that total expenditure equals total output (balanced trade), i.e.:

$$E_i = p_i Q_i. \tag{5}$$

Substituting the CES demand (equation 1) and supply equations (equations 2 or 3) into the market clearing and balanced trade conditions yields the following identical system of equilibrium equations for both variants of the model. In particular,

$$p_i^{1+\phi} \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j \in S} \tau_{ij}^{-\phi} P_j^{\phi} p_j \bar{c}_j \left(\frac{p_j}{P_j}\right)^{\psi} \ \forall i \in S \tag{6}$$

$$P_i^{-\phi} = \sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi} \ \forall i \in S, \tag{7}$$

where in the trade variant of the model $\psi \equiv \frac{1-\zeta}{\zeta}$ and $\bar{c}_i \equiv A_i L_i$, in the economic geography variant of the model $\psi \equiv -\frac{1+a}{a+b}$ and $\bar{c}_i \equiv \bar{A}_i^{\frac{b-1}{a+b}} \bar{u}_i^{-\frac{1+a}{a+b}}$, and in both models $\phi \equiv \sigma - 1$. Note in both models the constants $\{\bar{c}_i\}_{i \in S}$ are exogenous model location-specific fundamentals, which we refer to as *supply shifters* in what follows, and ϕ, ψ are global parameters. Given supply shifters, trade frictions, and the two parameters, one can use equations (6) and (7) to solve for output prices p_i and prices indices P_i (up-to-scale). One can then use a normalization that total world income is equal to one, i.e. $\sum_{i \in S} Y_i = 1$ and the gravity equation (equation 1) to calculate trade flows X_{ij}. Given trade flows, income Y_i can then be recovered from market clearing (equation 4). Note that although the endogenous scalar κ from the economic geography model does not enter the equilibrium system of equations (and hence does not affect trade flows or incomes), it does affect the level of output, a point we return to below.

This example highlights the close relationship between trade and geography models and suggests the possibility for a unified analysis of the properties of such spatial gravity models. In what follows, we present a framework comprising six simple economic conditions about aggregate trade flows of a representative good between many locations. We show that the equilibrium of any model that satisfies these conditions can be represented by the solution to equations (6) and (7).

To proceed with out universal gravity framework, it is helpful to first introduce some

case that welfare in unpopulated locations may be lower than populated locations. In Theorem 1 below, we provide sufficient conditions under which all equilibria are guaranteed to be interior.

terminology. Define the *output* $Q_i \geq 0$ to be the quantity of the representative good produced in location $i \in S$; the *quantity traded* $Q_{ij} \geq 0$ be the quantity of the representative good in location $i \in S$ that is consumed in location $j \in S$; the *output price* $p_i \geq 0$ to be the (factory gate) price per unit of the representative good in location $i \in S$; the *bilateral price* $p_{ij} \geq 0$ to be the cost of the representative good from location $i \in S$ in location $j \in S$; the *income* $Y_i \equiv p_i Q_i$ to be the total value of the representative good in location $i \in S$; the *trade flows* $X_{ij} \equiv p_{ij} Q_{ij}$ to be the value of the good in $i \in S$ sold to $j \in S$; the *expenditure* $E_i \equiv \sum_{j \in S} X_{ji}$ to be the total value of imports in $i \in S$; the *real expenditure* $W_i \equiv E_i / P_i$ is a measure of expenditure in location $i \in S$, where P_i is a *price index* defined below; and the *real output price* (referred to simply as "prices" in the introduction) to be p_i/P_i.[8]

We say that an equilibrium is *interior* if output and output prices are strictly positive in all locations, i.e. $Q_i > 0$ and $p_i > 0$ for all $i \in S$. In what follows, we focus our attention to interior equilibria and disregard the trivial equilibrium where $Q_i = 0$ for all $i \in S$. We provide sufficient conditions to ensure all equilibria are interior below and examine non-interior solutions in depth in Online Appendix B.2. Clearly, because of the presence of complementarities there is a possibility of multiple interior equilibria. This is true in the economic geography model because of labor mobility and agglomeration externalities or even in the trade model when complementarities in consumption are large (low σ).

We first start with a condition that describes the relationship between the output price in location i and the bilateral price:

Condition 1. The bilateral price is equal to the product of the output price and a bilateral scalar:

$$p_{ij} = p_i \tau_{ij}, \tag{8}$$

where, as above, $\{\tau_{ij}\}_{i,j \in S} \in \overline{\mathbb{R}}_{++}$ are referred to as *trade frictions*.[9]

Given prices, the next condition can be used to derive aggregate demand.

Condition 2. (CES Aggregate Demand). There exists an exogenous (negative of the) *demand elasticity* $\phi \in \mathbb{R}$ such that the expenditure in location $j \in S$ can be written as:

$$E_j = \left(\sum_{i \in S} p_{ij}^{-\phi} \right)^{-\frac{1}{\phi}} W_j, \tag{9}$$

where W_j is the real expenditure and the associated price index is $P_j \equiv \left(\sum_{i \in S} p_{ij}^{-\phi} \right)^{-\frac{1}{\phi}}$. By Shephard's lemma, condition 2 (or, for short, C.2 thereafter) implies that the trade

[8] Because the real output price is the ratio of the price of goods sold to the price index of goods purchased, it is closely related to the terms-of-trade, which is the ratio of export prices to import prices, differing only in that the price index also includes goods purchased domestically.

[9] $\overline{\mathbb{R}}_{++}$ is defined as $\mathbb{R}_{++} \cup \{\infty\}$. If $\tau_{ij} = \infty$, then there is no trade between i and j.

flows from $i \in S$ to $j \in S$ can be written as::

$$X_{ij} = \frac{p_{ij}^{-\phi}}{\sum_{k \in S} p_{kj}^{-\phi}} E_j. \tag{10}$$

We refer to equation (10) as the *aggregate demand* of the universal gravity model. The aggregate demand equation (10) combined with C.1 yields a *gravity equation* equivalent to equation (2) in Anderson and Van Wincoop (2004), Condition R3' in Arkolakis, Costinot, and Rodríguez-Clare (2012) and the CES factor demand specification considered in Adao, Costinot, and Donaldson (2017). Accordingly, we note that the demand elasticity ϕ is often referred to as the "trade elasticity" in the literature.

It is important to emphasize that real expenditure $W_i = \frac{E_i}{P_i}$ and real output prices $\frac{p_i}{P_i}$ are distinct concepts from welfare, as neither necessarily correspond to the welfare of the underlying factor of production (such as labor) of a particular model. In the models above, for example, the welfare of a worker corresponds to her real wage, which is equal to the marginal product of a worker divided by the price index. Because of the presence of roundabout production (in the trade model) or externalities (in the economic geography model), a workers marginal product is not equal to the price per unit (gross) output. [10]

We furthermore assume that output in a location is potentially endogenous and specify the following supply-side equation:

Condition 3. (CES Aggregate Supply) There exists exogenous *supply shifters* $\{\bar{c}_i\} \in \mathbb{R}^N_{++}$, an exogenous *aggregate supply elasticity* $\psi \in \mathbb{R}$, and an endogenous scalar $\kappa > 0$ such that output in each location $i \in S$ can be written as: (11)

$$Q_i = \kappa \bar{c}_i \left(\frac{p_i}{P_i} \right)^{\psi}. \tag{11}$$

In what follows, we refer to equation (11) as the *aggregate supply* of the universal gravity model and the pair of demand and supply elasticities $(-\phi, \psi)$ as the *gravity constants*.

In general, the value of the endogenous scalar κ will depend on the particular model; for example, as we saw above, in the trade model $\kappa = 1$, whereas in the economic geography model κ is endogenously determined. Without taking a particular stance on the underlying model (and the implied value of κ), the scale of output is unspecified.[11] However, we show below that we can still identify the equilibrium trade flows, incomes, and real output prices – including their level – without knowledge of κ.

[10] The relationship between real output prices and welfare for a number of seminal models are summarized in the last column of Table 1 and discussed in detail in Online Appendix B.11.

[11] While one can choose units of output to ensure $\kappa = 1$ in any given equilibrium, changes in model fundamentals given this choice of units will generally result in κ varying.

Finally, to close the model, we impose two standard conditions and choose our numeraire:

Condition 4. (Output market clearing). For all $i \in S$, $Q_i = \sum_{j \in S} \tau_{ij} Q_{ij}$.

Note that by multiplying both sides of C.4 by the output price we have that income is equal to total sales as in equation (4) in our example economy.[12]

Condition 5. (Exogenous deficits). For all $i \in S$, $E_i = \Xi \xi_i p_i Q_i$, where ξ_i is exogenous expenditure-output ratio for location i up to constant and Ξ is an endogenous scalar that ensures the world market clearing condition holds:

$$\Xi = \frac{\sum_i p_i Q_i}{\sum_i \xi_i p_i Q_i}. \tag{12}$$

We say that *trade is balanced* in the special case that $\xi_i = 1$ for all $i \in S$ (in which case $\Xi = 1$).While balanced trade is a standard assumption in (static) gravity models, we allow for (exogenous) trade imbalances in order to match observed trade data.

Our final condition is a normalization:

Condition 6. World income equals to one:

$$\sum_i Y_i = 1. \tag{13}$$

In the absence of a normalization, the level of prices are undetermined because equations (6) and (7) are homogeneous of degree 0 in $\{p_i, P_i\}_{i \in S}$. Moreover, without specifying κ in equation (11), the level of output is also unknown. The choice of normalizing world income to one in C.6 addresses both these issues simultaneously by pining down the product of the level of these two unknown scalars. As a result, we can determine the equilibrium level (i.e. including scale) of nominal incomes and trade flows. However, the cost of doing is that both the level of output (in quantities) and prices remain unknown. As a result, the primary focus in the following analysis is on three endogenous model outcomes for which we can pin down the levels: incomes, trade flows, and real output prices $\{p_i/P_i\}_{i \in S}$ (which are invariant to the both κ and the scale of prices and hence determined including scale).

Given any gravity constants $\{\phi, \psi\}$, supply shifters, $\{\bar{c}_i\}_{i \in S}$, and bilateral trade frictions $\{\tau_{ij}\}_{i,j \in S}$, we define an *equilibrium of the universal gravity framework* to be a set of endogenous outcomes determined up-to-scale, namely: outputs $\{Q_i\}_{i \in S}$, quantities traded $\{Q_{ij}\}_{i,j \in S}$, output prices $\{p_i\}_{i \in S}$, bilateral prices $\{p_{ij}\}_{i,j \in S}$, price indices $\{P_i\}_{i \in S}$, and

[12] As Anderson and Van Wincoop (2004) show, one can combine C.1, C.2, and C.4 to derive a gravity equation of the form $X_{ij} = \left(\frac{\tau_{ij}}{\Pi_i P_j}\right)^{-\phi} Y_i E_j$, where $\Pi_i^{-\phi} \equiv \sum_{j \in S} \left(\frac{\tau_{ij}}{P_j}\right)^{-\phi} E_j$ and $P_j^{-\phi} \equiv \sum_{i \in S} \left(\frac{\tau_{ij}}{\Pi_i}\right)^{-\phi} Y_i$ are outward and inward multilateral resistance terms, respectively.

real expenditures, as well as a set of endogenous outcomes for which the scale is known, namely: incomes $\{Y_i\}_{i\in S}$, expenditures $\{E_i\}_{i\in S}$, trade flows $\{X_{ij}\}_{i,j\in S}$ and real output prices $\{p_i/P_i\}_{i\in S}$ that together satisfy C.2-C.6.

As Table 1 summarizes, many well-known trade and economic geography models are contained within the universal gravity framework. On the demand side, it is well known (see e.g. Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012) and Adao, Costinot, and Donaldson (2017)) that many trade models imply an aggregate CES demand system as specified in C.2.[13] For example, in the Armington perfect competition model, a CES demand combined with linear production functions implies $\phi = \sigma - 1$, in the Eaton and Kortum (2002) model, a Ricardian model with endogenous comparative advantage across goods and Frechet distributed productivities across sectors with elasticity θ implies that $\phi = \theta$. Similarly, a class of monopolistic models with CES or non-CES demand, linear production function, and Pareto distributed productivities with elasticity θ, summarized in Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), also implies $\phi = \theta$. Economic geography models delivering gravity equations for trade flows such as Allen and Arkolakis (2014) and Redding (2016) also satisfy C.2.

As discussed in the example above, labor mobility across locations generates a CES aggregate supply satisfying C.3, with a supply elasticity of $\psi = -\frac{1+a}{a+b}$. In this case, the supply elasticity depends on the strength of the agglomeration / dispersion forces summarized by $a + b$. Assuming $a > -1$, if dispersion forces dominate ($a + b < 0$), the supply elasticity is positive, whereas when agglomeration forces dominate ($a + b > 0$), the supply elasticity is negative.

Perhaps more surprising, trade models incorporating "round-about" trade with intermediates goods also exhibit an aggregate CES supply, even though workers are immobile across locations. As discussed in the example above, the supply elasticity is $\psi = \frac{1-\zeta}{\zeta}$ and hence positive and increasing in the share of intermediates in the production. In the next two sections, we show that any trade and economic geography models sharing the same gravity constants will also share the same theoretical properties and counterfactual implications.

What types of models are not contained within the universal gravity framework? C.2 and C.3 are violated by models that do not exhibit constant demand and supply elasticities, which include Novy (2010), Head, Mayer, and Thoenig (2014), Melitz and Redding (2014), Fajgelbaum and Khandelwal (2013) and Adao, Costinot, and Donaldson (2017). Models with multiple factors of production with non-constant factor intensities will generally not admit a single aggregate good representation and hence are also not contained within the

[13] The class of trade models considered by Arkolakis, Costinot, and Rodríguez-Clare (2012) (under their CES demand assumption R3') are a strict subset of the models which fall within the universal gravity framework, corresponding to the case of $\psi = 0$.

universal gravity framework (although the tools developed below can often be extended to analyze such models depending on the particular functional forms). C.5 is violated both by dynamic models in which the trade deficits are endogenously determined and by models incorporating additional sources of revenue (like tariffs); hence these models are not contained within the universal gravity framework. However, we show in Online Appendix B.8 how the results below can be applied to a simple Armington trade model with tariffs.[14] Finally, while the universal gravity framework includes a single sector, the mathematical tools used to prove existence and uniqueness below can be extended to allow for multiple sectors of production as in e.g. Costinot, Donaldson, and Komunjer (2010); see Allen, Arkolakis, and Li (2014).

3 Existence, uniqueness, and interiority of equilibria

We proceed by deriving a number of theoretical properties of the equilibria of all models contained within the universal gravity framework.

To begin, we note that we can combine C.1 through C.**??** to write the equilibrium output prices and price indices (to-scale) as the solution to equations (6) and (7). These equations are sufficient to recover the equilibrium level of real output prices and – given the normalization in C.6 – the equilibrium level of incomes, expenditures, and trade flows as well as all other endogenous variables up-to-scale.[15] As a result, equations (6) and (7) (together with the normalization in C.6) are sufficient to characterize the equilibrium of the universal gravity framework.

Before proceeding, we impose two mild conditions on bilateral trade frictions $\{\tau_{ij}\}_{i,j\in S}$:

Assumption 1. *The following parameter restrictions hold:*

i) $\tau_{ii} < \infty$ *for all* $i \in S$.

ii) The graph of the matrix of trade frictions $\{\tau_{ij}\}_{i,j\in S}$ *is strongly connected.*

The first part of the assumption imposes strictly positive diagonal elements of the matrix of bilateral trade frictions. The second part of the assumption – strong connectivity – requires that there is a sequential path of finite bilateral trade frictions that can link

[14] It is important to note that while the universal gravity framework can admit tariffs, how tariffs affect the model implications will in general depend on the micro-economic foundations of a model. In particular, the Armington model presented in Online Appendix B.8 abstracts from two additional complications that may arise with the introduction of tariffs. First, the elasticity of trade to tariffs may be different than the elasticity of trade to trade frictions depending on the model; second, if one does not impose that tariffs are uniform for all trade flows between country pairs, the construction of (good-varying) optimal tariffs will depend on the particular micro-economic structure of the model; see Costinot, Rodríguez-Clare, and Werning (2016) for a detailed discussion of these issues.

[15] See Online Appendix B.1 for these derivations.

any two locations i and j for any $i \neq j$. This condition has been applied previously in general equilibrium analysis as a condition for existence in McKenzie (1959, 1961), Arrow, Hahn, et al. (1971), invertibility by Cheng (1985); Berry, Gandhi, and Haile (2013), and uniqueness by Arrow, Hahn, et al. (1971), Allen (2012). In our case these two assumptions are the weakest assumptions on the matrix of trade frictions we can accommodate in order to analyze existence and uniqueness of interior equilibrium.

We mention briefly (but do not need to assume) a third condition. We say that trade frictions are *quasi-symmetric* if there exist a pair of strictly positive vectors $\left(\tau_i^A, \tau_i^B\right) \in \mathbb{R}_{++}^{2N}$ such that for any $i, j \in S$, we can write $\tau_{ij} = \tilde{\tau}_{ij}\tau_i^A\tau_j^B$, where $\tilde{\tau}_{ij} = \tilde{\tau}_{ji}$. Quasi-symmetry is a common assumption in the literature (see for example Anderson and Van Wincoop (2003), Eaton and Kortum (2002), Waugh (2010), Allen and Arkolakis (2014)), and we prove in Online Appendix B.3 that C.1, C.2, C.4, and C.5 taken together imply that the origin and destination-specific terms in the bilateral trade flow expression are equal up to scale, i.e. $p_i^{-\phi} \propto p_i^{1+\psi}P_i^{\phi-\psi}\bar{c}_i$, which in turn implies that equilibrium trade flows will be symmetric, i.e. $X_{ij} = X_{ji}$ for all $i, j \in S$. The only way the trade can be balanced when trade frictions are quasi-symmetric is to make trade flows bilaterally balanced. As a result, equations (6) and (7) simplify to a single set of equilibrium equations, which allows allows us to relax the conditions on the following theorem regarding existence and uniqueness:

Theorem 1. *Consider any model contained within the universal gravity framework where trade is balanced (i.e. $\xi_i = 1$ for all $i \in S$) and Assumption 1 is satisfied. Then:*

(i) If $1 + \psi + \phi \neq 0$, then there exists an interior equilibrium.

(ii) If $\phi \geq -1$, and $\psi \geq 0$ then all equilibria are interior.

(iii) If $\{\phi \geq 0, \psi \geq 0\}$ or $\{\phi \leq -1, \psi \leq -1\}$ (or, if trade frictions are quasi-symmetric and either $\left\{\phi \geq -\frac{1}{2}, \psi \geq -\frac{1}{2}\right\}$ or $\left\{\phi \leq -\frac{1}{2}, \psi \leq -\frac{1}{2}\right\}$) then there is a unique interior equilibrium.

(iv) If $\{\phi > 0, \psi > 0\}$ or $\{\phi < -1, \psi < -1\}$ (or, if trade frictions are quasi-symmetric and either $\left\{\phi > -\frac{1}{2}, \psi > -\frac{1}{2}\right\}$ or $\left\{\phi < -\frac{1}{2}, \psi < -\frac{1}{2}\right\}$).

Proof. See Appendix A.1 for parts (i) and (iii) and Online Appendix B.2 for part (ii). □

A key advantage of Theorem 1 is that despite the large dimensionality of the parameter space (N supply shifters $\{\bar{c}_i\}_{i\in S}$ and N^2 trade frictions $\{\tau_{ij}\}_{i,j\in S}$), the conditions are only stated in terms of the two gravity constants. Of course, since we provide sufficient conditions, there may be certain parameter constellations such as particular geographies of trade frictions where uniqueness may still occur even if the conditions of Theorem 1 are not satisfied.[16,17]

The sufficient conditions for existence, interiority, and uniqueness from Theorem 1 are illustrated in Figure 1. In the case of existence, standard existence theorems (see e.g. Mas-Colell, Whinston, and Green (1995)) guarantee existence for endowment economies when preferences are strictly convex. This is also true in the universal gravity framework: existence of an interior equilibrium may fail only when $1+\psi+\phi=0$, which corresponds to the Armington trade model (without intermediate goods) where $\sigma=0$, i.e. with Leontief preferences that are not strictly convex. Moreover, in the economic geography example above, an interior equilibrium does not exist in the knife-edge case where $\sigma=\frac{1+a}{a+b}$, as agglomeration forces lead to the concentration of all economic activity in one location (see Allen and Arkolakis (2014)).

As long as the partial elasticity of aggregate demand with respect to own output price is greater than negative 1 and the partial elasticity of supply with respect to the real output price is positive, all equilibria are interior. For example, in the economic geography model above, if these conditions are satisfied, one can show that the welfare of an uninhabited location approaches infinity as its population approaches zero, ensuring that all locations will be populated in equilibrium.

An equilibrium is unique as long as the partial elasticity of aggregate demand to output prices is negative (i.e. $\phi\geq 0$) and the partial elasticity of aggregate supply is positive (i.e. $\psi\geq 0$). There is also a unique interior equilibrium the demand elasticity is positive and the supply elasticity is negative and both elasticities have magnitudes greater than one, although such parameter constellations are less economically meaningful (and there may also exist non-interior equilibria). Multiplicity of interior equilibria may arise in cases when supply and demand elasticities are both positive (which occurs e.g. in trade models when goods are complements) or when supply and demand elasticities are both negative (which occurs e.g. in economic geography models when agglomeration forces are stronger than dispersion forces). Such examples of multiplicity are easy to construct - Appendix B.7 provides examples of multiplicity in a two location world where either the demand elasticity is negative (in which case the relative demand and supply curves are both upward sloping)

[16] Alvarez and Lucas (2007) provide an alternative approach based on the gross substitute property to provide conditions for uniqueness of the Eaton and Kortum (2002) model. In Online Appendix B.6, we show that the gross substitutes property directly applied to our system may fail if the supply elasticity ψ is larger in magnitude than the demand elasticity ϕ, i.e. in ranges $\psi>\phi\geq 0$ or $\psi<\phi\leq -1$. Theorem 1 provides strictly weaker sufficient conditions in that regard. Such parameter constellations are consistent with economic geography models with weak dispersion forces or trade models with large intermediate goods shares. Importantly, in Section 5, we estimate that $\psi>\phi>0$ empirically.

[17] Theorem 1 generalizes Theorem 2 of Allen and Arkolakis (2014) in three ways: 1) it allows for asymmetric trade frictions; 2) it allows for infinite trade frictions between certain locations; and 3) it applies to a larger class of general equilibrium spatial model, including notably trade models with inelastic labor supplies (i.e. models in which $\psi=0$). Theorem 1 also provides a theoretical innovation, as it shows how to extend the mathematical argument of Karlin and Nirenberg (1967) to multi-equation systems of non-linear integral equations.

or the supply elasticity is negative (in which case the relative demand and supply curves are both downward sloping). Finally, quasi-symmetric trade frictions allow us to extend the range of gravity constants for which uniqueness is guaranteed, but do not qualitatively change the intuition for the results.

4 The network effects of a trade shock

We now turn to how the universal gravity framework can be used to make predictions of how a change in trade frictions alter equilibrium trade flows, incomes, and real output prices in each location.[18]

To begin, we define two $N \times 1$ vectors (which, with some abuse of language, we will call "curves"): define the supply curve $\mathbf{Q}^s$ to be the set of supply equations (11) from C.3 (multiplied by output prices and divided by κ); and define the demand curve $\mathbf{Q}^d$ to be the set of market clearing (demand) equations combining C.1, C.2, C.4, and C.5, i.e.:

$$\mathbf{Q}^s(\mathbf{p},\mathbf{P}) \equiv \left(p_i \bar{c}_i \left(\frac{p_i}{P_i} \right)^{\psi} \right)_{i \in S} \tag{14}$$

$$\mathbf{Q}^d(\mathbf{p},\mathbf{P},\Xi;\boldsymbol{\tau}) \equiv \left(\sum_{j \in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j \bar{c}_j \left(\frac{p_j}{P_j} \right)^{\psi} \Xi \xi_j \right)_{i \in S}, \tag{15}$$

where $\mathbf{p} \equiv (p_i)_{i \in S}$ and $\mathbf{P} \equiv \left(\left[\sum_j \tau_{ji}^{-\phi} p_j^{-\phi} \right]^{-\frac{1}{\phi}} \right)_{i \in S}$ are $N \times 1$ vectors and $\tau \equiv (\tau_{ij})_{i,j \in S}$ is an $N^2 \times 1$ vector.[19] Note that we express both the supply and demand curves in value terms, which will prove helpful in deriving the comparative statics in terms of observed trade flows.

In equilibrium, supply is equal to demand, i.e. $\mathbf{Q}^s(\mathbf{p},\mathbf{P}) = \mathbf{Q}^d(\mathbf{p},\mathbf{P};\boldsymbol{\tau})$, and equation (5) is expressed as follows:

$$\sum_{i \in S} \mathbf{Q}_i^s(\mathbf{p},\mathbf{P}) = \sum_{i \in S} \mathbf{Q}_i^d(\mathbf{p},\mathbf{P},\Xi;\boldsymbol{\tau}) \iff \sum_{i \in S} (1 - \Xi \xi_i)\, p_i \bar{c}_i \left(\frac{p_i}{P_i} \right)^{\psi} = 0.$$

For notational convenience, define $Z(\mathbf{p},\mathbf{P},\Xi)$ as $\sum_{i \in S} (1 - \Xi \xi_j)\, p_i C_i \left(\frac{p_i}{P_i} \right)^{\psi}$. We fully differentiate these equations, along with the definition of the price index, to yield the following system of $2N+1$ linear equations relating a small change in trade costs, $D \ln \boldsymbol{\tau}$, to a small

[18] In what follows, we focus on the policy shocks that alter bilateral trade frictions $\{\tau_{ij}\}_{i,j \in S}$. In Online Appendix B.8, we show how one can apply similar tools to characterize the theoretical properties and conduct counterfactuals in an Armington trade model with tariffs.

[19] One can also conduct comparative static w.r.t. ξ. See Dekle, Eaton, and Kortum (2008).

change in output prices and price indices, $D\ln\mathbf{p}$ and $D\ln\mathbf{P}$, respectively:

$$\left(\underbrace{\begin{pmatrix} D_{\ln\mathbf{p}}\mathbf{Q}^s & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{I} & 0 \\ \mathbf{0} & \mathbf{0} & D_{\ln\Xi}Z \end{pmatrix}}_{\equiv\mathbf{S}} - \underbrace{\begin{pmatrix} D_{\ln\mathbf{p}}\mathbf{Q}^d & D_{\ln\mathbf{P}}\mathbf{Q}^d - D_{\ln\mathbf{P}}\mathbf{Q}^s & D_{\ln\Xi}\mathbf{Q}^d \\ D_{\ln\mathbf{p}}\ln\mathbf{P} & \mathbf{0} & 0 \\ -D_{\ln\mathbf{p}}Z & -D_{\ln\mathbf{P}}Z & 0 \end{pmatrix}}_{\equiv\mathbf{D}}\right)\begin{pmatrix} D\ln\mathbf{p} \\ D\ln\mathbf{P} \\ D\ln\Xi \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} D_{\ln\boldsymbol{\tau}}\mathbf{Q}^d \\ D_{\ln\boldsymbol{\tau}}\ln\mathbf{P} \\ \mathbf{0} \end{pmatrix}}_{\equiv\mathbf{T}} D\ln\boldsymbol{\tau},$$

where $\mathbf{S}$ (*the supply matrix*) and $\mathbf{D}$ (*the demand matrix*) are $2N+1\times 2N+1$ matrices capturing the marginal effects of a change in the output price on the supply and demand curves (where the demand matrix also captures the net effect of a change in the price index), respectively, and $\mathbf{T}$ is a $2N+1\times N^2$ matrix capturing the marginal effects of a change in trade costs on the demand curve and price index.

Given expressions (14) and (15), we can write all three matrices solely as a function of the gravity constants and observables as follows:

$$\mathbf{S} = \begin{pmatrix} (1+\psi)\mathbf{Y} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{I} & 0 \\ \mathbf{0} & \mathbf{0} & -\sum_i E_i \end{pmatrix},\ \mathbf{D}= \begin{pmatrix} -\phi\mathbf{E} + (1+\psi)\mathbf{X}, & (\phi-\psi)\mathbf{X}+\psi\mathbf{E}, & (E_i)_i \\ \mathbf{E}^{-1}\mathbf{X}^T & \mathbf{0} & 0 \\ (1+\psi)(Y_i - E_i)_i^T & -\psi(Y_i-E_i)_i^T & 0 \end{pmatrix}, \tag{16}$$

$$\mathbf{T} = \begin{pmatrix} -\phi(\mathbf{X}\otimes\mathbf{1})\circ(\mathbf{I}\otimes\mathbf{1}) \\ \left(\mathbf{E}^{-1}\mathbf{X}^T\otimes\mathbf{1}\right)\circ(\mathbf{1}\otimes\mathbf{I}) \\ \mathbf{0} \end{pmatrix}, \tag{17}$$

where $\mathbf{X}$ is the (observed) $N\times N$ trade flow matrix whose $\langle i,j\rangle^{th}$ element is X_{ij}, $\mathbf{Y}$ is the $N\times N$ diagonal income matrix whose i^{th} diagonal element is Y_i, $\mathbf{E}$ is the $N\times N$ diagonal income matrix whose i^{th} diagonal element is E_i, $\mathbf{I}$ is the $N\times N$ identity matrix and $\mathbf{1}$ is an $1\times N$ matrix of ones, $\mathbf{I}_i$ is the standard i-th basis for $\mathbb{R}^N$, and where $\otimes$ represents the Kronecker product and $\circ$ represents the element-wise multiplication (i.e. Hadamard product). [20]

A simple application of the implicit function theorem allows us to characterize the elasticity of prices and price indices to any trade cost shock. Define the $2N+1\times 2N+1$

[20]In what follows (apart from part (iii) of Theorem 2), we do not assume that C.5 holds in the data, i.e. that income is necessarily equal to expenditure; rather, we allow for income and expenditure to differ by a location-specific scalar, i.e. we allow for (exogenous) deficits.

matrix $\mathbf{A} \equiv \mathbf{S} - \mathbf{D}$ and, with a slight abuse of notation, let $A^{-1}_{k,l}$ denote the $\langle k, l \rangle^{th}$ element of the (pseudo) inverse of $\mathbf{A}$. Then:

Theorem 2. *Consider any model contained in the universal gravity framework. Suppose that* $\mathbf{X}$ *satisfies strong connectivity. If* $\mathbf{A}$ *has rank* $2N$*, then:*

(i) The elasticities of output prices and output price indices are given by:

$$\frac{\partial \ln p_l}{\partial \ln \tau_{ij}} = -\phi X_{ij} A^{-1}_{l,i} + \frac{X_{ij}}{E_j} A^{-1}_{l,N+j} \quad \textit{and} \quad \frac{\partial \ln P_l}{\partial \ln \tau_{ij}} = -\phi X_{ij} A^{-1}_{N+l,i} + \frac{X_{ij}}{E_j} A^{-1}_{N+l,N+j}. \tag{18}$$

(ii) If the largest absolute value of eigenvalues of $\mathbf{S}^{-1}\mathbf{D}$ *is less than one, then* $\mathbf{A}^{-1}$ *has the following series expansion:*

$$\mathbf{A}^{-1} = \sum_{k=0}^{\infty} \left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1},$$

(iii) If trade frictions are quasi-symmetric, trade is balanced and $\phi, \psi \geq 0$*, then for all* $i, l \in S$ *and* $j \neq i, l$*,*

$$\frac{\partial \ln (p_i/P_i)}{\partial \ln \tau_{il}}, \frac{\partial \ln (p_l/P_l)}{\partial \ln \tau_{li}} < \frac{\partial \ln (p_j/P_j)}{\partial \ln \tau_{il}}$$
$$\frac{\partial \ln (p_i Q_i/P_i)}{\partial \ln \tau_{il}}, \frac{\partial \ln (p_l Q_l/P_l)}{\partial \ln \tau_{li}} < \frac{\partial \ln (p_j Q_j/P_j)}{\partial \ln \tau_{il}}.$$

and the inequalities have the opposite sign ($>$) *if* ($\phi, \psi \leq -1$).

Proof. See Appendix A.2. □

Recall from Section 3 that knowledge of the output prices and price indices up-to-scale is sufficient to recover real output prices and – along with the normalization C.6 – is sufficient to recover equilibrium trade flows, expenditures, and incomes.[21] As a result, part (i) of Theorem 2 states that given gravity constants and observed data, the (local) counterfactuals of these variables for all models contained in the university gravity framework are the same.[22]

[21] Because of homogeneity of degree 0, we can without loss of generality normalize one price; moreover, from Walras' law, if $2N-1$ equilibrium conditions hold, then the last equation holds as well. As a result, $\mathbf{A}$ will have at most $2N-1$ rank and $\mathbf{A}^{-1}$ can be calculated by simply eliminating one row and column of $\mathbf{A}$ and then calculating its inverse. The values of the eliminated row can then be determined using the normalization C.6. For example, if one removes the first row and column, $\frac{\partial \ln p_1}{\partial \ln \tau_{ij}}$ can be chosen to ensure that $\sum_{i \in S} \frac{\partial \ln Y_i}{\partial \ln \tau_{ij}} = 0$ so that C.6 is satisfied.

[22] In Online Appendix B.9, we show how the "exact hat algebra" (Dekle, Eaton, and Kortum (2008), Costinot and Rodriguez-Clare (2013)) can be applied to any model in the universal gravity framework to calculate the effect of any (possibly large) trade shock. The key takeaway – that counterfactual predictions depend only on observed data and the value of the gravity constants – remains true globally. However, if the uniqueness conditions of Theorem 1 do not hold, we are unaware of any procedure that guarantees that the solution found using the "exact hat algebra" approach corresponds to the counterfactual of the

The second part of Theorem 2 provides a simple interpretation of the counterfactuals as a shock propagating through the trade network. Consider a shock that decreases the trade cost between i and j by a small amount $\partial \ln \tau_{ij}$ and define $\left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}$ as the k^{th} *degree effect* of the shock. It turns out the k^{th} degree effect is simply the effect of the $k-1^{th}$ degree shock on the output prices and price indices of all locations' trading partners, holding constant their trading partners' prices and price indices. To see this, consider first the 0^{th} degree effect. Holding constant the prices and price indices in all other locations, the direct effect of a decrease in $\partial \ln \tau_{ij}$ is a shift of the demand curve upward in i by $\phi X_{ij} \times \partial \ln \tau_{ij}$ and a decrease in the price index in j by $\frac{X_{ij}}{E_j} \times \partial \ln \tau_{ij}$. To re-equilibriate supply and demand (holding constant prices and price indices in all other locations), we then trace along the supply curve to where supply equals demand by scaling the effect by $\mathbf{S}^{-1}$, for a total effect of $\mathbf{S}^{-1}\partial \ln \tau$. Consider now the 1^{st} degree effect. We first take the resulting changes in the price and price index from the 0^{th} degree effect and calculate how they shift the demand curve (and alter the price index) in all i and j trading partners by multiplying the 0^{th} degree effect by the demand matrix, i.e. $\mathbf{D}\left(\mathbf{S}^{-1}\partial \ln \tau\right)$. To find how this changes the price and price index in each trading partner, (holding constant the prices and price indices in the trading partners' trading partners), we then trace along the supply curve by again scaling the shock by $\mathbf{S}^{-1}$, for a combined effect of $\mathbf{S}^{-1}\mathbf{D}\mathbf{S}^{-1}\partial \ln \tau$. The process continues iteratively, with the k^{th} degree effect shifting the demand curve and price index according to the $k-1$ shock and then re-equilibriated supply and demand by tracing along the supply curve (holding constant the prices and price indices in all trading partners), for an effect of $\left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}\partial \ln \tau$, as claimed.[23] The total change in prices and price indices is the infinite sum of all k^{th} degree shocks.

The third part of Theorem 2 says that the direct impact of a symmetric decline in trade frictions $\partial \ln \tau_{il}$ and $\partial \ln \tau_{li}$ on real output prices (and real expenditure) in the directly affected locations i and l will be larger than the impact of that shock in any other indirectly affected location $j \neq i, l$. If the demand and supply elasticities are positive, then a decline in trade frictions will cause the real output prices in the directly affected locations to rise more than any indirectly affected location (the ordering is reversed if the demand and supply elasticities are negative). This analytical result characterizes the relative impact of a trade friction shock on different locations in a model with many locations and arbitrary

observed equilibrium. Indeed, it is straightforward to construct a simple example where in the presence of multiple equilibria, iterative algorithms used to solve the "exact hat algebra" system of equations will converge to qualitatively different equilibria than what is observed in the data even for arbitrarily small shocks, implying arbitrarily large counterfactual elasticities. In contrast, the elasticities in Theorem 2 will provide the correct local counterfactual elasticities around the observed equilibrium even in the presence of multiple equilibria.

[23] One can also derive the alternative representation $\mathbf{A}^{-1} = -\sum_{k=0}^{\infty} \mathbf{D}^{-1}\left(\mathbf{S}\mathbf{D}^{-1}\right)^k$, in which the ordering is reversed: the k^{th} degree effect is calculated by first shifting the supply curve by the $k-1$ degree shock and then tracing along the demand curve to re-equilibriate supply and demand.

bilateral frictions.[24]

5 Estimating the gravity constants

In the previous section, we saw that the impact of a trade friction shock on trade flows, incomes, expenditures, and real output prices in any gravity model can be determined solely from observed trade flow data and the value the demand and supply elasticities. In this section, we show how these gravity constants can be estimated. We use data on international trade flows, so for the remainder of the paper we refer to a location as a country.

5.1 Methodology

We first derive an equation that shows that the relationship between three observables – relative trade shares, relative incomes, and relative own expenditure shares – are governed by the two gravity constants. We then show how this relationship under minor assumptions can be used as an estimating equation to recover the gravity constants. We begin by combining C.1 and C.2 to express the expenditure share of country j on trade from i relative to its expenditure on its own goods as a function of the trade frictions, the output prices in i and j, and the aggregate demand elasticity:

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{jj} p_j}{\tau_{ij} p_i}\right)^{\phi}.$$

We then use the relationship $p_i = Y_i/Q_i$ to re-write this expression in terms of incomes and aggregate quantities and rely on C.3 to write the equilibrium output as a function of output prices and the output price index:

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{jj}\left(\frac{Y_j}{\bar{c}_j}\right)\left(\frac{p_i}{P_i}\right)^{\psi}}{\tau_{ij}\left(\frac{Y_i}{\bar{c}_i}\right)\left(\frac{p_j}{P_j}\right)^{\psi}}\right)^{\phi}. \tag{19}$$

We now define $\lambda_{jj} \equiv X_{jj}/E_j$ to be the fraction of income country j spends on its own goods (j's "own expenditure share"). By combining C.1 and C.2, we note j's own expenditure share can be written as $\lambda_{jj} = \left(\tau_{jj}\frac{p_j}{P_j}\right)^{-\phi}$, which allows us to write equation (19) (in log form) as:

$$\ln\frac{X_{ij}}{X_{jj}} = -\phi\ln\frac{\tau_{ij}}{\tau_{jj}} + \phi\ln\frac{Y_j}{Y_i} + \psi\ln\frac{\lambda_{jj}}{\lambda_{ii}} - \phi\ln\frac{\bar{c}_j}{\bar{c}_i} + \phi\psi\ln\frac{\tau_{jj}}{\tau_{ii}}. \tag{20}$$

[24] Mossay and Tabuchi (2015) prove a similar result in a three country world.

Equation (20) shows that the demand elasticity ϕ is equal to the partial elasticity of trade flows to relative incomes, whereas the supply elasticity ψ is equal to the partial elasticity of trade flows to the relative own expenditure shares. Intuitively, the greater j's income relative to i (holding all else equal, especially the relative supply shifters $\ln \frac{\bar{c}_j}{\bar{c}_i}$), the greater the price in j relative to i and hence the more it would demand from i relative to j; the greater the demand elasticity ϕ, the greater the effect of the price difference on expenditure. Conversely, because the real output price is inversely related to a country's own expenditure share, the greater j's own expenditure share relative to i, the lower the relative aggregate supply to j and hence the more j will consume from i relative to j; the larger the supply elasticity ψ, the more responsive supply will be to differences in own expenditure share.

Equation (20) forms the basis of our strategy for estimating the gravity elasticities ϕ and ψ. However, it also highlights two important challenges in estimation. First, because unobserved trade frictions act as a residual in equation (20), we require a moment condition along with observed trade flows in order to estimate the gravity elasticities.[25] Second, equation (20) highlights that the gravity elasticities are partial elasticities holding the (unobserved) relative supply shifters $\{\bar{c}_i\}_{i\in S}$ fixed. Because both income and own expenditure shares are correlated with supply shifters through the equilibrium structure of the model, any estimation procedure must contend with this correlation between observables and unobservables.

In order to address both concerns, we combine plausibly exogenous observed geographic variation with the general equilibrium structure of the model to estimate the gravity elasticities. We proceed in a two-stage procedure.[26] First, we re-write equation (20) as:

$$\ln \frac{X_{ij}}{X_{jj}} = -\phi \ln \frac{\tau_{ij}}{\tau_{jj}} - \ln \pi_i + \ln \pi_j,$$

where $\ln \pi_i \equiv \phi \ln Y_i + \psi \ln \lambda_{ii} - \phi \ln \bar{c}_i + \phi\psi \ln \tau_{ii}$ is a country-specific fixed effect. We assume relative trade frictions scaled by the trade elasticity can be written as a function of their continent of origin c, continent of destination d, and the decile of distance between

[25]Relatedly, Online Appendix B.10 shows how for any observed set of trade flows $\{X_{ij}\}$ and any assumed set of gravity elasticities $\{\phi, \psi\}$, own trade frictions $\{\tau_{ii}\}$, and supply shifters $\{\bar{c}_i\}$, there will exist a unique set of trade frictions $\{\tau_{ij}\}_{i\neq j}$ for which the observed trade flows are the equilibrium trade flows of the model..

[26]While the two step procedure we follow resembles the procedure used in Eaton and Kortum (2002) to recover the trade elasticity from observed wages, there are two important differences. First, our procedure applies to a large class of trade and economic geography models and allows us to simultaneously estimate both the demand (trade) elasticity and the supply elasticity (rather than assuming e.g. that the population of a country is exogenous and calibrating the model to a particular intermediate good share). Second, our procedure relies on the general equilibrium structure of the model to generate the identifying variation (rather than e.g. instrumenting for wages with the local labor supply, which would be inappropriate for economic geography models).

the origin and destination countries, l:

$$-\phi \ln \frac{\tau_{ij}}{\tau_{jj}} = \beta_{cd}^{l} + \varepsilon_{ij},$$

where ε_{ij} is a residual assumed to be independent across origin-destination pairs. The country-specific fixed effect can then be recovered from the following the following equation:

$$\ln \frac{X_{ij}}{X_{jj}} = \beta_{cd}^{l} - \ln \pi_i + \ln \pi_j + \varepsilon_{ij}, \tag{21}$$

where we estimate β_{cd}^{l} non-parametrically using a set of 360 dummy variables (10 distances deciles $\times$ 6 origin continents $\times$ 6 destination continents). Let $\ln \hat{\pi}_i$ denote the estimated fixed effect and define $\hat{\nu}_i \equiv \ln \hat{\pi}_i - \ln \pi_i$ to be its estimation error.

In the second stage, we write the estimated fixed effect as a function of income and own expenditure share:

$$\ln \hat{\pi}_i = \phi \ln Y_i + \psi \ln \lambda_{ii} + \nu_i, \tag{22}$$

where $\nu_i \equiv -\phi \ln \bar{c}_i + \phi\psi \ln \tau_{ii} + \hat{\nu}_i$ is a residual that combines the unobserved supply shifter, the unobserved own trade friction, and the estimation error from the first stage. As mentioned above, it is not appropriate to estimate equation (22) via ordinary least squares, as variation in the supply shifter will affect income and the own expenditure share through the equilibrium structure of the model, creating a correlation between the residual and the observed covariates. Intuitively, the larger the supply shifter of a country, the greater its output and hence the greater the trade flows for a given observed income; since the country-specific fixed effect $\ln \pi_i$ is decreasing in relative trade flows, the OLS estimate of ϕ will be biased downwards.

To overcome this bias, we pursue an instrumental variables (IV) strategy, where we use the general equilibrium structure of the model to construct a valid instrument. To do so, we calculate the equilibrium trade flows of a hypothetical world where the bilateral trade frictions and supply shifters depend only on observables. We then use the incomes and relative own expenditure shares of this hypothetical world as instruments for the observed incomes and own expenditure shares. These counterfactual variables are valid instruments as long as (1) they are correlated with their observed counterparts (which we can verify); and (2) the observable components of the bilateral trade frictions and supply shifters are uncorrelated with unobserved supply shifters.

Because the first-stage estimation of (21) provides an unbiased estimate of $-\phi \ln \frac{\tau_{ij}}{\tau_{jj}}$, we use the estimated origin-continent-destination-continent-decile coefficients $\hat{\beta}_{cd}^{l}$ to create our counterfactual measure of bilateral trade frictions (normalizing own trade frictions $\tau_{jj} = 1$). In the simplest version of our procedure, we then calculate the equilibrium income and own expenditure share given these bilateral trade frictions, assuming that the supply shifter $\bar{c}_i$ is equal in all countries. In this version of the procedure, the instrument is valid as long as the the general equilibrium effects of distance on the origin fixed effects of a gravity equation are uncorrelated with unobserved heterogeneity in supply shifters (or own trade frictions). Because we calculate the equilibrium of the model in a counterfactual world where there is no heterogeneity in supply shifters, it seems reasonable to assume that the resulting equilibrium income and own expenditure shares that we use as instruments are uncorrelated with any real world heterogeneity. However, our instrument would be invalid if there were a correlation between unobserved supply shifters and the observed geography of a country (e.g. if countries more remotely located were also less productive or less attractive places to reside).

To mitigate such a concern (and to allow for more realistic variation across countries in supply), we extend the approach to allow the supply shifter to vary across countries depending on a vector of (exogenous) observables X_i^c, e.g. land controls like the amount of fertile land, geographic controls like the distance to nearest coast, institutional controls like the rule of law, historical controls like the population in 1400, and schooling and R&D controls like average years of schooling. Given a set of supply shifters $\{\bar{c}_i\}$ that depend only these observables and the set of trade frictions that depend only on our non-parametric estimates from above, we re-calculate the equilibrium income and own expenditure share in each country. We then use the equilibrium values from this hypothetical world as our instruments, while and control directly for the observables X_i^c in equation (22). As a result, the identifying variation from the instruments only arises through the general equilibrium structure of the model.[27] Intuitively, differences in observables like land area in neighboring countries generates variation in the demand that a country faces for its production, as well as variation in the price it faces for its consumption, even conditional on its own observables.

There are two things to note about the above procedure. First, to construct the hypothetical equilibrium incomes and own expenditure shares requires assuming values of the gravity constants ϕ and ψ for the hypothetical world. In what follows, we choose a demand elasticity $\phi = 8.28$ and a supply elasticity $\psi = 3.76$, which correspond to the

[27]Calculating the counterfactual equilibrium income and own expenditure share in each country when the supply shifters depend on observables requires assuming a particular mapping between the observables X_i^c and the supply shifter $\bar{c}_i$. We assume that $\bar{c}_i = X_i^c \beta^c$ and note that the theory implies the following equilibrium condition:

$$\ln Y_i = \frac{\phi}{\phi - \psi} \ln \bar{c}_i + \frac{1+\psi}{\psi - \phi} \ln \gamma_i + \frac{\psi}{\psi - \phi} \ln \delta_i.$$

As a result, we choose the β^c that arise from the OLS regression $\ln Y_i = \frac{\phi}{\phi - \psi} X_i^c \beta^c + \epsilon_i$. Although our estimates of β^c may be biased due to the correlation between X_i^c and ϵ_i, this bias only affects the strength of the instrument, because if each X_i^c is uncorrelated with the residual ν_i in equation (22) (i.e. X_i^c is exogenous), then any linear combination of X_i^c will also be uncorrelated with the residual.

(estimated) demand elasticity estimated and (implicitly calibrated) supply elasticity in Eaton and Kortum (2002). We should note that while the particular choice of the these parameters will affect the strength of the constructed instruments, they will not affect the consistency of our estimates of the gravity constants under the maintained assumption that bilateral distances are uncorrelated with the unobserved supply shifters conditional on observables.[28]

The second thing to note about the estimation procedure is more subtle. As mentioned in Section 3 and discussed in detail in Online Appendix B.3, when bilateral trade frictions are "quasi-symmetric" the equilibrium origin and destination shifters will be equal up to scale. In this case, there will be a perfect log linear relationship between the income of a country, its own expenditure share and its supply shifter.[29] As a result, if we were to impose quasi-symmetric bilateral trade frictions in the hypothetical world, the equilibrium income and expenditure shares generated would be perfectly collinear, preventing us from simultaneously identifying the demand and supply elasticities in the second stage. Intuitively, identification of the demand elasticity requires variation in a country's supply curve (its destination fixed effect), whereas identification of the supply elasticity requires variation in a country's demand curve (its origin fixed effect); when trade frictions are quasi-symmetric, however, the two co-vary perfectly. Our choice to allow distance to affect trade frictions differently depending on the continent of origin and continent of destination introduces the necessary asymmetries in the trade frictions to allow the model constructed instruments to vary separately, allowing for identification of both the supply and demand elasticities simultaneously. To address concerns about the extent to which these asymmetries are sufficient to separately identify the two, we report the Sanderson-Windmeijer F-test (see Sanderson and Windmeijer (2016)) in the results that follow.

5.2 Data

We now briefly describe the data we use to estimate the gravity constants.

Our trade data comes from the Global Trade Analysis Project (GTAP) Version 7 (Narayanan, 2008). This data provides bilateral trade flows between 94 countries for the year 2004. To construct own trade flows, we subtract total exports from the total sales of domestic product, i.e. $X_{ii} = X_i - \sum_{j \neq i} X_{ij}$. We use the bilateral distances between countries from the CEPII gravity data set of Head, Mayer, and Ries (2010) to construct deciles of distance between two countries. We rely on the data set of Nunn and Puga (2012) to provide a number of country level characteristics that plausibly affect supply shifters,

[28] In principle, we could search over different values of the gravity constants to find the constellation that maximizes the power of our instruments. In practice, however, our estimates vary only a small amount across different values of the gravity constants.

[29] In particular, $(1 + 2\phi) \ln E_i = (2\phi) \ln \bar{c}_i + (1 - 2\psi) \ln \lambda_{ii} + C$.

including "land controls" (land area interacted with the fraction of fertile soil, desert, and tropical areas), "geographic controls" (distance to the nearest coast and the fraction of country within 100 kilometers of an ice free coast), "historical controls" (log population in 1400 and the percentage of the population of European descent), "institutional controls" (the quality of the rule of law). Finally, following Eaton and Kortum (2002), we also consider "schooling and R&D controls" including the average years of schooling from UNESCO (2015) and the R&D stocks from Coe, Helpman, and Hoffmaister (2009), where a dummy variable is included if the country is not in each respective data set.

5.3 Estimation results

Table 2 presents the results of our estimation of equation (20). The first column presents the ordinary least squares regression; we estimate a positive supply elasticity and negative demand elasticity, consistent with the discussion above that the OLS estimate of the demand elasticity is biased downward. Column 2 presents the instrumental variable estimation where the counterfactual income and own expenditure shares comprising our instrument are constructed assuming equal supply shifters. After correcting for the bias arising from the correlation between the unobserved supply shifters and observed incomes and own expenditure shares, we find positive supply and demand elasticities, although the demand elasticity is not statistically significant. Columns 3 through 7 sequentially allows the supply shifter in the construction of the instrument to vary across countries depending on an increasing number of observables (while including these same observables as controls in both the first and second stages of the IV estimation of equation (20)). Including these observables both increases the strength of the instruments and reduces the concern that the instruments are correlated with unobserved supply shifters. Reassuringly, our estimated demand and supply elasticities vary only slightly with the inclusion of additional controls.[30]

In our preferred specification (column 7), we estimate a demand elasticity of $\phi = 3.72$ (95% confidence interval [1.14,6.29] and a supply elasticity $\psi = 68.49$ (95% confidence interval [5.38,131.60]).[31] Hence, our demand elasticity estimate is somewhat lower than the preferred estimate of Eaton and Kortum (2002) of 8.28 (although similar to their estimate using variation in wages of 3.6), as well as similar to estimates of trade elasticity around 4 in Anderson and Van Wincoop (2004), Simonovska and Waugh (2014), and Donaldson (forthcoming). Unlike these papers, however, we also estimate the supply elasticity. Our point estimate, while noisily estimated, is substantially larger than and statistically dif-

[30]Figure 4 in the online appendix shows that our instrumental variables of counterfactual income and own expenditure shares are positively correlated with their observed counterparts, even after differencing out the observables in the supply shifters.

[31]While the p-value of the Sanderson-Windmeijer F-test is statistically significant in the first stage for income, it is only marginally statistically significant for expenditure shares, suggesting that the wide confidence interval for the supply elasticity may be due in part to a weak instrument.

ferent (at the 5% level) from the supply elasticity to which Eaton and Kortum (2002) implicitly calibrate. Moreover, our estimated value is consistent with recent estimates of labor mobility from the migration literature. To see this, consider an economic geography framework with intermediate goods, agglomeration forces, and Frechet distributed preferences over location (see the last row of Table 1). If we match the labor share in production of 0.21 in Eaton and Kortum (2002) and the agglomeration force of $\alpha = 0.10$ in Rosenthal and Strange (2004), then our point estimate of ψ is consistent with a migration elasticity (Fréchet shape parameter) of 1.4. This is similar to estimates from the migration literature using observed labor flows and about one-third to one-half the size of within-country estimates.[32]

6 The impact of a U.S.-China trade war

We now apply the estimates from Section 5 to evaluate the impact of a trade war between the U.S. and China. We model the trade war as an increase in the trade frictions between the U.S. and China (holding constant all other trade frictions). We then characterize how such a trade war propagates through the trade network using the methodology developed in Section 4.[33]

There are two 0^{th} degree effects of the trade war: first, the U.S. and China export less to each other, causing the output prices in both countries to fall; second, the the cost of importing increases, causing the price index in both countries to rise. Both effects cause the real output price to decline, with a greater decline in China because both its export and import shares with the U.S. are relatively larger.

The top panel of Figure 2 depicts the 1^{st} degree effect on the real output price in all countries. The effect in the U.S. and China is positive, as the degree 0 decline in output price reduces the cost of own expenditure (causing the price index to fall in both countries). In other countries, however, the degree 1 effect is negative, as the U.S. and China demand less of their goods, causing their trading partner's output prices to fall. The most negatively affected countries are those who export the most to the U.S. and China.

Summing across all degree shocks yields the total elasticity of real output prices in

[32] Ortega and Peri (2013) estimates an migration elasticity to destination country income of 0.6 using international migration flows and an estimate of 1.8 for the sub-sample of migration flows within the European Union, albeit not using a log-linear gravity specification. Within countries (aud with log-linear gravity specifications), Monte, Redding, and Rossi-Hansberg (2015) estimate a migration elasticity of 4.4 in the U.S.; Tombe, Zhu, et al. (2015) estimate a migration elasticity of 2.54 in China, and Morten and Oliveira (2014) estimate a migration elasticity of 3.4 in Brazil.

[33] In the counterfactuals that follow, we accommodate the deficits observed in the data by assuming that the observed ratio of expenditure to income for each country remains remains constant and impose an aggregate market clearing condition that total income is equal to total expenditure. The results are qualitatively similar if we instead solve for the (unique) set of balanced trade flows that match the observed import shares and treat these balanced trade flows as the data.

each country to the trade war shock, which the bottom panel of Figure 2 depicts.[34] Not surprisingly, the two countries hurt most by a trade war are the U.S. and China. Moreover, while all countries are made worse off, the countries who are closely linked through the trading network with the U.S. and China (e.g. Canada, Mexico, Vietnam, and Japan) are hurt more than those countries that are less connected (e.g. India). All told, we estimate that a 10% increase in bilateral trade frictions is associated with a decline in real output price of 0.04% in the U.S. and 0.14% in China. These modest changes in the real output price are due to the large supply elasticity, causing the aggregate output to reallocate away from the U.S. and China in response to the trade war. The converse of this result, however, is that the reallocation of the aggregate output results in large changes to *total* real expenditure: for example, in the Armington trade model interpretation, a 10% increase in bilateral trade frictions causes the total real expenditure to fall by 2.7% in the U.S. and by 9.8% in China.[35]

There are two potential concerns about these estimated effects. First, because the elasticities correspond to an infinitesimal shock, one may worry that the effects of a large trade war may differ. To address this concern, we calculate the effect of a 50% increase in bilateral trade frictions using the methodology discussed in Online Appendix B.9. The correlation between the local elasticities and global changes exceeds 0.99, indicating that the local *relative* effect of the trade war is virtually the same as the global effect.[36] However, the local effect does overstate the global effect of such a shock, as we find that log first differences implied by the global shock are roughly 80% the size of those implied by the local elasticities. Second, the effects of the trade war above were calculated given the gravity constants estimated in Section 5; one may be concerned that the effects of the trade wars may differ substantially across alternative values of these elasticities. To address this concern, we calculate the effects of a trade war for a large number of different combinations of supply and demand elasticities.[37] Across all constellations in the 95% confidence interval of the two estimated gravity constants, the calculated elasticities are quite similar, with a 10% increase in bilateral trade frictions associated with a decline in real output price between 0.03% and 0.05% in the U.S. and 0.07% and 0.26% in China. Of course, as Section 4 emphasizes, the particular value of the gravity constants may substantially affect the impact of counterfactuals more generally.

[34]Figures 5 through 9 in the Online Appendix depict the impact of the degrees 0, 1, 2, and higher on the relative prices, relative output, income, the relative price index, and real output prices in each country.

[35]Recall from Section 2 that while the changes in real output prices are identified from the value of trade flows alone, without specifying κ in equation (11), the change in total real expenditure is only identified up to scale. In Armington trade models with intermediates, however, this is not a problem, as $\kappa = 1$.

[36]See Figure 10 in the Online Appendix.

[37]See Figure 11 in the Online Appendix.

7 Conclusion

In this paper, we provide a framework that unifies a large set of trade and geography models. We show that the properties of models within this framework depend crucially on the value of two gravity constants: the aggregate supply and demand elasticities. Sufficient conditions for the existence and uniqueness of the equilibria depend solely on the gravity constants. Moreover, given observed trade flows, these gravity constants are sufficient to determine the effect of a trade friction shock on trade flows, incomes, and real output price without needing to specify a particular underlying model.

We then develop a novel instrumental variables approach for estimating the gravity constants using the general equilibrium structure of the framework. Using our estimates, we find potentially large losses may arise due to a trade war between U.S. and China occur.

By providing a universal framework for understanding the general equilibrium forces in trade and geography models, we hope that this paper provides a step toward unifying the quantitative general equilibrium approach with the gravity regression analysis common in the empirical trade and geography literature. Toward this end, we have developed a toolkit that operationalizes all the theoretical results presented in this paper.[38] We also hope the tools developed here can be extended to understand other general equilibrium spatial systems, such as those incorporating additional types of spatial linkages beyond trade frictions.

[38]The toolkit is available for download on Allen's website.

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Table 1: Examples of models in the universal gravity framework

Model	Demand elasticity (ϕ)	Supply elasticity (ψ)	Model parameters	Welfare of labor
Armington (1969), Anderson (1979), Anderson and Van Wincoop (2003) (with intermediates)	$\sigma - 1$	$\frac{1-\zeta}{\zeta}$	σ subs. param. ζ labor share	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Krugman (1980) (with intermediates)	$\sigma - 1$	$\frac{1-\zeta}{\zeta}$	σ subs. param. ζ labor share	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Eaton and Kortum (2002) (with intermediates)	θ	$\frac{1-\zeta}{\zeta}$	θ heterogeneity param. ζ labor share	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Melitz (2003), Di Giovanni and Levchenko (2013)	$\theta\left(1 + \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\right)$	$\frac{1-\zeta}{\zeta}$	σ subs. param. θ heterogeneity param.	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Allen and Arkolakis (2014)	$\sigma - 1$	$-\frac{1+a}{a+b}$	σ subs. param. a productivity spillover b amenity spillover	$\left(\sum_i B_i \left(\frac{p_i}{P_i}\right)^{-\frac{1}{a+b}}\right)^{-(a+b)}$
Redding (2016)	θ	$\frac{\alpha\varepsilon}{1+\varepsilon(1-\alpha)}$	θ heterogeneity (goods) param. ε heterogeneity (labor) param. α goods expenditure share	$\left[\sum_i B_i \left(\frac{p_i}{P_i}\right)^{\alpha\varepsilon}\right]^{\frac{1}{\varepsilon}}$
Redding and Sturm (2008) (A variant of Helpman (1998))	$\sigma - 1$	$\frac{\alpha}{(1-\alpha)(\sigma-1)-\alpha}$	σ subs. param. α share spent on goods	$\left(\sum_i B_i \left(\frac{p_i}{P_i}\right)^{\psi\phi}\right)^{\frac{1+\psi}{1+\psi\phi+\psi}}$
Economic geography with intermediate goods and spillovers	θ	$\frac{1-\zeta}{\zeta} - \frac{1}{\zeta b+a}\left(\frac{a+\zeta}{\zeta}\right)$	θ heterogeneity (goods) param. ζ intermediate good share a productivity spillover b amenity spillover	$\left(\sum_i B_i \left(\frac{p_i}{P_i}\right)^{\frac{1}{\zeta}}\right)$
Economic geography with intermediate goods, idiosyncratic preferences, and spillovers	θ	$\frac{1-\zeta}{\zeta} + \frac{\varepsilon}{\zeta-\varepsilon(a+\zeta\beta)}\left(\frac{a+\zeta}{\zeta}\right)$	θ heterogeneity (goods) param. ε heterogeneity (labor) param. ζ intermediate good share a productivity spillover b amenity spillover	$\left[\sum_i B_i \left[\left(\frac{p_i}{P_i}\right)^{\frac{1}{\zeta}}\right]^{\frac{\varepsilon}{1-\varepsilon\left(\frac{a}{\zeta}+b\right)}}\right]^{\frac{1-\varepsilon\left(\frac{a}{\zeta}+b\right)}{\varepsilon}}$

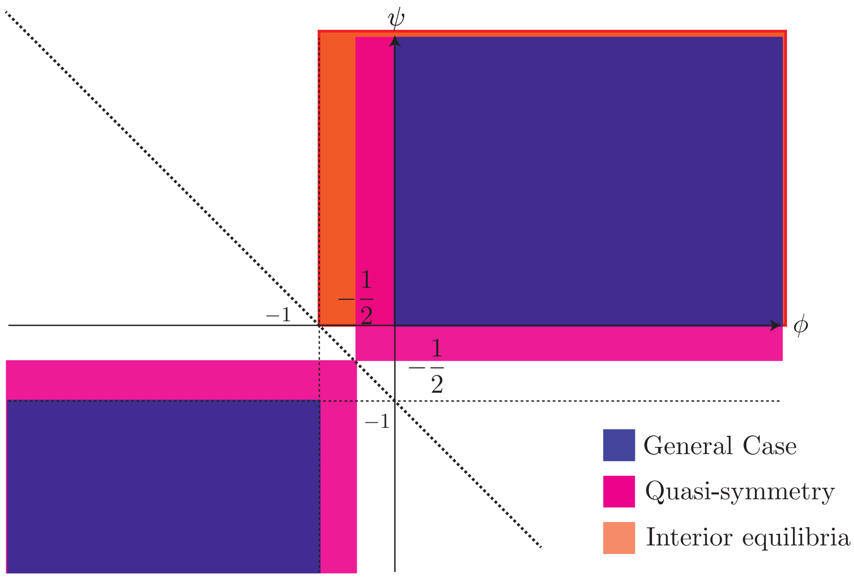
Notes: This table includes a (non-exhaustive) list of trade and economic geography models that can be examined within the universal gravity framework, the mapping of their structural parameters to the gravity constants, and the relationship between the welfare of workers and the real output prices. B_i is an exogenous location specific parameter whose interpretation depends on the particular model. λ is an endogenous variable which affects every country simultaneously.

Table 2: Estimating the gravity constants

	(1) OLS	(2) IV	(3) IV	(4) IV	(5) IV	(6) IV	(7) IV
Log Income (Demand	-0.403**	1.484	3.278	4.364*	3.882**	3.539***	3.715***
elasticity)	(0.171)	(1.157)	(2.674)	(2.371)	(1.838)	(1.356)	(1.312)
Log Own Expenditure	3.381**	92.889***	108.592**	116.649**	71.859**	64.968**	68.488**
Share (Supply Elasticity)	(1.600)	(13.417)	(48.104)	(47.944)	(35.883)	(33.014)	(32.198)
Land controls	No	No	Yes	Yes	Yes	Yes	Yes
Geographic controls	No	No	No	Yes	Yes	Yes	Yes
Historical controls	No	No	No	No	Yes	Yes	Yes
Institutional controls	No	No	No	No	No	Yes	Yes
Schooling and R&D controls	No	No	No	No	No	No	Yes
First stage Sanderson-Windmeijer F-test:							
Income		25.909	3.994	6.349	20.095	34.198	25.763
(p-value)		0.004	0.102	0.053	0.007	0.002	0.004
Own expenditure share		72.702	4.388	4.923	3.561	4.577	5.205
(p-value)		0.000	0.090	0.077	0.118	0.085	0.071
Observations	94	94	94	94	94	94	94

Notes: The dependent variable is the estimated country fixed effect of a gravity regression of the log ratio of bilateral trade flows to destination own trade flows on categorical deciles of distance variables, where the coefficient is allowed to vary by continent of origin and destination. Hence, each observation in the regressions above is a country. Instruments for income and own expenditure share are the equilibrium values from a trade model where the bilateral trade frictions are those predicted from the same gravity equation and countries are either identical in their supply shifters (column 2) or their supply shifters are estimated from a regression of observed income on observables (columns 3 through 7). In the latter case, the observables determining the supply shifters are controlled for directly in the first and second stage regressions, so identification of the demand and supply elasticities arise only from the general equilibrium effect on income and own expenditure shares. Land controls include land area interacted with fraction fertile soil, desert, and tropical areas. Geographic controls include the distance to nearest coast and the fraction of country within 100 km of an ice free coast. Historical controls include the log population in 1400 and the percentage of the population of European descent. Institutional controls include the quality of the rule of law. Schooling and R&D controls are average years of schooling (from UNESCO) and the R&D stocks (from Coe et al. (2009)), where a dummy variable is included if the country is not in each respective data set. Land, geographic, and historical control are from Nunn and Puga (2012). Standard errors clustered at the continent level are reported in parentheses. Stars indicate statistical significance: * p¡.10 ** p¡.05 *** p¡.01.

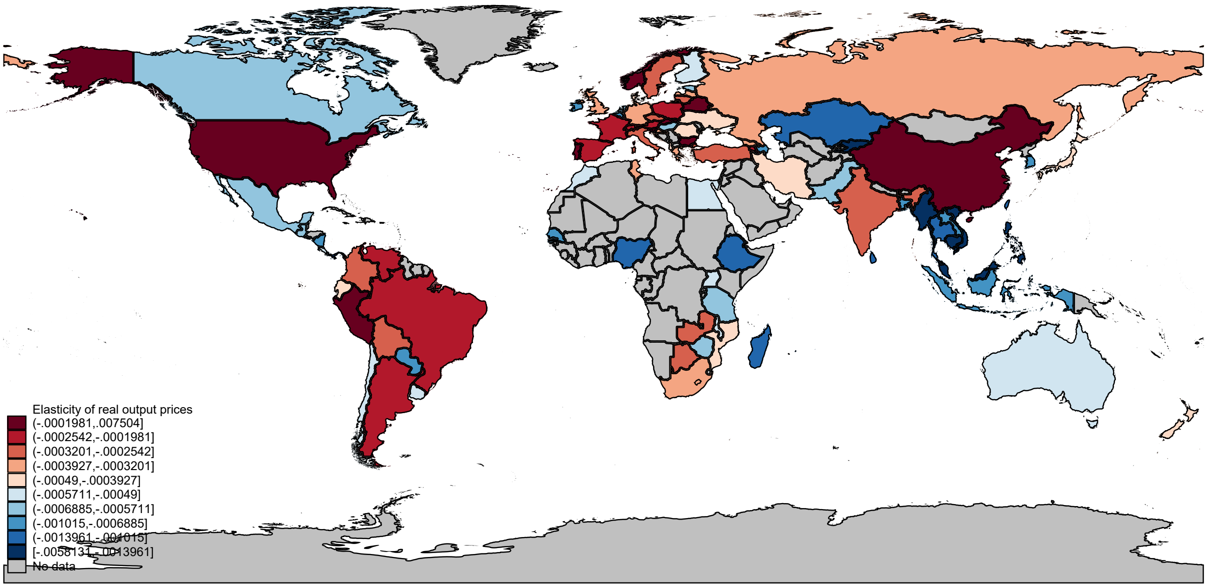
Figure 1: Existence and uniqueness



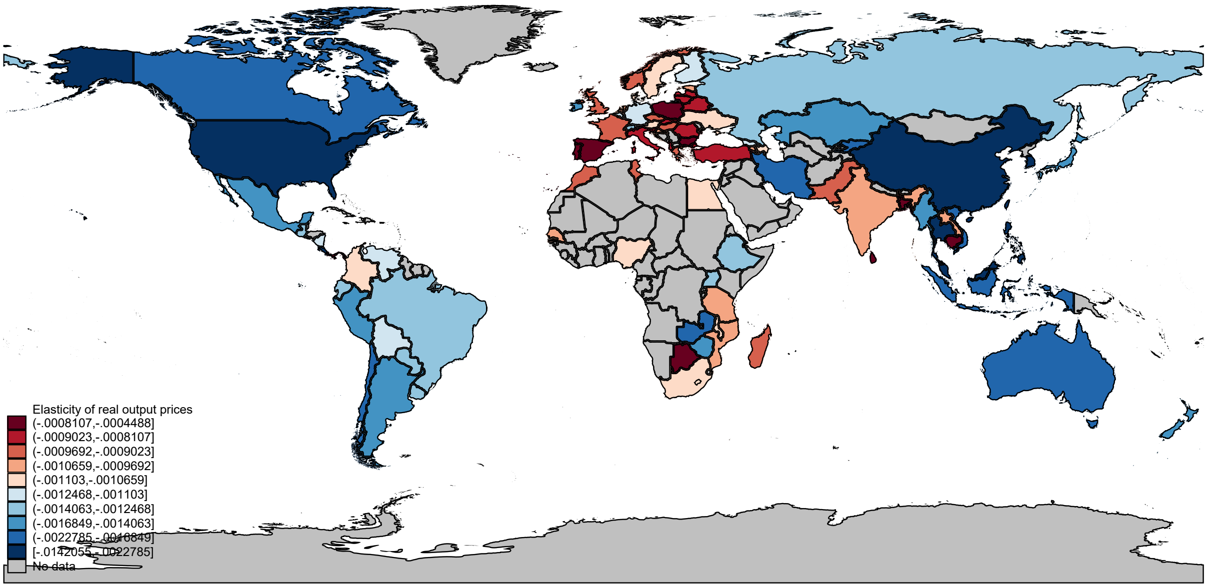
Notes: This figure shows the regions in (ϕ, ψ) space for which the gravity equilibrium is unique and interior. Existence can be guaranteed throughout the entire region except for the case $1 + \phi + \psi = 0$.

Figure 2: The network effect of a U.S.-China trade war

(a) Degree 1 Effect



(b) Total Effect



Notes: This figure depicts the elasticity of real output prices to an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The top panel depicts the "Degree 1" effect, which is the effect of the direct shock on the U.S. and China on all countries through the trade network, holding constant the output prices and quantities of their trading partners fixed. The bottom panel shows the total effect of the trade war on the real output price in each country.

A Proofs

A.1 Proof of Theorem 1

Proof. **Part i)** The proof proceeds as follows. First we transform the equilibrium conditions to the associated non-linear integral equations form. However, we cannot directly apply the fixed point theorem for the non-linear integral equations since the system does not map to a compact space. Therefore we need to "scale" the system so that we can apply the fixed point, which implies that there exists a fixed point for the scaled system. Finally we construct a fixed point for the original non-linear integral equations. In this subsection, we show how to set up in the associated integral equation form, and apply the fixed point theorem. The other technical parts are proven in Online Appendix B.4. Note that our result proposition is a natural generalization of Karlin and Nirenberg (1967) to a system of non-linear integral equations.

Define z as follows:

$$z \equiv \begin{pmatrix} (x_i)_i \\ (y_i)_i \end{pmatrix} \equiv \begin{pmatrix} \left(p_i^{1+\psi+\phi} P_i^{-\psi}\right)_i \\ \left(P_i^{-\phi}\right)_i \end{pmatrix}.$$

Then the system of equations (6) and (7) of the general equilibrium gravity model is re-written in vector form:

$$\begin{pmatrix} (x_i)_i \\ (y_i)_i \end{pmatrix} = \begin{pmatrix} \sum_j K_{ij} \bar{c}_i^{-1} \bar{c}_j x_j^{a_{11}} y_j^{a_{12}} \\ \sum_j K_{ji} x_j^{a_{21}} y_j^{a_{22}} \end{pmatrix}, \tag{23}$$

where $A = (a_{ij})_{i,j}$ is given by

$$A = \begin{pmatrix} \frac{1+\psi}{1+\psi+\phi} & -\frac{1+\phi}{1+\psi+\phi} \\ -\frac{\phi}{1+\psi+\phi} & \frac{\psi}{1+\psi+\phi} \end{pmatrix}.$$

Also the kernel, K_{ij}, is given by $K_{ij} = \tau_{ij}^{-\phi}$. Notice that we cannot directly apply Browser's fixed point theorem for equation (23) since there are no trivial compact domain for equation (23). Therefore consider the following "scaled" version of equation (23).

$$z = \begin{pmatrix} (x_i)_i \\ (y_i)_i \end{pmatrix} = \begin{pmatrix} \frac{\sum_j K_{ij} \bar{c}_i^{-1} \bar{c}_j x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j} K_{ij} \bar{c}_i^{-1} \bar{c}_j x_j^{a_{11}} y_j^{a_{12}}} \\ \frac{\sum_j K_{ji} x_j^{a_{21}} y_j^{a_{22}}}{\sum_{i,j} K_{ji} x_j^{a_{21}} y_j^{a_{22}}} \end{pmatrix} \equiv F(z), \tag{24}$$

and F is defined over the following compact set C:

$$C = \left\{ x \in \Delta\left(R_+^N\right); x_i \in [\underline{x}, \overline{x}]\, \forall i \right\} \times \left\{ y \in \Delta\left(R_+^N\right); y_i \in [\underline{y}, \overline{y}]\, \forall i \right\}, \tag{25}$$

where the bounds for x and y are respectively given as follow:

$$\bar{x} \equiv \max_{i,j} \frac{K_{ij} \bar{c}_i^{-1} \bar{c}_j}{\sum_{i,j} K_{ij} \bar{c}_i^{-1} \bar{c}_j} \quad \underline{x} \equiv \min_{i,j} \frac{K_{ij} \bar{c}_i^{-1} \bar{c}_j}{\sum_{i,j} K_{ij} \bar{c}_i^{-1} \bar{c}_j}$$
$$\bar{y} \equiv \max_{i,j} \frac{K_{ji}}{\sum_{i,j} K_{ji}} \quad \underline{y} = \min_{i,j} \frac{K_{ji}}{\sum_{i,j} K_{ji}}.$$

It is trivial to show that F maps from C to C and continuous over the following compact set C, so that we can apply Brouwer's fixed point and there exists an fixed point $z^* \in C$.

There are two technical points needed to be proven; first, there exists a fixed point for the original (un-scaled) system (23); second, the equilibrium z^* is strictly positive. These two claims are proven in Lemma 1 and Lemma 2 in Online Appendix B.4, respectively.

Part (iii) It suffices to show that there exists a unique interior solution for equation (23). Suppose that there are two strictly positive solutions (x_i, y_i) and $(\widehat{x}_i, \widehat{y}_i)$ such that there does not exist $t, s > 0$ satisfying

$$(x_i, y_i) = (t\widehat{x}_i, s\widehat{y}_i)\,.$$

Namely, two solutions are "linearly independent." First note that for any $i \in S$, we can evaluate one of equation (23).

$$\frac{x_i}{\widehat{x}_i} = \frac{1}{\widehat{x}_i}\sum_{j\in S} K_{ij}\bar{c}_i^{-1}\bar{c}_j \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} (\widehat{x}_j)^{\alpha_{11}} (\widehat{y}_j)^{\alpha_{12}} \tag{26}$$

$$\leqslant \max_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \max_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}. \tag{27}$$

Taking the maximum of the left hand side,

$$\max_{i\in S}\frac{x_i}{\widehat{x}_i} \leqslant \max_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \max_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}. \tag{28}$$

Lemma 3 in Online Appendix B.4 shows that the inequality is actually strict. Analogously, we obtain

$$\min_{i\in S}\frac{x_i}{\widehat{x}_i} \geqslant \min_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \min_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}. \tag{29}$$

Dividing equation (28) by equation (29), it is shown that

$$1 \leqslant \mu_x \equiv \frac{\max_{i\in S}\frac{x_i}{\widehat{x}_i}}{\min_{i\in S}\frac{x_i}{\widehat{x}_i}} < \frac{\max_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}}}{\min_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}}} \times \frac{\max_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}}{\min_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}} = \mu_x^{|\alpha_{11}|} \times \mu_y^{|\alpha_{12}|},$$

where

$$\mu_y \equiv \frac{\max_{i\in S}\frac{y_i}{\widehat{y}_i}}{\min_{i\in S}\frac{y_i}{\widehat{y}_i}}.$$

The same argument is applied to obtain the following inequality

$$1 \leqslant \mu_y \equiv \frac{\max_{i\in S}\frac{y_i}{\widehat{y}_i}}{\min_{i\in S}\frac{y_i}{\widehat{y}_i}} < \frac{\max_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{21}}}{\min_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{21}}} \times \frac{\max_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{22}}}{\min_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{22}}} = \mu_x^{|\alpha_{21}|} \times \mu_y^{|\alpha_{22}|}.$$

Taking logs in the two inequalities and exploiting the restriction we can write

$$\begin{pmatrix}\ln\mu_x\\ \ln\mu_y\end{pmatrix} < \underbrace{\begin{pmatrix}|\alpha_{11}| & |\alpha_{12}|\\ |\alpha_{21}| & |\alpha_{22}|\end{pmatrix}}_{=|A|}\begin{pmatrix}\ln\mu_x\\ \ln\mu_y\end{pmatrix}, \tag{30}$$

which from the Collatz–Wielandt formula, equation (30) implies that the largest eigenvalue of $|A|$ is greater than one:

$$\rho\left(|A|\right) > 1.$$

However, we prove in Lemma 4 in Online Appendix B.4 that the sufficient condition in part (ii) of Theorem 1 guarantees that the largest absolute eigenvalue is 1. As a result, this is a contradiction.

Quasi-symmetry) When the bilateral trade frictions satisfy quasi-symmetry, then we can reduce the system to$N$ dimensional integral system (see Online Appendix B.3). Then the same logic used above can be applied to show there exists a unique strictly positive solution. As mentioned above, this result follows directly from Karlin and Nirenberg (1967) and is summarized in Theorem 2.19 of Zabreyko, Koshelev, Krasnosel'skii, Mikhlin, Rakovshchik, and Stetsenko (1975). The same argument for (iv) is used for convergence. □

A.2 Proof of Theorem 2

Proof. **Part (i)** Equation (18) is a direct application of the implicit function theorem. Define a function $F : R^{2N} \to R^{2N}$ as follows.

$$F_i\left(\left(\ln p_i\right)_{i=1}^N, \left(\ln P_i\right)_{i=1}^N\right) = \kappa \bar{c}_i p_i^{1+\psi} P_i^{-\psi} - \kappa \sum_k \tau_{ik}^{-\phi} p_i^{-\phi} \bar{c}_k P_k^{\phi-\psi} p_k^{1+\psi} \Xi \xi_k$$

$$F_{N-1+i}\left(\left(\ln p_i\right)_{i=1}^N, \left(\ln P_i\right)_{i=1}^N\right) = P_i^{-\phi} - \sum_k \tau_{ki}^{-\phi} p_k^{-\phi}$$

Applying the implicit function theorem for F, we obtain the comparative static (18). As in Dekle, Eaton, and Kortum (2008), the matrix $\mathbf{A}$ and $\mathbf{T}$ can be expressed in terms of observables.

Part (ii) Notice that $\mathbf{A}$ is written as follows:

$$\mathbf{A} = \mathbf{S}\left(\mathbf{I} - \mathbf{S}^{-1}\mathbf{D}\right),$$

where $\mathbf{S}$ and $\mathbf{D}$ are defined by equation (16). If the largest absolute eigenvalue for $\mathbf{S}^{-1}\mathbf{D}$ is less than 1, then $\mathbf{A}^{-1}$ is expressed as $\sum_{k=0}^{\infty}\left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}$. Note that we could have similarly written $\mathbf{A} = -\left(\mathbf{I} - \mathbf{S}\mathbf{D}^{-1}\right)\mathbf{D}$, so that if the largest eigenvalue for $\mathbf{S}\mathbf{D}^{-1}$ is less than 1, $\mathbf{A}^{-1}$ can be expressed as $-\sum_{k=0}^{\infty}\mathbf{D}^{-1}\left(\mathbf{S}\mathbf{D}^{-1}\right)^k$, as noted in footnote 23.

Part (iii) When quasi-symmetric assumption and balanced trade are imposed, destination effects are proportional to the associated origin effects. Therefore as shown in Online Appendix B.3, the equilibrium is characterized by the following single non-linear system of equations:

$$\underbrace{p_i^{1+\psi-\psi\frac{1+\psi+\phi}{\psi-\phi}}\left(\frac{\tau_i^A}{\tau_i^B}\right)^{-\psi\frac{\phi}{\psi-\phi}}\left(\bar{c}_i\right)^{\frac{\phi}{\psi-\phi}}}_{=Y_i/\kappa} = \sum_{j\in S}\underbrace{\tilde{\tau}_{ij}^{-\phi} p_i^{-\phi}\left(\tau_i^A\right)^{-\phi}\left(\tau_j^A\right)^{-\phi} p_j^{-\phi}}_{=X_{ij}/\kappa} \tag{31}$$

As before, define z_i for all $i \in S$ as follows:

$$z_i\left(p;\tau\right) = \kappa p_i^{1+\psi-\psi\frac{1+\psi+\phi}{\psi-\phi}}\left(\frac{\tau_i^A}{\tau_i^B}\right)^{-\psi\frac{\phi}{\psi-\phi}}\left(\bar{c}_i\right)^{\frac{\phi}{\psi-\phi}} - \kappa\sum_{j\in S}\tilde{\tau}_{ij}^{-\phi} p_i^{-\phi}\left(\tau_i^A\right)^{-\phi}\left(\tau_j^A\right)^{-\phi} p_j^{-\phi}.$$

Then apply the implicit function theorem to (31),

$$\frac{\partial \ln p}{\partial \ln \tau_{il}} = -2\left(\underbrace{\frac{\partial z}{\partial \ln p}}_{N\times N}\right)^{-1}\underbrace{\frac{\partial z}{\partial \ln \tau_{il}}}_{N\times 1}. \tag{32}$$

Note that numerical number 2 shows up to preserve quasi-symmetry of trade frictions. As in the general trade friction case, $\frac{\partial z}{\partial \ln p}$ is expressed as observables:

$$\frac{\partial z}{\partial \ln p} = \left[\phi\frac{1+\psi+\phi}{\phi-\psi}\right]\left[\mathbf{Y} + \frac{\phi-\psi}{1+\psi+\phi}\mathbf{X}\right],$$

where $\mathbf{Y} = diag\,(Y_i)$ and $\mathbf{X} = (X_{ij})_{i,j \in S}$. Define $\mathbf{A}$ as follows:

$$\mathbf{A} = \mathbf{Y} + \frac{\phi - \psi}{1 + \psi + \phi}\mathbf{X}.$$

From Lemma 5, $\mathbf{A}$ has positive diagonal elements and is dominant of its rows. Equation (32) is

$$\frac{\partial \ln p_i}{\partial \ln \tau_{il}} = -2\frac{\phi - \psi}{1 + \psi + \phi}A_{ii}^{-1}X_{il}, \quad \frac{\partial \ln p_j}{\partial \ln \tau_{il}} = -2\frac{\phi - \psi}{1 + \psi + \phi}A_{ji}^{-1}X_{il}.$$

Since the price index is log-linear w.r.t. the associated output price, we have

$$\frac{\partial \ln P_i}{\partial \ln \tau_{il}} = \frac{1 + \psi + \phi}{\psi - \phi}\frac{\partial \ln p_i}{\partial \ln \tau_{il}}.$$

Therefore, the real output price is

$$\frac{\partial \ln (p_i/P_i)}{\partial \ln \tau_{il}} = \left(\frac{2\phi + 1}{\phi - \psi}\right)\frac{\partial \ln p_i}{\partial \ln \tau_{il}} = -2\frac{2\phi + 1}{1 + \psi + \phi}A_{ii}^{-1}X_{il}.$$

Then the ordering of the real output price follows from part (iii) of Theorem 2 , $A_{ii}^{-1} > A_{ji}^{-1}$ for $j \in S - i$. The result for real expenditure then follows immediately from C.5 and equation (11), as $E_i/P_i \propto \bar{c}_i\,(p_i/P_i)^{1+\psi}$:

$$\frac{\partial \ln (p_i Q_i/P_i)}{\partial \ln \tau_{il}} = -2\frac{2\phi + 1}{1 + \psi + \phi}(1 + \psi)\,A_{ii}^{-1}X_{il} + \underbrace{\frac{\partial \ln \kappa}{\partial \ln \tau_{il}}}_{\text{common}}.$$

By the same argument, the ordering of $\left(\frac{\partial \ln(p_i Q_i/P_i)}{\partial \ln \tau_{il}}\right)$ follows. □

B Online Appendix (not for publication)

This Online Appendix provides some additional results referenced in the paper.

B.1 Recovering the equilibrium variables from the Universal Gravity conditions

In this subsection, we show how the universal gravity conditions C.1-C.5 can be combined to derive equations (6) and (7), which can be used to solve for equilibrium prices and price indices up to scale. We then show how information of these prices and price indices up-to-scale can be used to solve for the level of real output prices $\{p_i/P_i\}_{i\in S}$ and, combined with the numeraire in C.6, to determine the equilibrium level of income $\{Y_i\}_{i\in S}$, expenditure $\{E_i\}_{i\in S}$, and trade flows $\{X_{ij}\}_{i,j\in S}$. Finally, we show how all other endogenous variables can be recovered up-to-scale if the equilibrium prices and price indices are known up to scale.

B.1.1 From Universal Gravity C.1-C.5 to Equations (6) and (7)

We first show Universal Gravity C.1-C.5 imply equations (6) and (7).

Combing C.1 and C.2 (in particular the gravity equation (10)):

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j, \tag{33}$$

where recall from C.2 that the price index can be written as:

$$P_i^{-\phi} \equiv \sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi} \tag{34}$$

Combining equation (33) with C.(4) and C.(5) yields:

$$p_i Q_i = \sum_{j\in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j Q_j \tag{35}$$

Finally, we substitute C.3 into equation (35) to yield:

$$p_i \left(\bar{c}_i \left(\frac{p_i}{P_i} \right)^{\psi} \right) = \sum_{j\in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j \left(\bar{c}_j \left(\frac{p_j}{P_j} \right)^{\psi} \right) \tag{36}$$

Note that equations (34) and (36) are equivalent to equations (6) and (7). Hence, C.1-C. 5 imply equations (6) and (7), as claimed. There are two things to note about equilibrium equations (34) and (36): first, they depend only on output prices $\{p_i\}$, the price indices $\{P_i\}_{i\in S}$, and exogenous model fundamentals (in particular, they do not depend on the endogenous scalar κ); second, they are homogeneous of degree zero with respect to $\{p_i, P_i\}_{i\in S}$, so the scale of prices (and price indices) are undetermined.

B.1.2 From Equations (6) and (7) to endogenous variables

We now show that given a solution to equations (6) and (7), we can construct all endogenous variables in the models. We divide the derivations into endogenous variables determined up to scale and endogenous variables for which the scale is known (given the choice of numeraire in C.6. Suppose that we have a set of prices $\{p_i\}_{i\in S}$ and price indices $\{P_i\}_{i\in S}$ that solve equations (6) and (7). Note that because equations (6) and (7) are homogeneous of degree zero with respect to $\{p_i, P_i\}_{i\in S}$, for any scalar α, the normalized prices $\tilde{p}_i \equiv \frac{1}{\alpha} p_i$ and price indices $\tilde{P}_i \equiv \frac{1}{\alpha} P_i$ continue to satisfy equations (6) and (7).

We first solve for the real output price. Note that for any choice of α, the real output price $\{p_i/P_i\}_{i\in S}$ remains unchanged, so its level is unaffected by the unknown scalar.

We now solve for quantities. From equation (11), the quantity in location i does not depend on α, but it does depend on the unknown scalar κ as follows:

$$Q_i = \kappa \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi}.$$

Hence, equilibrium quantities are only determine up-to-scale.

We now solve for income and expenditure. From C.4 and C.5 we have:

$$E_i = Y_i = p_i Q_i.$$

Applying the numeraire in C.6 then yields:

$$\begin{aligned}
\sum_{i\in S} Y_i = 1 &\iff \\
\sum_{i\in S} p_i Q_i = 1 &\iff \\
\kappa\alpha \sum_{i\in S} \tilde{p}_i \bar{c}_i \left(\frac{\tilde{p}_i}{\tilde{P}_i}\right)^{\psi} = 1 &\iff \\
\kappa\alpha = \left(\sum_{i\in S} \tilde{p}_i \bar{c}_i \left(\frac{\tilde{p}_i}{\tilde{P}_i}\right)^{\psi}\right)^{-1}&,
\end{aligned}$$

which, as claimed, pins down the product of the unknown quantity scalar and unknown price scalar. Given $\kappa\alpha$, we can now determine the level of income and expenditure as follows:

$$\begin{aligned}
E_i = Y_i = p_i Q_i &\iff \\
E_i = Y_i = \frac{\tilde{p}_i \bar{c}_i \left(\frac{\tilde{p}_i}{\tilde{P}_i}\right)^{\psi}}{\left(\sum_{j\in S} \tilde{p}_j \bar{c}_j \left(\frac{\tilde{p}_j}{\tilde{P}_j}\right)^{\psi}\right)}&,
\end{aligned}$$

as claimed.

We now determine the level of trade flows using equation (33):

$$\begin{aligned}
X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j &\iff \\
X_{ij} = \frac{\tau_{ij}^{-\phi} \tilde{p}_i^{-\phi}}{\sum_{k\in S} \tau_{kj}^{-\phi} \tilde{p}_k^{-\phi}} \left(\frac{\tilde{p}_j C_j \left(\frac{\tilde{p}_j}{\tilde{P}_j}\right)^{\psi}}{\left(\sum_{k\in S} \tilde{p}_k C_k \left(\frac{\tilde{p}_k}{\tilde{P}_k}\right)^{\psi}\right)}\right)&.
\end{aligned}$$

Other than real output prices $\{p_i/P_i\}_{i\in S}$, income $\{Y_i\}_{i\in S}$, expenditure $\{E_i\}_{i\in S}$, and trade flows $\{X_{ij}\}_{i,j\in S}$, all other endogenous variables are determined only up-to-scale, as they depend either on the price scalar α (i.e. output prices $\tilde{p}_i$, price indices $\tilde{P}_i$, bilateral prices $p_{ij} = \tau_{ij}\tilde{p}_i$, and the quantity traded $Q_{ij} = X_{ij}/\tau_{ij}p_i$) or the quantity scalar κ (i.e. quantities Q_i).

B.2 Proof of Theorem 1 part (ii)

We first provide a general mathematical formulation to incorporate non-interior solutions. Let the equilibrium be a duple $(p_i, Q_i) \in \overline{\mathbb{R}}_+^N \times \mathbb{R}_+^N$ such that for all $i \in S$,

$$Q_i = \sum_j \frac{\tau_{ij}^{-\phi-1} p_i^{-\phi-1}}{\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}} p_j Q_j \tag{37}$$

$$(p_i, Q_i) \in F_i(p, Q) \tag{38}$$

where F is a supply condition, which might be a correspondence. (The fact that F might be correspondence allows us to extend the framework to allow for non-interior solutions). In particular, we define F as follows: We say $(p_i, Q_i) \in F_i(p, Q)$ if and only if

$$sign(\psi)\left[Q_i - \kappa\left(\frac{p_i}{P_i(p)}\right)^{\psi}\right] \geq 0 \tag{39}$$

$$Q_i = \kappa\left(\frac{p_i}{P_i(p)}\right)^{\psi} \quad \text{if } Q_i > 0, \tag{40}$$

and where $\left(\frac{0}{0}\right)$ is defined as 0. That is, if $Q_i = 0$, then we replace C.3 with an inequality. For example, in an economic geography model, inequality constraint (39) corresponds to welfare equalization. If there are people living in location i, then Q_i is given by equality (40). If not, then the welfare living in location i should be lower than one obtained in other places, which is represented as the inequality (39).

As we mentioned in Section 3, we restrict our attention to non-trivial equilibria where there is positive production in at least one location. To show that all (non-trivial) equilibria are interior, it then suffices to show that if some locations produces nothing, then all other locations must also produce nothing.

Suppose that $Q_l = 0$ for some $l \in S$. Then from equation (37) for l:

$$0 = \sum_j \underbrace{\frac{\tau_{lj}^{-\phi} p_l^{-\phi-1}}{\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}} p_j Q_j}_{\geq 0}, \tag{41}$$

which in turn implies that for all $j \in S$,

$$\frac{\tau_{lj}^{-\phi} p_l^{-\phi-1}}{g_j} p_j Q_j = 0, \tag{42}$$

where $g_j = \sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}$.

Note that there are two reasons why equation (42) is zero for all j; either (1) ; or (2) for all $j \in S$, $\tau_{lj}^{-\phi} \frac{p_j Q_j}{g_j} = 0$. We will prove a contradiction in both cases.

First assume that (1) $p_l^{-\phi-1} = 0$, which if $\phi > -1$ implies that $p_l = \infty$. While $(p_l, Q_l) = (\infty, 0)$ satisfies equation (41), it does not satisfy equation (38). To see this, note:

$$0 = Q_i < \kappa\left(\frac{p_i}{g_i^{-\frac{1}{\phi}}}\right)^{\psi} = \infty,$$

which contradicts with equation (39) since $\psi \geq 0$. Therefore p_l needs to be finite, $p_l < \infty$.

Now assume that (2) for all $j \in S$, $\tau_{lj}^{-\phi} \frac{p_j Q_j}{g_j} = 0$. Since the price for country l is finite, equation (42) is reduced to

$$\tau_{lj}^{-\phi}\frac{p_jQ_j}{g_j}=0$$

for all $j\in S$. An equivalent expression is that for all countries connected with l, $j\in S_l=\{k\in S;\tau_{lk}<\infty\}$,

$$p_jQ_j=0 \quad \text{or} \quad g_j=\infty. \tag{43}$$

Fix any country $j\in S_l$. Suppose that $p_j,Q_j>0$ Then equation (43), $g_j=\infty$. Then for all $(p_j,Q_j)\in\overline{\mathbb{R}}_+\times\mathbb{R}$ if $\psi\geq 0$ we have

$$\infty=\kappa\left(\frac{p_j}{g_j^{-\frac{1}{\phi}}}\right)^{\psi}\leq Q_j=0,$$

which is a contradiction. Therefore in order to satisfy equation (43), p_j or Q_j needs to be zero. Suppose that $p_j=0$. Then we have

$$0=\kappa\left(\frac{p_j}{g_j^{-\frac{1}{\phi}}}\right)^{\psi}\leq Q_j.$$

If $Q_j>0$, then C. (3). Therefore, $Q_j=0$. Therefore Q_j needs to be zero for all $j\in S_l$.

So far, we have shown that if $Q_l=0$ then the connected countries $j\in S_l$ produce nothing, $Q_j=0$. Because of strong connectedness, any country n is connected with l through third countries. Therefore, by repeating the argument along with the chain, we have $Q_n=0$ for all $n\in S$.

As a result, if $\phi\geq -1$, and $\psi\geq 0$ then all equilibria are interior, as claimed.

B.3 Quasi-symmetric trade frictions

In this subsection, we show that when trade frictions are quasi-symmetric, then balanced trade implies that the origin and destination fixed effects of the gravity trade flow expression are equal up to scale.

We first formally define "quasi-symmetry." We say that the set of trade frictions $\{\tau_{ij}\}_{i,j\in S}$ are *quasi-symmetric* if there exists a set of origin scalars $\left\{\tau_i^A\right\}_{i\in S}\in\mathbb{R}_{++}^N$, destination scalars $\left\{\tau_i^B\right\}_{i\in S}\in\mathbb{R}_{++}^N$, and a symmetric matrix $\{\tilde{\tau}_{ij}\}_{i,j\in S}$ where $\tilde{\tau}_{ij}=\tilde{\tau}_{ji}$ for all $i,j\in S$ such that we can write:

$$\tau_{ij}=\tau_i^A\tau_i^B\tilde{\tau}_{ij}\ \forall i,j\in S.$$

Loosely speaking, quasi-symmetric trade frictions are those that are reducible to a symmetric component and exporter- and importer-specific components. While restrictive, it is important to note that the vast majority of papers which estimate gravity equations assume that trade frictions are quasi-symmetric; for example Eaton and Kortum (2002) and Waugh (2010) assume that trade frictions are composed by a symmetric component that depends on bilateral distance and on a destination or origin fixed effect.

Combining the universal gravity conditions C. 1 and C. 2 allows us to write the value of bilateral trade flows from i to j as:

$$X_{ij}=\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j,$$

which we now re-write as:

$$X_{ij}=\kappa\tau_{ij}^{-\phi}\gamma_i\delta_j, \tag{44}$$

where we call $\gamma_i \equiv p_i^{-\phi}$ the *origin fixed effect* and $\delta_i \equiv P_i^{\phi} E_i = \bar{c}_i P_i^{\phi-\psi} p_i^{1+\psi}$ the *destination fixed effect.*

Proposition 1. *If trade frictions are quasi-symmetric, then in any model within the universal gravity framework, the product of the equilibrium origin fixed effect and the origin scalar will be equal to the product of the equilibrium destination fixed effects and the destination fixed effect up to scale, i.e.: for some scalar* $\lambda \geqslant 0$,

$$\left(\tau_i^A\right)^{-\phi} \gamma_i = \lambda \left(\tau_i^B\right)^{-\phi} \delta_i \; \forall i \in S.$$

Proof. We first note that market clearing condition C.4 and balanced trade condition C.5 together imply that: $\sum_{j \in S} X_{ij} = \sum_{j \in S} X_{ji} \; \forall i \in S$. Combining this with the gravity expression (44) and quasi-symmetry implies:

$$\sum_j \underbrace{\kappa \tau_{ij}^{-\phi} \gamma_i \delta_j}_{=X_{ij}} = \sum_j \underbrace{\kappa \tau_{ji}^{-\phi} \gamma_j \delta_i}_{X_{ji}} \iff$$

$$\frac{\left(\tau_i^A\right)^{-\phi} \gamma_i}{\left(\tau_i^B\right)^{-\phi} \delta_i} = \frac{\sum_{j \in S} \tilde{\tau}_{ij}^{-\phi} \left(\tau_j^A\right)^{-\phi} \gamma_j}{\sum_{j \in S} \tilde{\tau}_{ij}^{-\phi} \left(\tau_j^B\right)^{-\phi} \delta_j} = \sum_{j \in S} \frac{\tilde{\tau}_{ij}^{-\phi} \left(\tau_j^B\right)^{-\phi} \delta_j}{\sum_{k \in S} \tilde{\tau}_{ik}^{-\phi} \left(\tau_k^B\right)^{-\phi} \delta_k} \times \frac{\left(\tau_j^A\right)^{-\phi} \gamma_j}{\left(\tau_j^B\right)^{-\phi} \delta_j}.$$

It is easy to show that $\frac{\left(\tau_i^A\right)^{-\phi} \gamma_i}{\left(\tau_i^B\right)^{-\phi} \delta_i} = 1$ is a solution to this problem for any kernel. From the Perron-Frobenius theorem, the solution is unique up to scale. Therefore we have:

$$\left(\tau_i^A\right)^{-\phi} = \lambda \left(\tau_i^B\right)^{-\phi} \delta_i \; \forall i \in S, \tag{45}$$

as required. □

Proposition 1 has a number of important implications. First, Proposition 1 allows one to simplify the equilibrium system of equations 6 and 7 into a single non-linear equation when $\phi \neq \psi$:

$$\left(p_i^{\frac{1+\psi+\phi}{\psi-\phi}}\right)^{-\phi} = (\lambda)^{\frac{\phi}{\psi-\phi}} \left(\bar{c}_i\right)^{\frac{\phi}{\psi-\phi}} \sum_{j \in S} \tilde{\tau}_{ij}^{-\phi} \left(\tau_i^A\right)^{\frac{\phi^2}{\psi-\phi}} \left(\tau_i^B\right)^{-\frac{\phi\psi}{\psi-\phi}} \left(\tau_j^A\right)^{-\phi} p_j^{-\phi}, \quad i \in S, \tag{46}$$

which simplifies the characterization of the theoretical and empirical properties of the equilibrium. Notice that λ is an endogenous scalar. Since (46) holds for any location $i \in S$, λ is expressed as

$$\lambda^{\frac{\phi}{\psi-\phi}} = \frac{\sum_i \left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}}}{\sum_i \sum_{j \in S} \tau_{ij}^{-\phi} \left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} \bar{c}_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}}.$$

Substituting above expression, we obtain:

$$\frac{\left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}}}{\sum_i \left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}}} = \sum_{j \in S} \frac{\tau_{ij}^{-\phi} \left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} \bar{c}_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}}{\sum_i \sum_{j \in S} \tau_{ij}^{-\phi} \left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} \bar{c}_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}}.$$

Notice that the system is now homogeneous degree 0. Therefore, if $\phi \notin \left\{-\frac{1}{2}, \psi, 0\right\}$, then we can normalize $\lambda = 1$ without loss of generality.

Second, by showing that the origin and destination fixed effects are equal up to scale, Proposition 1 provides offers an analytical characterization of the equilibrium. For example, given the definition of the origin and destination fixed effects, Proposition 1 can equivalently be expressed as:

$$p_i P_i \propto \frac{\tau_i^B}{\tau_i^A} E_i^{-\frac{1}{\phi}}, \tag{47}$$

i.e. there is a log-linear relationship between output prices, the price index and total expenditure in a location.

Third, it is straightforward to show that quasi-symmetry implies that equilibrium trade flows will be bilaterally symmetric, i.e. $X_{ij} = X_{ji}$ for all $i, j \in S$, allowing one to test whether trade frictions are quasi-symmetric directly from observed trade flow data.

Finally, we should note that the results of Proposition 1 have already been used in the literature for particular models, albeit implicitly. The most prominent example is Anderson and Van Wincoop (2003), who use the result to show the bilateral resistance is equal to the price index.[39] To our knowledge, Head and Mayer (2013) are the first to recognize the importance of balanced trade and market clearing in generating the result for the Armington model; however, Proposition 1 shows that the result applies more generally to any model with quasi-symmetrical trade frictions in the universal gravity framework.

B.4 Proofs of the lemmas used in Theorem (1)

There are 4 lemmas which are not proven in the paper. In this section, we discuss them carefully. Before proving these lemmas, we discuss how we use them in the proof. In the proof, we show a fixed point for the "scaled" system, not the actual system. Therefore it needs to be shown that there exists a fixed point for the actual system, which is shown in Lemma 1. Then we argue that the solution we obtain is strictly positive, which is guaranteed by Assumption 1. We emphasize the connectivity assumption is crucial here. These two lemmas are used in **Part i)** Theorem 1.

Part ii) shows that there exists an unique solution. During the proof, we argue that 28 should hold with strict inequality. Again the connectivity allows us to show this result (Lemma 3). After establishing this strict inequality, we follow the argument by Allen, Arkolakis, and Li (2014), which requires that the largest absolute eigenvalues for $|A|$ are less than 1. Since A is a 2-by-2 matrix, we can compute the eigenvalues by hand and show that one of them is exactly 1, and the other is less than 1 if the conditions in **Part ii)** are satisfied.

Lemma 1. *Suppose that z solves (24). Then there exists $\hat{z}$ solving (23).*

Proof. First it is easy to show[40]

$$\sum_{i,j \in S} K_{ij} \bar{c}_i^{-1} \bar{c}_j x_j^{a_{11}} y_j^{a_{12}} = \sum_{i,j \in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}. \tag{50}$$

[39] The result is also used in economic geography by Allen and Arkolakis (2014) to simplify a set on non-linear integral equations into a single integral equation.

[40] To see this, multiply $\bar{c}_i x_i^{a_{21}} y_i^{a_{22}} = \bar{c}_i p_i^{-\phi}$, to the first equations of (24) and sum over i;

$$\sum_i \bar{c}_i p_i^{1+\psi} P_i^{-\psi} = \frac{\sum_i \sum_j K_{ij} \bar{c}_j x_i^{a_{21}} y_i^{a_{22}} x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j} K_{ij} \bar{c}_i^{-1} \bar{c}_j x_j^{a_{11}} y_j^{a_{12}}}. \tag{48}$$

Also multiply $\bar{c}_i x_i^{a_{11}} y_i^{a_{12}} = \bar{c}_i P_i^{\phi-\psi} p_i^{1+\psi}$ to the second equations (24) and sum over i;

$$\sum_{i \in S} \bar{c}_i p_i^{1+\psi} P_i^{-\psi} = \frac{\sum_{i \in S} \sum_{j \in S} K_{ij} \bar{c}_j x_i^{a_{21}} y_i^{a_{22}} x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i \in S, j \in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}}. \tag{49}$$

Notice that the LHS is the same as one in (48). Also the numerator of the RHS in (48) is the same as one

Guess a solution

$$\widehat{z}=\begin{pmatrix}(\widehat{x}_i)_i\\(\widehat{y}_i)_i\end{pmatrix}=\begin{pmatrix}t^{-1}(x_i)_i\\t^{-1}(y_i)_i\end{pmatrix}, \tag{51}$$

where $t=\left(\sum_{i,j\in S}K_{ij}\bar{c}_i^{-1}\bar{c}_jx_j^{a_{21}}y_j^{a_{22}}\right)^{\frac{1}{1-a_{11}-a_{12}}}=\left(\sum_{i,j\in S}K_{ji}x_j^{a_{21}}y_j^{a_{22}}\right)^{\frac{1}{1-a_{21}-a_{22}}}$.[41] Then it is easy to verify that (51) solves (23); in particular, note that

$$\begin{aligned}\widehat{x}_i&=t^{-1}\frac{\sum_{j\in S}K_{ij}\bar{c}_i^{-1}\bar{c}_jx_j^{a_{11}}y_j^{a_{12}}}{\sum_{i,j\in S}K_{ij}\bar{c}_i^{-1}\bar{c}_jx_j^{a_{11}}y_j^{a_{12}}}=t^{1-a_{11}-a_{12}}\frac{\sum_{j\in S}K_{ij}\bar{c}_i^{-1}\bar{c}_j\left(\widehat{x}_j\right)^{a_{11}}\widehat{y}_j^{a_{12}}}{\sum_{i,j\in S}K_{ij}\bar{c}_i^{-1}\bar{c}_jx_j^{a_{11}}y_j^{a_{12}}}\\&=\sum_{j\in S}K_{ij}\bar{c}_i^{-1}\bar{c}_j\widehat{x}_j^{a_{11}}\widehat{y}_j^{a_{12}}.\end{aligned}$$

We can also show that the second equations in (23) are also solved in the same vein:

$$\begin{aligned}\widehat{y}_i&=t\frac{\sum_{j\in S}K_{ji}x_j^{a_{21}}y_j^{a_{22}}}{\sum_{i,j\in S}K_{ji}x_j^{a_{21}}y_j^{a_{22}}}=t^{1-a_{21}-a_{22}}\frac{\sum_{j\in S}K_{ji}\widehat{x}_j^{a_{21}}\widehat{y}_j^{a_{22}}}{\sum_{i,j\in S}K_{ji}x_j^{a_{21}}y_j^{a_{22}}}\\&=\sum_{j\in S}K_{ji}\widehat{x}_j^{a_{21}}\widehat{y}_j^{a_{22}}.\end{aligned}$$

The above two equations confirm that $\widehat{x}_i$ and $\widehat{y}_i$ is a solution to (23). □

Lemma 2. *If $\{\tau_{ij}\}_{i,j}$ satisfies Assumption 1, then the fixed point for (24) is strictly positive.*

Proof. We need to consider four different cases for the combinations of a_{11},a_{12} satisfying different inequalities. We will consider the case $a_{11},a_{12}>0$ since the logic in the other cases is the same. We proceed by contradiction. Suppose that there is a solution x to equation (24) such that for some $i\in S$ $x_i=0$. Consider an arbitrary location $n\neq i$ and consider a connected path, $K_{in}^c\equiv K_{i\pi_1}\times...\times K_{\pi_mn}>0$ for some $m(*)$. Then, from the first of equations in (23) notice that

$$x_i=\sum_{j\in S}K_{ij}x_j^{a_{11}}y_j^{a_{12}}\geq\underbrace{K_{i\pi_1}}_{\neq0}x_{\pi_1}^{a_{11}}y_{\pi_1}^{a_{12}}.$$

Note that $K_{i\pi_1}$ is strictly positive due to $(*)$. Then either x_n or y_n or both are zero if a_{11} and $a_{11}>0$. If$x_n=0$ this argument holds for any$n$ so this is a contradiction with the non-zero equilibrium proved above. Else if $y_n=0$ we can repeat the argument the second of the equations in (23) to establish another contradiction. Notice that if either of $\alpha_{11},\alpha_{12}=0$ a contradiction is also easy to establish. □

Lemma 3. *Equation 28 holds with strict inequality.*

To that end, define the set of directly connected countries to each location $i\in S$ as $S_i^c\equiv\{j\in S:K_{ij}>0\}$. Then notice that equation (26) combined with our equality assumption on equation (28) yields

$$\frac{x_i}{\widehat{x}_i}=\frac{1}{\widehat{x}_i}\sum_{j\in S_i^c}K_{ij}\bar{c}_i^{-1}\bar{c}_j\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}\left(\widehat{x}_j\right)^{\alpha_{11}}\left(\widehat{y}_j\right)^{\alpha_{12}}=\max_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}}\max_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}.$$

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in (49). Therefore the following double sum terms should be the same:

$$\sum_{i,j}K_{ij}\bar{c}_i^{-1}\bar{c}_jx_j^{a_{11}}y_j^{a_{12}}=\sum_{i\in S,j\in S}K_{ji}x_j^{a_{21}}y_j^{a_{22}}.$$

[41]Notice that $a_{11}+a_{12}=a_{21}+a_{22}$.

Notice that given that $\hat{x}_i$ is a solution, this implies that the following has to be true for all $j \in S_i^c$

$$\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} = \max_{j\in S}\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \quad \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} = \max_{j\in S}\left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}.$$

Now notice that if $\alpha_{11} \neq 0$ then for all $n \in S_i^c$,$x_j/\widehat{x}_j = x_n/\widehat{x}_n$. However, because of C. 1, we assume that there exists an indirectly connected path from any location to any other location, so that repeating this argument for all j and using the indirect connectivity we can prove that $x_j/\widehat{x}_j = x_n/\widehat{x}_n$ for all $j, n \in S$ i.e. the solutions are the same up-to-scale, a contradiction.

Lemma 4. *If $\phi, \psi \geqslant 0$ or $\phi, \psi \leqslant -1$, the eigenvalue for $|A|$ is*

$$\lambda = \frac{\phi - \psi}{1 + \phi + \psi}, 1,$$

and

$$\left|\frac{\phi - \psi}{1 + \phi + \psi}\right| < 1.$$

Proof. Notice that

$$|A| = \begin{pmatrix} \left|\frac{1+\psi}{1+\psi+\phi}\right| & \left|\frac{1+\phi}{1+\psi+\phi}\right| \\ \left|\frac{\phi}{1+\psi+\phi}\right| & \left|\frac{\psi}{1+\psi+\phi}\right| \end{pmatrix} = \begin{pmatrix} \frac{1+\psi}{1+\psi+\phi} & \frac{1+\phi}{1+\psi+\phi} \\ \frac{\phi}{1+\psi+\phi} & \frac{\psi}{1+\psi+\phi} \end{pmatrix}.$$

Then we can solve the following characteristic functions

$$\lambda^2 - \left(\frac{1+\psi}{1+\psi+\phi} + \frac{\psi}{1+\psi+\phi}\right)\lambda + \frac{1+\psi}{1+\psi+\phi}\frac{\psi}{1+\psi+\phi} - \frac{1+\phi}{1+\psi+\phi}\frac{\phi}{1+\psi+\phi} = 0.$$

Then

$$\lambda = \frac{\phi - \psi}{1 + \phi + \psi}, 1.$$

We need to show that $\left|\frac{\phi-\psi}{1+\phi+\psi}\right| < 1$. To show it, it suffices to show

$$g = |1 + \phi + \psi| - |\phi - \psi| > 0$$

Suppose that $\phi, \psi \geq 0$. Then g is strictly positive as follows:

$$\begin{aligned} g &= 1 + \phi + \psi - |\phi - \psi| \\ &\geq 1 + \phi + \psi - (|\phi| + |\psi|) = 1 > 0. \end{aligned}$$

Suppose that $\phi, \psi \leq -1$. Then g is given by

$$g = -1 - \phi - \psi - |\phi - \psi|.$$

If $\phi \leq \psi$, then

$$\begin{aligned} g &= -1 - \phi - \psi + \phi - \psi \\ &= -1 - 2\psi \geq 1. \end{aligned}$$

If $\phi \geq \psi$, then

$$\begin{aligned} g &= -1 - \phi - \psi - \phi + \psi \\ &= -1 - 2\psi \geq 1, \end{aligned}$$

which completes the proof. □

B.5 Lemmas and Proposition used in Theorem 2 (iii)[42]

In this section, we prove the lemma and proposition used in Theorem 2 (iii).

Lemma 5. *If $\phi, \psi \geq 0$ or $\phi, \psi \leq -1$, then A has strictly positive diagonal elements and is diagonal dominant in its rows; namely, for all $i \in S$*

$$A_{ii} > 0, \tag{52}$$

$$|A_{ii}| > \sum_{j \in S-i} |A_{ij}|. \tag{53}$$

Proof. Recall that A matrix is

$$A = \mathbf{Y} + \frac{\phi - \psi}{1 + \psi + \phi}\mathbf{X},$$

and from Lemma 4,

$$\left|\frac{\phi - \psi}{1 + \phi + \psi}\right| < 1.$$

Then the diagonal elements for A are positive; for all $i \in S$,

$$\begin{aligned} A_{ii} &= Y_{ii} + \frac{\phi - \psi}{1 + \psi + \phi}X_{ii} \\ &= Y_{ii} - \left|\frac{\phi - \psi}{1 + \psi + \phi}\right| X_{ii} \\ &> Y_{ii} - X_{ii} \geq 0. \end{aligned}$$

Also, for all $i \in S$,

$$\begin{aligned} &|A_{ii}| - \sum_{l \in S-i} |A_{il}| \\ =&\left|\underbrace{Y_{ii} + \frac{\phi - \psi}{1 + \psi + \phi}X_{ii}}_{>0}\right| - \left|\frac{\phi - \psi}{1 + \psi + \phi}\right| \sum_{l \in S-i} X_{il} \\ =&Y_{ii} + \frac{\phi - \psi}{1 + \psi + \phi}X_{ii} - \left|\frac{\phi - \psi}{1 + \psi + \phi}\right| (Y_i - X_{ii}) \\ =&\left(\underbrace{1 - \left|\frac{\phi - \psi}{1 + \psi + \phi}\right|}_{>0}\right) Y_{ii} + \left[\underbrace{\frac{\phi - \psi}{1 + \psi + \phi} + \left|\frac{\phi - \psi}{1 + \psi + \phi}\right|}_{\geq 0}\right] X_{ii} > 0, \end{aligned}$$

which is equation (53). □

The next proposition plays a crucial role in the proof for Theorem 2 (iii).

Proposition 2. *If A has strictly positive diagonal elements and is dominant of its rows, then for all $i \neq j$,*

$$A^{-1}_{ii} > A^{-1}_{ji} > 0.$$

[42] A similar argument is found in Johnson and Smith (2011).

Proof. The co-factor expansion of A^{-1} is[43]

$$A^{-1}_{ii} - A^{-1}_{ji} = \frac{det\left(A\left[S-i\right]\right) - (-1)^{i+j} det\left(A\left[S-i, S-j\right]\right)}{det\left(A\right)}$$
$$= \frac{det\left(T\right)}{det\left(A\right)},$$

where T is defined as follows:

$$\widetilde{T} = A + \left(\underbrace{\mathbf{0}, \cdots, \mathbf{0}}_{N\times(j-1)}, \underbrace{A_i}_{N\times 1}, I \underbrace{\mathbf{0}, \cdots, \mathbf{0}}_{N\times(N-j)} \right).$$

T is a principal component of $\widetilde{T}$:

$$T = \widetilde{T}\left[S-i, S-i\right].$$

If a matrix C has positive diagonal elements, and is diagonally dominant of its rows, then $det\left(C\right) > 0$. [44] Then if T has such properties, then

$$\frac{det\left(T\right)}{det\left(A\right)} > 0$$

since A is assumed to have these properties. Thus it suffices to show that T has positive diagonal elements and is dominant of its rows.

By construction of T, it suffices to show

$$A_{kk} > 0 \quad k \in S-i-j \tag{54}$$
$$A_{kk} + A_{ki} > 0 \quad k = j \tag{55}$$
$$|A_{kk}| > \sum_{l\in S-i-k} |A_{kl} + 1_{l=j} A_{ki}| \quad k \in S-i-j \tag{56}$$
$$|A_{kk} + A_{ki}| > \sum_{l\in S-i-k} |A_{kl}| \quad k = j. \tag{57}$$

First we show equation (54) and equation (55). since A has a strictly positive diagonal, for all $k \in S$,

$$A_{kk} > 0,$$

which is equation (54) . Also since A is diagonal dominant,

$$A_{jj} + A_{ji} > \sum_{l\neq j} |A_{jl}| + A_{ji} \geq |A_{ji}| + A_{ji} \geq 0,$$

which is equation (55).

Second, we show equation (56) and equation (57). Fix $k \in N-i-j$. Since A is diagonally dominant,

---

[43] Remember

$$A^{-1}_{ij} = (-1)^{i+j} \frac{det\left(A\left[N-j, N-i\right]\right)}{det\left(A\right)}.$$

[44] See also Theorem 3 of Evmorfopoulos (2012).

$$
\begin{aligned}
|A_{kk}| &> \sum_{l \in S-k} |A_{kl}| \\
&= \sum_{l \in S-k-i-j} |A_{kl}| + |A_{ki}| + |A_{kj}| \\
&\geq \sum_{l \in S-i-k-j} |A_{kl}| + |A_{ki} + A_{kj}| \quad (\because \text{triangle inequality}) \\
&= \sum_{l \in S-i-k} |A_{kl} + 1_{l=j} A_{ki}|,
\end{aligned}
$$

which is equation (56). Fix $k = j$. Since A has positive diagonal elements, and is diagonally dominant,

$$
\begin{aligned}
|A_{kk} + A_{ki}| &\geqslant ||A_{kk}| - |A_{ki}|| \\
&= |A_{kk}| - |A_{ki}| \quad \left(\because |A_{kk}| \geq \sum_{l \in S-k} |A_{kl}| \geqslant |A_{ki}|\right) \\
&= \sum_{l \in S-k-i} |A_{kl}| + |A_{ki}| - |A_{ki}| \\
&= \sum_{l \in S-k-i} |A_{kl}|,
\end{aligned}
$$

which is equation (57). □

B.6 Existence and Uniqueness using Gross Substitutes Methodology (a la Alvarez and Lucas (2007))

In this subsection, we prove the existence and uniqueness of an equilibrium in our universal gravity framework using the gross substitutes methodology employed by Alvarez and Lucas (2007). As we show below, the sufficient conditions here are stronger than we provide in Theorem 1 above.

Proposition 3. *Consider any model within the universal gravity framework. If $\phi > \psi > 0$ and $\tau_{ij} \in (0, \infty)$ for all $i, j \in S$, then the excess demand system of the model satisfies gross substitutes and, as a result, the equilibrium exists and is unique.*

Proof. Recall the equilibrium conditions of the universal gravity framework from equations (6) and

$$
p_i \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j \in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j \bar{c}_j \left(\frac{p_j}{P_j}\right)^{\psi} \ \forall i \in S \tag{58}
$$

$$
P_i^{-\phi} = \sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi} \ \forall i \in S \tag{59}
$$

Substituting equation (59) into (58) yields a single equilibrium system of equations that depends only on the output prices in every location:

$$
p_i^{1+\phi+\psi} \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} \bar{c}_i = \sum_{j \in S} \tau_{ij}^{-\phi} \bar{c}_j p_j^{1+\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} \ \forall i \in S
$$

We define the corresponding excess demand function as:

$$
\begin{aligned}
Z_i(\mathbf{p}) =& \frac{1}{p_i}\left(\frac{1}{\sum_{k\in S}\bar{c}_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}(\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}}(\beta p_k)^{\psi}}\right)\times \\
& \left[\sum_{j\in S}\tau_{ij}^{-\phi}\bar{c}_j p_i^{-\phi}p_j^{1+\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi}\left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{\frac{\psi}{\phi}}\bar{c}\right], \qquad (60)
\end{aligned}
$$

whereP_i is defined by equation (59).This system written as such needs to satisfy 6 properties to be an excess demand system and the gross substitute property to establish existence and uniqueness. The six conditions are:

1.$Z(\mathbf{p})$ is continuous for$\mathbf{p}\in\Delta\left(R_+^N\right)$

2.$Z(\mathbf{p})$ is homogeneous of degree zero.

3.$Z(\mathbf{p})\cdot\mathbf{p}=0$ (Walras' Law).

4. There exists a$k>0$ such that$Z_j(\mathbf{p})>-k$ for all$j$.

5. If there exists a sequence $p^m\to p^0$, where $p^0\neq 0$ and $p_i^0=0$ for some i, then it must be that:

$$max_j\left\{Z_j\left(p^m\right)\right\}\to\infty \qquad (61)$$

and the gross-substitute property:

6. Gross substitutes property:$\frac{\partial Z(p_j)}{\partial p_k}>0$ for all$j\neq k$.

We verify each of these properties in turn. Property 1 is trivial given equation (60) for excess demand. To see property 2, consider multiplying output prices by a scalar $\beta>0$, which immediately yields $Z_i(\beta\mathbf{p})=Z_i(\mathbf{p})$.as required. Property 3 can be seen as follows:

$$
\begin{aligned}
Z(\mathbf{p})\cdot\mathbf{p} =& = \sum_{i\in S}Z_i(\mathbf{p})\,p_i \iff \\
&= \left(\frac{1}{\sum_{k\in S}\bar{c}_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}p_l^{-\phi}\right)^{\frac{\psi}{\phi}}p_k^{\psi}}\right)\times \\
&= \sum_{i\in S}\left(\sum_{j\in S}\tau_{ij}^{-\phi}\bar{c}_j p_i^{-\phi}p_j^{1+\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi}\left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{\frac{\psi}{\phi}}\bar{c}_i\right) \iff \\
&= 0,
\end{aligned}
$$

as required. Property 4 can be seen as follows:

$$
\begin{aligned}
Z_i(\mathbf{p}) &= \frac{1}{p_i}\frac{\sum_{j\in S}\tau_{ij}^{-\phi}\bar{c}_j p_i^{-\phi}p_j^{1+\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S}\bar{c}_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}(\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}}(\beta p_k)^{\psi}} - Q_i \implies \\
Z_i(\mathbf{p}) &> -Q_i > \bar{Q}
\end{aligned}
$$

since $\frac{1}{p_i}\left(\frac{1}{\sum_{k\in S}\bar{c}_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}(\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}}(\beta p_k)^{\psi}}\right)\sum_{j\in S}\tau_{ij}^{-\phi}\bar{c}_j p_i^{-\phi}p_j^{1+\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}>0$ for all $\mathbf{p}\gg 0$ and $Q_i\leq\bar{Q}$ from C. 3. Property 5 can be seen as follows: consider any $\mathbf{p}\in\Delta\left(R_+^N\right)$ such that there exists an$l\in S$ where$p_l=0$ and an$l'\in S$ where$p_{l'}>0$. Consider any sequence of output prices such that $\mathbf{p}^n \to \mathbf{p}$ as$n \to \infty$. Then we need to show that:

$$\max_{i \in S} Z_i(\mathbf{p}) \to \infty.$$

To see this note that:

$$\max_{i \in S} Z_i(\mathbf{p}^n) = \max_{i \in S} \frac{\frac{1}{p_i} \sum_{j \in S} (\tau_{ij} p_i)^{-\phi} \bar{c}_j p_j^{1+\psi} \left(\sum_{k \in S} (\tau_{kj} p_k)^{-\phi} \right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} \bar{c}_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} p_l^{-\phi} \right)^{\frac{\psi}{\phi}} p_k^{\psi}} - Q_i \implies$$

$$\max_{i \in S} Z_i(\mathbf{p}^n) > \max_{i,j \in S} \frac{p_j}{p_i} \tau_{ij}^{-\phi} \frac{\bar{c}_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi} \right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} \bar{c}_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} p_l^{-\phi} \right)^{\frac{\psi}{\phi}} p_k^{\psi}} - \bar{Q}.$$

Hence, if it is the case that $\max_{i,j \in S} \frac{p_j}{p_i} \tau_{ij}^{-\phi} \frac{\bar{c}_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi} \right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} \bar{c}_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} p_l^{-\phi} \right)^{\frac{\psi}{\phi}} p_k^{\psi}} \to \infty$, then because $\max_{i \in S} Z_i(\mathbf{p}^n)$ is bounded below it, it must be that $\max_{i \in S} Z_i(\mathbf{p}^n) \to \infty$ as well. Note that:

$$\max_{i,j \in S} \frac{p_j}{p_i} \tau_{ij}^{-\phi} \frac{\bar{c}_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi} \right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} \bar{c}_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} p_l^{-\phi} \right)^{\frac{\psi}{\phi}} p_k^{\psi}} > \max_{i,j \in S} \frac{p_j}{p_i} \tau_{ij}^{-\phi} \frac{\bar{c}_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} \left(p^{\min}\right)^{-\phi} \right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} \bar{c}_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} \left(p^{\min}\right)^{-\phi} \right)^{\frac{\psi}{\phi}} \left(p^{\max}\right)^{\psi}} \implies$$

$$> C_{ij} \min_{l \in S} p_l^{-(\phi-\psi)},$$

where $p^{\min} \equiv \min_{l \in S} p_l$,$p^{\max} \equiv \max_{l \in S}$, and $C_{ij} \equiv \tau_{ij}^{-\phi} \frac{\bar{c}_j \left(\sum_{k \in S} \tau_{kj}^{-\phi} \left(p^{\min}\right)^{-\phi} \right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} \bar{c}_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} \left(p^{\min}\right)^{-\phi} \right)^{\frac{\psi}{\phi}} \left(p^{\max}\right)^{\psi}}$. Since $\phi > \psi > 0$ and there exists an$l \in S$ such that $p_l^n \to \infty$ as $n \to \infty$, then we have $\max_{i \in S} Z_i(\mathbf{p}^n) \to \infty$ as well.

Finally, we verify gross-substitutes. Without loss of generality, we differentiate only the bracketed term (as the term outside the bracket will be multiplied by zero since the bracket term is equal to zero in the equilibrium). We have:

$$\frac{\partial Z_i(\mathbf{p})}{\partial p_j} = \frac{\partial}{\partial p_j} \left[\sum_{j \in S} \tau_{ij}^{-\phi} \bar{c}_j p_i^{-\phi} p_j^{1+\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi} \right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi} \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi} \right)^{\frac{\psi}{\phi}} \bar{c}_i \right] \iff$$

$$= (1+\psi) \tau_{ij}^{-\phi} \bar{c}_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi} \right)^{\frac{\psi-\phi}{\phi}} +$$

$$(\phi-\psi) p_j^{-\phi-1} \sum_{l \in S} \tau_{il}^{-\phi} \bar{c}_l p_i^{-\phi} p_l^{\psi} \left(\sum_{k \in S} \tau_{kl}^{-\phi} p_k^{-\phi} \right)^{\frac{\psi-\phi}{\phi}-1} + \psi p_j^{-\phi-1} p_i^{1+\psi} \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi} \right)^{\frac{\psi}{\phi}-1} > 0$$

because $\phi > \psi > 0$ and prices, trade frictions, and supply shifters $\bar{c}_l$ are strictly positive. Because properties 1-6 hold, by Propositions 17.B.2, 17.C.1 and 17.F.3 of Mas-Colell, Whinston, and Green (1995), the equilibrium exists and unique. □

Note that in the case where $\psi > \phi > 0$ – which is the ordering we find when we estimate the gravity constants in Section 5 – Theorem 1 still proves existence and uniqueness of the equilibrium. The following example shows that gross substitutes may not be satisfied in this case.

Example 1. (Gross substitution) Consider the three location economy. Take p_3 as the numeraire The gross substitute is violated if there exists $\overline{p}_1$ such that $Z_1(\overline{p}_1, p_2, 1)$ is not monotonic w.r.t. p_2. Consider the following parameter values:

$$\begin{aligned}(\phi, \psi) &= (2, 5)\\ \tau_{ij} &= 1 \quad \text{for } i, j \in \{1, 2, 3\}\\ \bar{c}_i &= (.9, .6, .1)^{\mathbf{T}}.\end{aligned}$$

Figure 12 shows that with these parameter values, $Z_1(\overline{p}_1, p_2, 1)$ is not monotonic w.r.t. p_2 when $\overline{p}_1 = .5$.

B.7 Examples of multiplicity in two location world

In this subsection, we derive the equilibrium conditions of a two location world and provide examples for different combinations of the gravity constants (i.e. the demand elasticity ϕ and supply elasticity ψ).

We first derive equations for the demand and supply of the representative good in each location as a function of parameters and prices in all other locations. Combining C. 2 (aggregate demand) and C. 3 (market clearing) yields the following aggregate demand equation:

$$Q_i^d = p_i^{-(1+\phi)} \times \left(\sum_{j \in S} \frac{\tau_{ij}^{-\phi}}{\sum_k \tau_{kj}^{-\phi} p_k^{-\phi}} p_j Q_j^d \right), \tag{62}$$

where we denote the quantity of the representative good demanded in location i as Q_i^d. Similarly, C. 3 (aggregate supply) yields the following aggregate supply equation:

$$Q_i^s = \kappa \bar{c}_i \left(\frac{p_i}{\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}} \right)^{\psi}, \tag{63}$$

where we denote the quantity of the representative good supplied in location i as Q_i^d.

Now consider the two-location case (i.e. $S \equiv \{1, 2\}$) where $\tau_{12} = \tau_{21} = \tau \geq 1$ and $\bar{c}_1 = \bar{c}_2 = 1$. Dividing Q_1^d by Q_2^d using equation (62) delivers the following relative demand equation:

$$\frac{Q_1^d}{Q_2^d} = \left(\frac{p_1}{p_2}\right)^{-(1+\phi)} \times \frac{\left(\frac{\tau^{-\phi}\left(\frac{p_1}{p_2}\right)^{-\phi} + 1}{\left(\frac{p_1}{p_2}\right)^{-\phi} + \tau^{-\phi}} \right) \times \frac{p_1}{p_2} \times \frac{Q_1^d}{Q_2^d} + \tau^{-\phi}}{\tau^{-\phi} \left(\left(\frac{\tau^{-\phi}\left(\frac{p_1}{p_2}\right)^{-\phi} + 1}{\left(\frac{p_1}{p_2}\right)^{-\phi} + \tau^{-\phi}} \right) \times \frac{p_1}{p_2} \times \frac{Q_1^d}{Q_2^d} \right) + 1} \tag{64}$$

Similarly, dividing Q_1^s by Q_2^s delivers the following relative supply equation:

$$\frac{Q_1^s}{Q_2^s} = \left(\frac{p_1}{p_2}\right)^{\psi} \times \left(\frac{\tau^{-\phi}\left(\frac{p_1}{p_2}\right)^{-\phi} + 1}{\left(\frac{p_1}{p_2}\right)^{-\phi} + \tau^{-\phi}} \right)^{-\frac{\psi}{\phi}} \tag{65}$$

Note that given the trade friction τ and gravity constants, the relative demand and relative supply can be solved solely as a function of relative output price $\frac{p_1}{p_2}$ using equations (64) and (65), allowing us to analytically characterize the equilibria using standard (relative) supply and demand curves.

Figure 3 depicts example equilibria possible for different combinations of gravity constants; the points where the two curves intersect are possible equilibria. The top left figure shows that when the supply and demand elasticities are both positive (corresponding to a case where the relative aggregate supply is increasing and the relative aggregate demand is decreasing), there is a unique equilibrium. The top right figure shows that when the supply elasticity is positive but the demand elasticity is negative, both the relative aggregate supply and demands are increasing, potentially resulting in multiple equilibria. Similarly, the bottom left figure shows that when the supply elasticity is negative and the demand elasticity is positive, both the relative aggregate supply and demand curves are decreasing, also potentially resulting in multiple equilibria. Finally, the bottom right figure shows that when both the supply and demand elasticities are negative and suitably large in magnitude, the relative aggregate supply curve is downward sloping and the relative aggregate demand curve is upward sloping, allowing for a unique equilibria (albeit one without much economic relevance). These examples are consistent with the sufficient conditions for uniqueness presented in Theorem 1.

B.8 Tariffs in the universal gravity framework

In this subsection, we show how one can use the tools developed above to analyze the effect of tariffs in a simple Armington trade model.

Because tariffs introduce an additional source of revenue, they are are not strictly contained within the universal gravity framework. However, it turns out that the equilibrium structure of an Armington trade model with tariffs is mathematically equivalent to the equilibrium structure of the universal gravity framework. As a result, we can apply Theorems 1 and 2 almost immediately to the case of tariffs in this model.

To see this, consider a simple Armington trade model with N locations.[45] Each location $i \in S$ is endowed with its own differentiated variety and L_i workers who supply their unit labor inelastically and consume varieties from all locations with CES preferences and an elasticity of substitution σ. Suppose that trade is subject to technological iceberg trade frictions $\tau_{ij} \geq 1$ and ad-valorem tariffs $\tilde{t}_{ij} \geq 0$. Define $t_{ij} \equiv 1 + \tilde{t}_{ij}$. Then we can write the value of trade flows from i to j (excluding the tariffs) as:

$$X_{ij} = \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j, \tag{66}$$

where A_i is the productivity in location $i \in S$, w_i is the wage, P_j is the ideal Dixit-Stiglitz price index, and E_j is expenditure.

Income in location i from trade is equal to its total sales (excluding tariffs):

$$Y_i = \sum_{j \in S} X_{ij}. \tag{67}$$

Total income (and hence expenditure) also includes the revenue earned from tariffs T_i:

$$E_i = Y_i + T_i, \tag{68}$$

where tariff revenue is equal to the bilateral tariff charged on all trade being sent[46]:

$$T_i = \sum_{j \in S} \tilde{t}_{ji} X_{ji}. \tag{69}$$

The total expenditure by consumers in location i is also equal to its total imports plus the tariffs

[45] We consider an Armington model in order to have an explicit welfare function, the results that follow will hold for any general equilibrium model where the aggregate supply elasticity $\psi = 0$.

[46] If we had instead supposed that tariffs are only levied on goods that actually arrive, we would have $T_i = \sum_j \frac{\tilde{t}_{ji}}{\tau_{ji}} X_{ji}$, which does not change the following analysis in any substantive way.

incurred:

$$E_i = \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji}. \tag{70}$$

Combining equations (68), (69), (70), we can demonstrate that trade flows are balanced:

$$\begin{aligned} E_i &= \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji} \iff \\ Y_i + \sum_{j \in S} \tilde{t}_{ji} X_{ji} &= \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji} \iff \\ Y_i &= \sum_{j \in S} X_{ji} \end{aligned} \tag{71}$$

Finally, total expenditure is equal to the payment to workers plus tariff revenue:

$$\begin{aligned} E_i &= w_i L_i + T_i \iff \\ Y_i &= w_i L_i \end{aligned} \tag{72}$$

Define $K_{ij} \equiv \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma}$ as the bilateral "kernel", $B_i \equiv A_i L_i$ as the "income shifter", $\gamma_i \equiv A_i^{\sigma-1} w_i^{1-\sigma}$ as the origin fixed effect, $\delta_j \equiv P_j^{\sigma-1} E_j$ as the destination fixed effect, and $\alpha \equiv \frac{1}{1-\sigma}$. Combining equations (67), (71), and (72) yields the following system of equilibrium equations:

$$\begin{aligned} w_i L_i &= \sum_{j \in S} X_{ij} \iff \\ B_i \gamma_i^{\alpha} &= \sum_{j \in S} K_{ij} \gamma_i \delta_j \end{aligned} \tag{73}$$

$$\begin{aligned} w_i L_i &= \sum_{j \in S} X_{ji} \iff \\ B_i \gamma_i^{\alpha} &= \sum_{j \in S} K_{ji} \gamma_j \delta_i. \end{aligned} \tag{74}$$

Equations (73) and (74) can be jointly solved to recover the equilibrium $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$; given $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$, in turn, we can solve for all endogenous variables, as wages can be written as $w_i = \gamma_i^{\frac{1}{1-\sigma}} A_i$, the price index can be written as $P_i = \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)^{\frac{1}{1-\sigma}}$, expenditure can be written as $E_i = \delta_i \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)$, and real expenditure can be written as $W_i \equiv \frac{E_i}{P_i} = \delta_i \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)^{\frac{\sigma}{\sigma-1}}$. As we note at the beginning of Section 3, this equilibrium system is identical in mathematical structure to the universal gravity equilibrium equations 6 and 7. Hence, Theorem 1 applies directly (with existence as long as $\sigma \neq 0$ and uniqueness as long as $\sigma \geq 1$). Moreover, a similar methodology as employed in Theorem 2 can be used to determine how the equilibrium variables γ_i and δ_i respond to shocks that alter the kernel K_{ij} (be they due to changes in iceberg trade frictions or tariffs). In particular:

$$\frac{\partial \ln \gamma_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A^{+}_{l,i} + A^{+}_{N+l,j} - c\right) \tag{75}$$

$$\frac{\partial \ln \delta_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A^{+}_{N+l,i} + A^{+}_{l,j} - c\right), \tag{76}$$

where $\tilde{A}^{-1}_{i,j}$ is the $\langle i, j\rangle$ element of the $2N \times 2N$ matrix the (pseudo) inverse $\tilde{\mathbf{A}}^{-1}$:[47]

$$\tilde{\mathbf{A}}^{-1} = \begin{pmatrix} \frac{\sigma}{1-\sigma}\mathbf{Y} & -\mathbf{X} \\ \frac{1}{1-\sigma}\mathbf{Y} - \mathbf{X}^T & -\mathbf{Y} \end{pmatrix}^{-1}, \tag{77}$$

Because all endogenous variables in the model are simple functions $\{\gamma_i\}_{i\in S}$ and $\{\delta_i\}_{i\in S}$, one can apply equations (75) and (76) to immediately derive any elasticity of interest, e.g. the effect of welfare in location l from changing the tariffs j impose on goods coming from i.

B.9 Global shocks

In this subsection we show that the "exact hat algebra" pioneered by Dekle, Eaton, and Kortum (2008) and extended by Costinot and Rodriguez-Clare (2013) can be applied to any model in the universal gravity framework to calculate the effect of any (possibly large) trade shock. (Note that Section 4 instead showed how to calculate the *elasticity* of endogenous variables to any trade friction shock). We show that the key takeaway from Section 4 holds for all trade shocks: Given observed data, all the gravity models with the same gravity constants imply the same counterfactual predictions for all endogenous variables (i.e. output prices, price indices, nominal incomes, real expenditures, and trade flows).

Consider an arbitrary change in the trade friction matrix $\{\tau_{ij}\}_{S\times S}$. In what follows, we denote with a hat the ratio of the counterfactual to initial value of the variable, i.e. $\hat{x}_i \equiv \frac{x_i^{counterfactual}}{x_i^{initial}}$. The following proposition provides an analytical expression relating the change in the output price and the associated price index to the change in trade frictions and the initial observed trade flows:

Proposition 4. *Consider any given set of observed trade flows* $\mathbf{X}$*, gravity constants* ϕ *and* ψ*, and change in the trade friction matrix* $\hat{\tau}$*. Then the percentage change in the exporter and importer shifters,* $\{\hat{p}_i\}$ *and* $\{\hat{P}_i\}$*, if it exists, will solve the following system of equations:*

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \sum_{j\in S}\frac{X_{ij}}{Y_i}\widehat{\tau}_{ij}^{-\phi}\widehat{P}_j^{\phi}\widehat{p}_j\left(\frac{\widehat{p}_j}{\widehat{P}_j}\right)^{\psi} \quad and \quad \widehat{P}_i^{-\phi} = \sum_{j\in S}\left(\frac{X_{ji}}{E_j}\right)\hat{\tau}_{ji}^{-\phi}\hat{p}_j^{-\phi},\ \forall i\in S \tag{78}$$

Proof. We first note that equilibrium equations (10) and (7) must hold for both the initial and counterfactual equilibria. Taking ratios of the counterfactual to initial values yields:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \frac{\sum_{j\in S}\left(\tau'_{ij}\right)^{-\phi}\left(P'_j\right)^{\phi}p'_j\bar{c}_j\left(\frac{p'_j}{P'_j}\right)^{\psi}}{\sum_{j\in S}\tau_{ij}^{-\phi}P_j^{\phi}p_j\bar{c}_j\left(\frac{p_j}{P_j}\right)^{\psi}}\forall i\in S$$

$$\widehat{P}_i^{-\phi} = \frac{\sum_{j\in S}\left(\tau'_{ji}\right)^{-\phi}\left(p'_j\right)^{-\phi}}{\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}},\ \forall i\in S$$

where we denote the counterfactual equilibrium variables with a prime and the initial equilibrium variables as unadorned. Note that from the gravity equation (10) (and C. 3 - C. 5) we have

[47] The psuedo-inverse can be calculated simply by removing the first row and column and taking the inverse; see footnote 21.

$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j C_j \left(\frac{p_j}{P_j}\right)^{\psi}$, where $p_j \bar{c}_j \left(\frac{p_j}{P_j}\right)^{\psi} = E_j$, so that the above equations become:

$$\widehat{p}_i^{1+\phi+\psi} \widehat{P}_i^{-\psi} = \frac{\sum_{j\in S} \left(\tau_{ij}'\right)^{-\phi} \left(P_j'\right)^{\phi} p_j' \bar{c}_j \left(\frac{p_j'}{P_j'}\right)^{\psi}}{p_i^{\phi} \sum_{j\in S} X_{ij}} \forall i \in S$$

$$\widehat{P}_i^{-\phi} = \frac{\sum_{j\in S} \left(\tau_{ji}'\right)^{-\phi} \left(p_j'\right)^{-\phi}}{P_i^{-\phi} \frac{1}{E_i} \sum_{j\in S} X_{ji}}, \ \forall i \in S$$

Finally, note that from C. 2 and C. 4 we have $E_i = \sum_{j\in S} X_{ij}$ and $Y_i = \sum_{j\in S} X_{ij}$, respectively. Then using our definition $\hat{x}_i \equiv \frac{x_i^{counterfactual}}{x_i^{initial}} \iff x_i^{counterfactual} = \hat{x}_i x_i^{initial}$ we have:

$$\widehat{p}_i^{1+\phi+\psi} \widehat{P}_i^{-\psi} = \sum_{j\in S} \left(\frac{X_{ij}}{Y_i}\right) \hat{\tau}_{ij}^{-\phi} \hat{P}_j^{\phi} \hat{p}_j \left(\frac{\widehat{p}_j}{\widehat{P}_j}\right)^{\psi} \forall i \in S$$

$$\widehat{P}_i^{-\phi} = \sum_{j\in S} \left(\frac{X_{ji}}{E_j}\right) \hat{\tau}_{ji}^{-\phi} \hat{p}_j^{-\phi} \forall i \in S,$$

as required. □

Note that equation (78) inherits the same mathematical structure as equations (6) and (7). As a result, part (i) of Theorem 1 proves that there will exist a solution to equation (78) and part (ii) of Theorem 1 provides conditions for its uniqueness.

B.10 Identification

In this subsection, we show how one can always choose a set of bilateral trade frictions to match observed trade flows for any choice of gravity constants, own trade frictions, and supply shifters. We first state the result as a proposition before providing a proof.

Proposition 5. *Take as given the set of observed trade flows $\{X_{ij}\}_{i,j\in S}$, an assumed set of supply shifters $\{\bar{c}_i\}_{i\in S}$, an aggregate scalar κ, and own trade frictions $\{\tau_{ii}\}_{i\in S}$, and the gravity constants ϕ and ψ. Then there exists a unique set of trade frictions $\{\tau_{ij}\}_{i\neq j}$, output prices $\{p_i\}_{i\in S}$, price indices $\{P_i\}_{i\in S}$, and output $\{Q_i\}_{i\in S}$ such that the following equilibrium conditions hold:*

1. *For all locations $i \in S$, income is equal to the product of the output price and the output:*

$$Y_i = p_i Q_i$$

2. *For all location pairs $i, j \in S$, the value of trade flows from i to j can be written in the following gravity equation form:*

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j$$

3. *For all locations $i \in S$, output satisfies the following supply condition:*

$$Q_i = \kappa \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi}$$

Proof. First, note that the income $Y_i = \sum_{j\in S} X_{ij}$, expenditure $E_i = \sum_{j\in S} X_{ji}$, and own expenditure share $\lambda_{jj} \equiv \frac{X_{jj}}{E_j}$, are all immediately derived from the observed trade flow data.

Second, let us define our unknown parameters and endogenous variables as functions of data and known parameters. The trade frictions are defined follows:

$$\tau_{ij} = \tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{\bar{c}_i}{\bar{c}_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}$$

for all $i, j \in S$ such that $i \neq j$.

The output prices are defined as

$$p_i = Y_i\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\frac{\psi}{\phi}}/\kappa\bar{c}_i$$

for all $i \in S$.

Given the output prices and trade frictions, the price index is defined as: for all $i \in S$,

$$P_i = \left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{-\frac{1}{\phi}}.$$

Finally, the output in each location is defined as: for all $i \in S$,

$$Q_i = \kappa\bar{c}_i\left(\frac{p_i}{P_i}\right)^{\psi}.$$

It is first helpful to note that given the above definitions of the trade frictions and output price indices, we have the following convenient relationship between own expenditure shares and prices:

$$\lambda_{jj} = \left(\tau_{jj}\frac{p_j}{P_j}\right)^{-\phi}$$

To see this, note that we can write:

$$\begin{aligned}
\lambda_{jj} &= \left(\tau_{jj}\frac{p_j}{P_j}\right)^{-\phi} \iff \\
\frac{X_{jj}}{E_j} &= \frac{\tau_{jj}^{-\phi}p_j^{-\phi}}{\sum_{i\in S}\tau_{ij}^{-\phi}p_i^{-\phi}} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \left(\frac{X_{jj}}{E_j}\right)\sum_{i\in S}\tau_{ij}^{-\phi}p_i^{-\phi} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \left(\frac{X_{jj}}{E_j}\right)\sum_{i\in S}\left(\tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{\bar{c}_i}{\bar{c}_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}\right)^{-\phi}p_i^{-\phi} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)\left(\frac{(Y_i/\bar{c}_i)^{\phi}\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\psi}}{(Y_j/\bar{c}_j)^{\phi}\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\psi}}\right)\tau_{jj}^{-\phi}p_i^{-\phi} \iff \\
(Y_j/C_j)^{\phi}\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\psi}p_j^{-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)(Y_i/\bar{c}_i)^{\phi}\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\psi}p_i^{-\phi} \iff \\
p_j^{\phi-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)p_i^{\phi-\phi} \iff \\
E_j &= \sum_{i\in S}X_{ij},
\end{aligned}$$

which is the definition of E_j.

We now confirm each of the three equilibrium conditions. To see that income is equal to the product of the output price and the output, we write:

$$\begin{aligned}
p_i \times Q_i &= Y_i \times \left(\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\frac{\psi}{\phi}} / \kappa\bar{c}_i\right) \times Q_i \iff \\
p_i \times Q_i &= Y_i \times \left(\kappa\bar{c}_i\left(\frac{p_i}{P_i}\right)^{\psi}\right)^{-1} \times Q_i \iff \\
p_i \times Q_i &= Y_i \times \frac{Q_i}{Q_i} \iff \\
p_i \times Q_i &= Y_i,
\end{aligned}$$

as required.

To see that the value of trade flows can be written in the gravity equation form, we write the gravity equation as follows:

$$\begin{aligned}
\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j &= \left(\tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{\bar{c}_i}{\bar{c}_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}\right)^{-\phi} p_i^{-\phi}P_j^{\phi}E_j \iff \\
\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j &= X_{ij}\left(\frac{(Y_i/\bar{c}_i)^{\phi}\,\lambda_{ii}^{\psi}\tau_{ii}^{\phi\psi}}{(Y_j/\bar{c}_j)^{\phi}\,\lambda_{jj}^{\psi}\tau_{jj}^{\phi\psi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}
\end{aligned}$$

Recall from above that we have the following relationship between prices and own expenditure shares:

$$\lambda_{ii} = \left(\tau_{ii}\frac{p_i}{P_i}\right)^{-\phi}$$

so that:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i)^{\phi}\left(\left(\frac{p_i}{P_i}\right)^{\psi}\bar{c}_i\right)^{-\phi}}{(Y_j)^{\phi}\left(\left(\frac{p_j}{P_j}\right)^{\psi}\bar{c}_j\right)^{-\phi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

Furthermore, recall that we have defined our quantities as follows:

$$Q_i = \kappa\bar{c}_i\left(\frac{p_i}{P_i}\right)^{\psi},$$

which implies that:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i/Q_i)^{\phi}}{(Y_j/Q_j)^{\phi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

We have shown above that $p_iQ_i = Y_i$, so that we have:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

We claim that this implies that observed trade flows are explained by the gravity equation, i.e.:

$$X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j$$

To see this, suppose not. Then we have

$$\tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j = X_{ij} \frac{\tau_{jj}^{-\phi} p_j^{-\phi} P_j^{\phi} E_j}{X_{jj}}$$

but $X_{ij} \neq \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j$. Then without loss of generality we can write $X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_{ij}$, where $\varepsilon_{ij} \neq 1$.

$$\begin{aligned}
\tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j &= \left(\tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_{ij}\right) \frac{\tau_{jj}^{-\phi} p_j^{-\phi} P_j^{\phi} E_j}{\left(\tau_{jj}^{-\phi} p_j^{-\phi} P_j^{\phi} E_j \varepsilon_{jj}\right)} \iff \\
1 &= \frac{\varepsilon_{ij}}{\varepsilon_{jj}} \iff \\
\varepsilon_{ij} &= \varepsilon_{jj} \equiv \varepsilon_j \ \forall i \in S
\end{aligned}$$

which then implies that we have:

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_j$$

however, we know that:

$$\begin{aligned}
\sum_{i \in S} X_{ij} &= E_j \iff \\
\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_j &= E_j \iff \\
\frac{\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi}}{\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi}} &= \frac{1}{\varepsilon_j} \iff \\
\varepsilon_j &= 1,
\end{aligned}$$

which is a contradiction. Hence, the observed trade flows are explained by the gravity equation.

Finally, we note that the third equilibrium condition trivially holds by the definition of Q_i:

$$Q_i = \kappa \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi}.$$

Hence, given our definitions, we have found a unique set of trade frictions $\{\tau_{ij}\}_{j \neq i}$, output prices $\{p_i\}_{i \in S}$, price indices $\{P_i\}_{i \in S}$, and output $\{Q_i\}_{i \in S}$ such that the equilibrium conditions hold for any set of observed trade flows $\{X_{ij}\}_{i,j \in S}$, an assumed set of supply shifters $\{\bar{c}_i\}_{i \in S}$ and own trade frictions $\{\tau_{ii}\}_{i \in S}$, and the gravity constants (ϕ, ψ). □

B.11 Real output prices, welfare, and the openness to trade

In this section, we explore the relationship between the real output E_i/P_i and real output price p_i/P_i in the universal gravity framework and the welfare in a number of seminal models. We then show how the real output price in the universal gravity framework relates to the observed own expenditure share. Combining the two results allow ones to write the welfare in each of these models as a function of observed own expenditure share, as in Arkolakis, Costinot, and Rodríguez-Clare (2012).

B.11.1 Real output prices and welfare

In this subsection, we provide a mapping between real output prices and the welfare of a unit of labor for the trade introduced and the economic geography model in Section 2.

The trade model In the trade model, the output price p_i is $w_i^{\zeta} P_i^{1-\zeta}/A_i$. As a result we have the welfare of each worker Ω_i can be expressed as a function of the real output price in the universal gravity framework as follows:

$$\frac{w_i}{P_i} = \underbrace{\left(\frac{p_i A_i}{P_i^{1-\gamma}}\right)^{\frac{1}{\zeta}}}_{=w_i} \frac{1}{P_i} = A_i^{\frac{1}{\gamma}} \left(\frac{p_i}{P_i}\right)^{\frac{1}{\zeta}}.$$

Or equivalently, we can express the welfare in terms of the supply elasticity.

$$\frac{w_i}{P_i} = A_i^{1+\psi} \left(\frac{p_i}{P_i}\right)^{1+\psi}.$$

The economic geography model In the economic geography model, the welfare is $\frac{w_i}{P_i} u_i$,and the price p_i is $\frac{w_i}{\overline{A}_i L_i^a}$. Therefore the welfare is

$$\Omega = \overline{A}_i \overline{u}_i L_i^{a+b} \left(\frac{p_i}{P_i}\right).$$

Welfare equalization and the labor market clearing condition implies

$$\Omega = \left(\overline{L}\right)^{a+b} \left[\sum_{i \in S} \left[\overline{A}_i \overline{u}_i \left(\frac{p_i}{P_i}\right)\right]^{-\frac{1}{a+b}}\right]^{-(a+b)}.$$

B.11.2 Real expenditure, real output prices and the openness to trade

In this subsection, we show we can express real expenditure and real output prices in any model within the universal gravity framework as a function of openness to trade and the gravity constants, as in Arkolakis, Costinot, and Rodríguez-Clare (2012).

We begin by defining $\lambda_{ii} \equiv \frac{X_{ii}}{E_i}$ as location i's own expenditure share. From equation (10), we can express the real output price $\frac{p_i}{P_i}$ in a location as a function of its own expenditure share:

$$X_{ij} = \frac{p_{ij}^{-\phi}}{\sum_{k \in S} p_{kj}^{-\phi}} E_j \implies$$
$$\frac{p_i}{P_i} = \lambda_{ii}^{-\frac{1}{\phi}}. \tag{79}$$

Moreover, given C. 3, C. 4 and C. 5, we can write total real expenditure $W_i \equiv \frac{E_i}{P_i}$ as a function of its own expenditure share as well:

$$\begin{aligned}
W_i &= \frac{E_i}{P_i} \iff \\
W_i &= \left(\frac{p_i}{P_i}\right) Q_i \iff \\
W_i &= \left(\frac{p_i}{P_i}\right) \left(\kappa \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi}\right) \iff \\
W_i &= \kappa \bar{c}_i \left(\frac{p_i}{P_i}\right)^{1+\psi}.
\end{aligned} \tag{80}$$

Combining equations (79) and (80) yields:

$$W_i = \kappa \bar{c}_i \left(\lambda_{ii}\right)^{-\frac{1+\psi}{\phi}}.$$

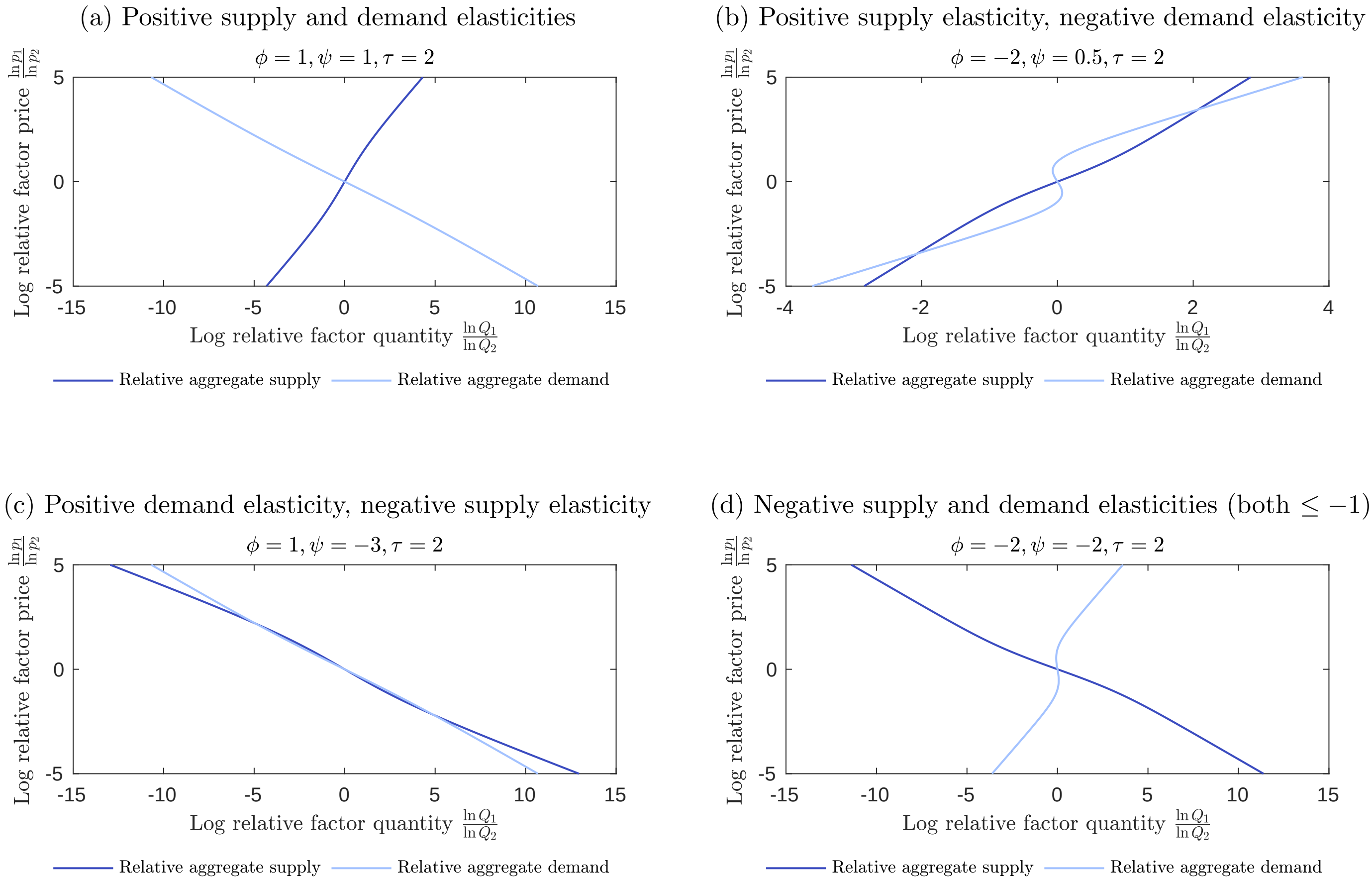
Note that a positive aggregate supply elasticity ($\psi > 0$) increases the elasticity of total real expenditure to own expenditure share, thereby amplifying the gains from trade. Note too that the derivations above imply that:

$$\frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = (\psi + 1) \frac{\partial \ln \left(\frac{p_i}{P_i}\right)}{\partial \ln \tau_{ij}} + \frac{\partial \ln \kappa}{\partial \ln \tau_{ij}},$$

i.e. we can recover the elasticity of the total real expenditure (to-scale) to the trade friction shock from the elasticity of the real output price to the trade friction shock by simply multiplying by $\psi + 1$.

B.12 Additional Figures

Figure 3: Examples of multiplicity and uniqueness in two locations

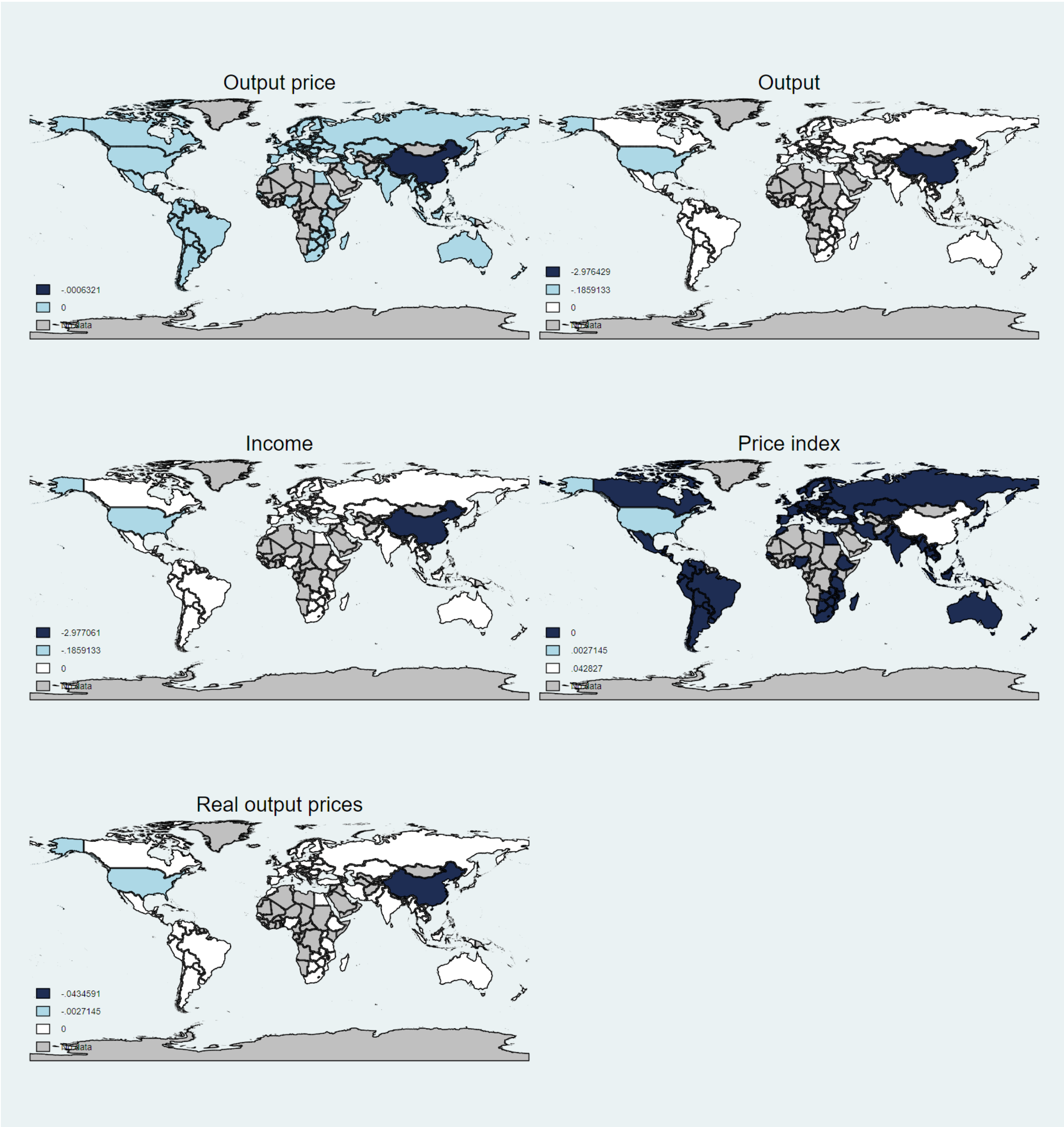


Notes: This figure shows examples of relative supply curve and relative demand curves for a two location world for different combinations of supply and demand elasticities; see Section B.7 for a discussion.

Figure 4: Correlation between observed income and own expenditure shares and the equilibrium values from the gravity model

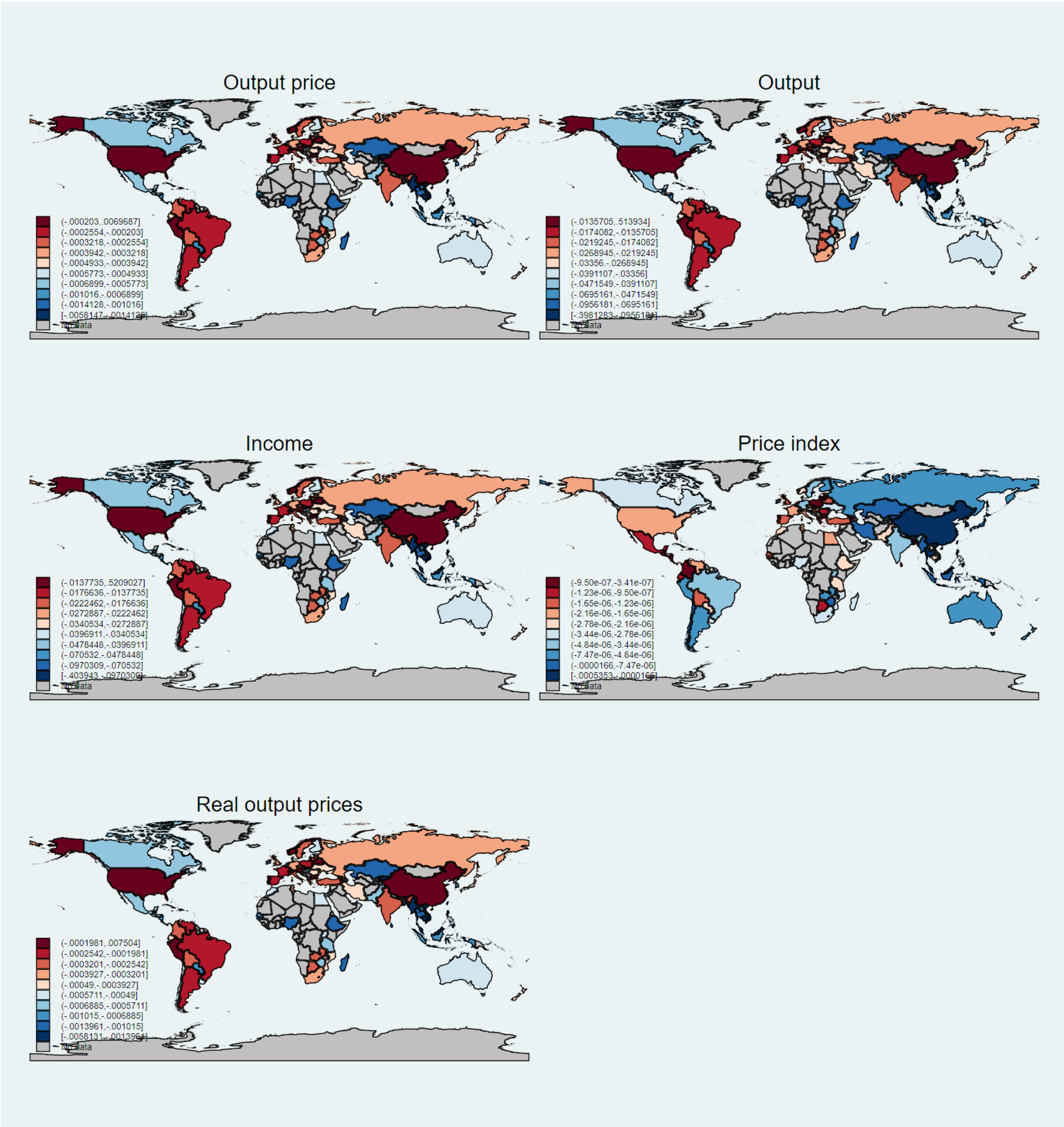
Notes: This figure shows the relationship between the observed and predicted income and own expenditure shares, respectively. The predicted incomes and own expenditure shares are the equilibrium values from the general equilibrium gravity model where bilateral frictions are those estimated from a fixed effects gravity regression and the supply shifters are estimated from a regression of log income on geographic and institutional controls. The scatter plots are plots of the residuals after controlling for the direct effect of the geographic, historical, and institutional observables.

Figure 5: The network effect of a U.S.-China trade war: Degree 0



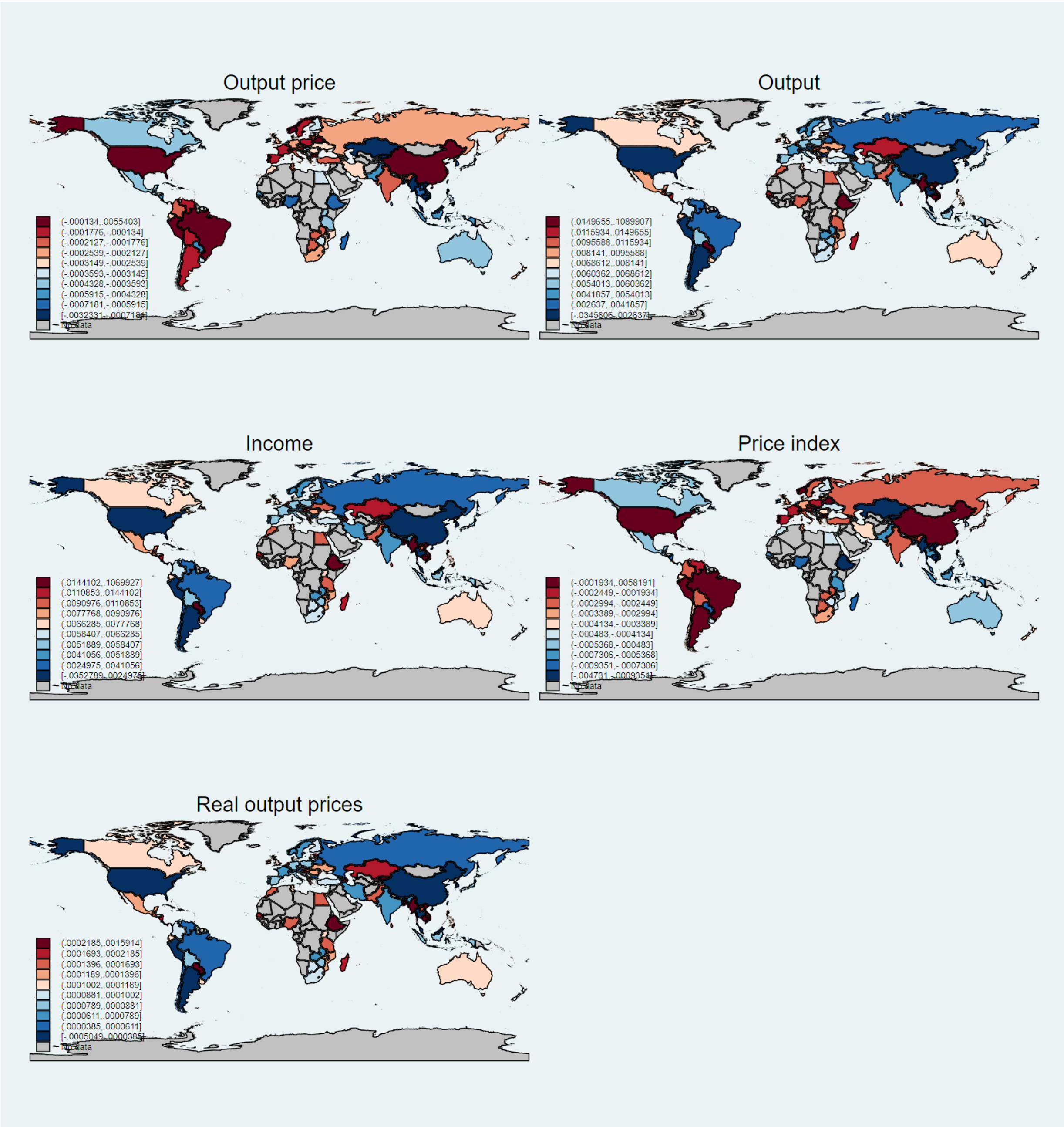
Notes: This figure depicts the "degree 0" effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The "degree 0" effect is the direct impact of the trade war on the U.S. and China, holding constant the price and output in all other countries. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 6: The network effect of a U.S.-China trade war: Degree 1



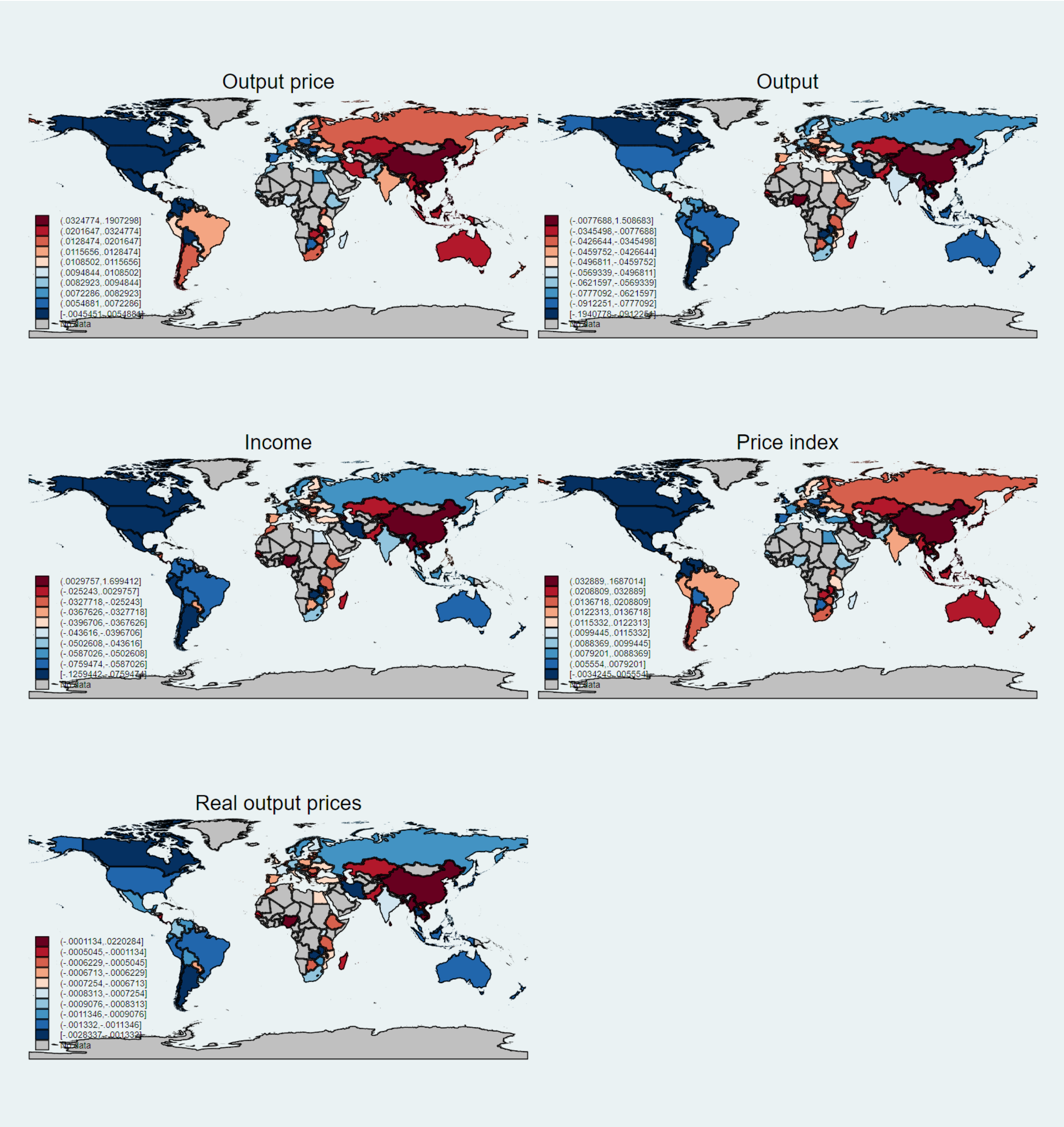
Notes: This figure depicts the “degree 1” effect of an increase in the bilateral trade frictions between the U.S. and China (a “trade war”) in all countries. The “degree 1” effect is the impact of the “degree 0” shock on all countries through the trade network, holding constant the prices and output of their trading partners. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 7: The network effect of a U.S.-China trade war: Degree 2



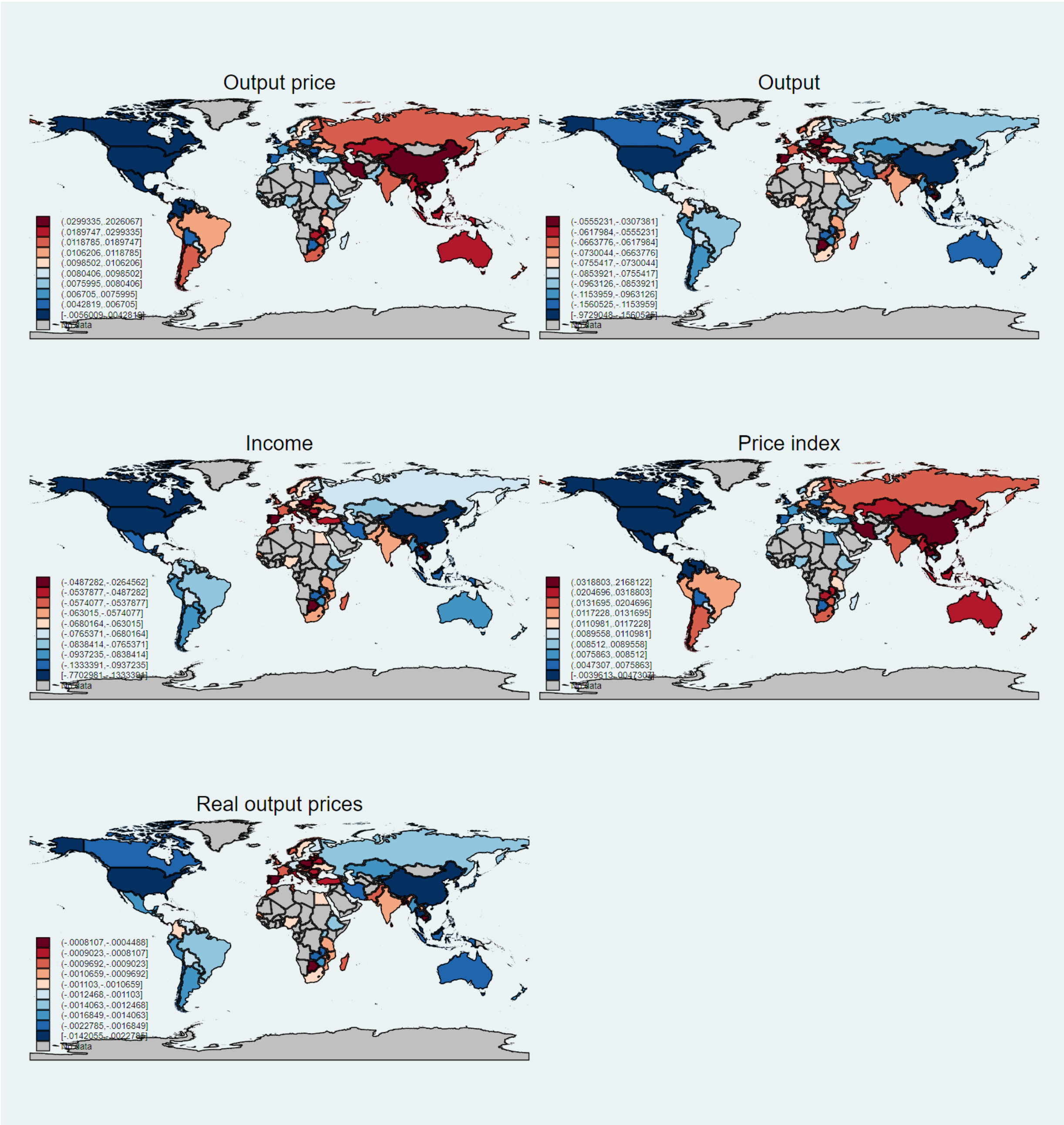
Notes: This figure depicts the "degree 2" effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The "degree 2" effect is the impact of the "degree 1" shock on all countries through the trade network, holding constant the prices and output of their trading partners. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 8: The network effect of a U.S.-China trade war: Degrees >2



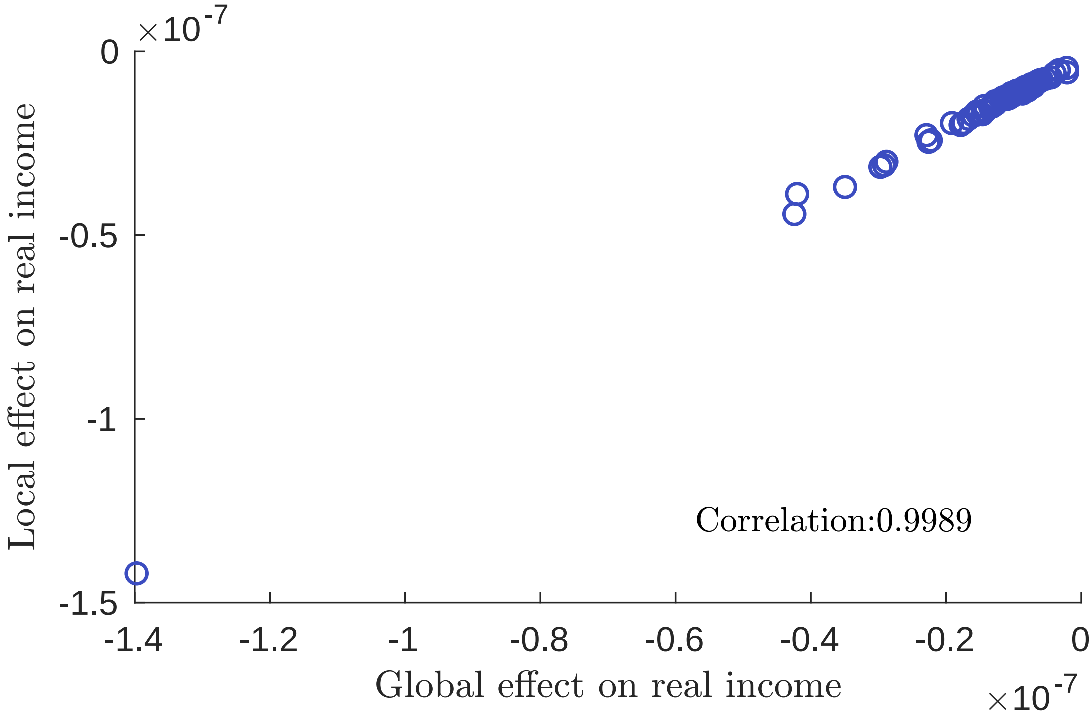
Notes: This figure depicts the cumulative effect of all degrees greater than two of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. A degree k effect is the impact of a degree $k-1$ shock on all countries through the trade network, holding constant the prices and output of their trading partners. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 9: The network effect of a U.S.-China trade war: Total effect



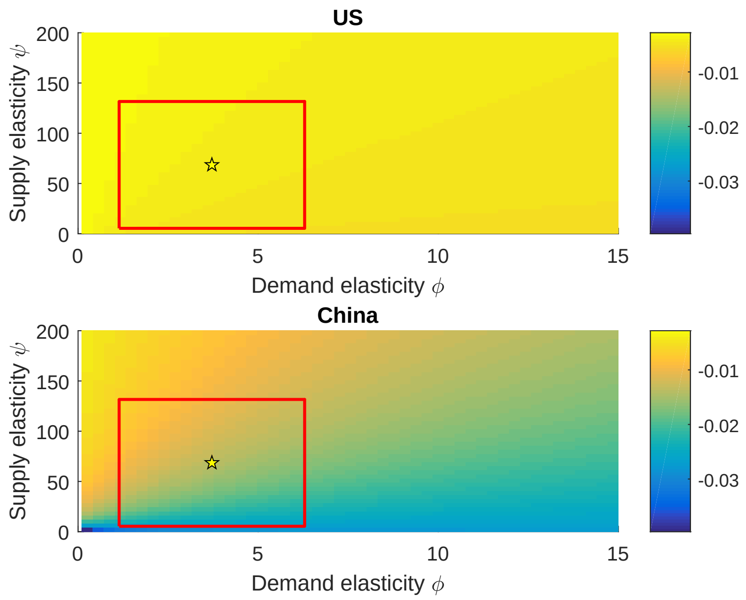
Notes: This figure depicts the total effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. This is the infinite sum of all degree k effects. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 10: Local versus global effects of a U.S.-China trade war



Notes: This figure depicts the correlation of the local (infinitesimal) elasticities and the global (50% increase) impacts of a trade war on the real output price in each country.

Figure 11: The effect of a U.S.-China trade war on real output prices in the U.S. and China: Robustness



Notes: This figure depicts the elasticity of real output prices to an increase bilateral trade frictions between the U.S. and China (a "trade war") for many constellations of demand and supply elasticities ϕ and ψ, respectively. The star indicates the estimated supply and demand elasticity constellation, and the red box outlines the 95% confidence interval of the two parameters.

Figure 12: Excess non-monotonic demand function for 1, $Z_1(p_2)$