Math 105: Collected Homework #6 – due Thursday, March 30

For the following questions, imagine we have a **30-tone** equally tempered scale (that is, a "30-TET" tuning system) rather than the usual "12-TET."

For this assignment, T will denote a transposition by n semitones in this 30-TET system. So, for example, on this assignment we would say $T_{30} = T_0$, since 30 semitones (in 30-TET) would be equal to one octave. (Note: this rule applies only to this assignment. In general, all variations are based on 12-TET unless stated otherwise!)

Answer each of the following questions. Write your work and answers **on a separate sheet of paper** (i.e., not crammed into the small spaces on this sheet).

1. How would we define "inversions" in a 30-tone system? (Think by analogy – take what we do with our usual 12-tone system, and extend the same reasoning to a 30-tone system.) It may help to "name" the notes numerically – say, [0] for the base note, then [1], [2], etc., according to their relative positions on a 30-tone "musical clock."

   Note: #1 needs to be answered before we can do #2 and #3 below!

2. Suppose a melody (in 30-TET) consists of the following tones, in order, where each tone is named as suggested in #1 above:

   [0] , [20] , [12] , [25] , [15] , [10]

   Find each of the following variations of this melody.

   a. T_{10} b. $\mathcal{T}_{20}$ c. IT_{10} d. $T_{10}I$ e. $T_{20}I$

   Hint: if you were asked to find $\mathcal{T}_{15}$ of the given melody, the correct answer would be as follows:
   [15] , [5] , [10] , [20] , [0] , [25]

3. Find the *cyclic subgroup* of variations *generated* by each of the following. Show your work.

   a. $\mathcal{T}_6$ b. $\mathcal{T}_{10}$ c. $\mathcal{T}_{18}$ d. $\mathcal{T}_{20}$ e. $\mathcal{T}_{25}$ f. $\mathcal{T}_{20}I$

   Hint: in 30-TET, transpositions should combine according to a "mod 30" rule, rather than "mod 12." So, for example, $\mathcal{T}_{20}\mathcal{T}_{20} = \mathcal{T}_{40} = \mathcal{T}_{10}$, since in 30-TET there are 30 tones to the octave.

   For example: The cyclic subgroup generated by $\mathcal{T}_5$ would be $\{\mathcal{T}_5 , \mathcal{T}_{10} , \mathcal{T}_{15} , \mathcal{T}_{20} , \mathcal{T}_{25} , \mathcal{T}_0\}$.

Note: the main idea of this assignment is to do something similar to what we have been doing in class, but in the universe of "mod 30" arithmetic rather than "mod 12" arithmetic. It is interesting to consider what is similar about the two systems, and what is different!