Math 105, Fall 2016
Collected Homework – Combinations and Permutations
Solutions and Comments

Write all work and answers on a separate sheet of paper.

1. Evaluate each of the following, using the formulas developed in class. Show all work; in particular, for the combinations formula, show the setup and cancellation of factors that leads to your result. Write each anwer as a product (e.g. $2 \times 7 \times 3 \times 5$) and also as a decimal (e.g. 210).

$P(10,3) = 10 \times 9 \times 8 = 720$

$P(10,7) = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 604800$

$$C(10,3) = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 10 \times 3 \times 4 = 120$$

$$C(10,7) = \frac{10\times9\times8\times7\times6\times5\times4}{7\times6\times5\times4\times3\times2\times1} = \frac{10\times9\times8}{3\times2\times1} = 10 \times 3 \times 4 = 120$$

$$C(16,2) = \frac{16 \times 15}{2 \times 1} = 8 \times 15 = 120$$

$$C(16,14) = \frac{16 \times 15 \times 14 \times 13 \times \ldots \times 4 \times 3}{14 \times 13 \times \ldots \times 4 \times 3 \times 2 \times 1} = \frac{16 \times 15}{2 \times 1} = 8 \times 15 = 120$$

Note: As a general rule, $C(n,k)$ is equal to $C(n,n-k)$. This can help when k is “large” relative to n.

2. For the following questions, assume we’re selecting all notes from the set {A, B, C, C#, D, E, F#, G}. (Note that there are eight notes in this set.)

Try to answer each of the following. Show/explain how you came up with your answers, either by listing all possible ways to complete the given task, or by explaining how to predict how many there must be without listing them all.

(Reminder: a “melody” is an ordered selection of notes; a “chord” is an unordered selection.)

a. How many ways are there to select a three-note <u>chord</u>?
Answer: $C(8,3) = \frac{(8\times7\times6)}{3\times2\times1} = 8 \times 7 = 56$

b. How many ways are there to select a three-note <u>chord</u>, if one of the notes *must* be D?
Answer: If one note must be a D, then we’re only selecting the other two notes, and that selection is being made from the seven notes other than D in the given set. So, this is equivalent to selecting a two-note chord from a set of seven notes; there are $C(7,2) = 21$ ways to select such a chord.

c. How many ways are there to select a three-note <u>chord</u>, if *none* of the notes may be D?
Answer: In this case, we’re selecting all three notes from the seven notes other than D, so there are $C(7,3) = \frac{7\times6\times5}{3\times2\times1} = 7 \times 5 = 35$ ways to select such a chord.

> Comment on parts (b) and (c): Another way to answer part (c) would have been as follows – since we know there are 56 three-note chords total, and we just found out (in part (b)) that 21 of these chords do not contain a D, it stands to reason that all of the other three-note chords must contain a D. That would mean there are 56-21=35 three-note chords containing a D. (Same answer, different thought process!)

d. How many ways are there to select a three-note melody, if no repetition of notes is allowed?
Answer: Since no repetition is allowed and order matters, this is simply a permutation of 3 notes from a set of 8 notes; there are $8 \times 7 \times 6 = 336$ such melodies.

e. How many ways are there to select a three-note melody, if exactly one note must be a D, and no other repetition of notes is allowed?
Answer: This is not simply a permutation or combination, so we must devise a selection process in order to count the number of ways to make this selection…

Selection Process:

1. Decide where to locate the D in the three-note melody – first, second, or third?
   There are 3 ways to make this decision.

2. Choose the other two notes for the melody. Since this is an ordered selection of two notes from a set of seven notes (seven since we can't use the D again), there are $7 \times 6 = 42$ ways to choose these notes.

Total: There are $3 \times 42 = 126$ ways to select such a melody.

f. How many ways are there to select a four-note melody, if exactly two notes must be D's, and no other repetition of notes is allowed?
Answer: Again, we must devise a selection process.

Selection Process:

1. Decide where to locate the two D's in the four-note melody. We're choosing two locations (out of four) to hold the D's, and this is an unordered selection without repetition; therefore, we're choosing a combination of two locations out of four locations. The number of ways to make this selection is $C(4,2) = 6$.

2. Choose the other two notes for the melody. Since this is an ordered selection of two notes from a set of seven notes (seven since we can't use the D again), there are $7 \times 6 = 42$ ways to choose these notes.

Total: There are $6 \times 42 = 252$ ways to select such a melody.