Math 105: Music & Mathematics
April 5, 2017
Test #2

You may not use a calculator for this test. (None of the test problems should require a great deal of computation; if you find yourself needing a calculator, you're probably on the wrong track.)

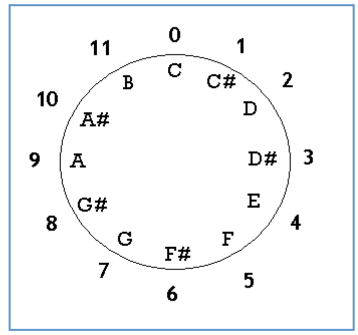
For each question, show your work and/or explain your answer. Always write *something* to justify your answer, unless specifically instructed otherwise; <u>you will not receive full credit for an answer with insufficient supporting work or explanation, even if it is correct</u>. Also, keep in mind that partial credit (for an incorrect answer) can be given only if your supporting work or explanation is shown.

If you need more space for your work on a problem, please use the back of the page on which the problem appears rather than a separate sheet of paper.

1. For each part of this problem, consider the melody consisting of the notes: **C E A G F D.**

For each of the following, find the result of applying the given variation to the above melody.

a) T_7



b) IT_5

c) T_{40}

d) I_D (an inversion centered at D rather than C)

2. Simplify each of the following combinations of variations. Write each answer in one of the following forms: $T_n, T_nR, T_nI,$ or T_nIR, with n between 0 and 11. Show your work.

a) $T_8RT_1RT_5R$

b)) $T_8IT_1IT_5$

c) IRT_2RIT_4

3. Determine whether each of the following sets, with the given operation, is a group. Justify your answers.

a) {0, 10, 5} under mod 15 addition

b) $\{T_0,\ T_4R,\ T_8\}$ using the usual rules for combining variations

4. Consider the group {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} under mod 11 multiplication
(Note: this *does* turn out to be a group. You are *not* being asked to verify this!)

a) Find the cyclic subgroup $\langle 3 \rangle$.

b) Find the cyclic subgroup $\langle 5 \rangle$.

5. Consider the group of all transpositions: $\{T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}\}$

a. Show that $\{T_0, T_3, T_6\}$ is *not* a subgroup of this group.

b. Show that $\{T_0, T_8, T_4\}$ *is* a subgroup of this group.

c. Find all of the coset(s) of $\{T_0, T_8, T_4\}$ in this group.

6. How many elements are there in each of the following groups? Briefly explain each of your answers. (Note: to receive full credit, your explanation must consist of more than “I remember this from class”…)

a. The cyclic subgroup $\langle T_3 I \rangle$

b. The group of *all* possible variations involving transpositions, inversions, and/or retrogrades

c. The group of all actions on a square (that is, the number of different ways a square can be moved) generated by 90-degree right-rotations and/or horizontal flips