Math 201, Test #2B
October 26, 2015

This test covers sections 2.8-3.6 of the textbook.

You may NOT use a calculator or any reference materials during this test.

Read all instructions carefully! Little credit, if any, will be given for a problem on which instructions are not followed.

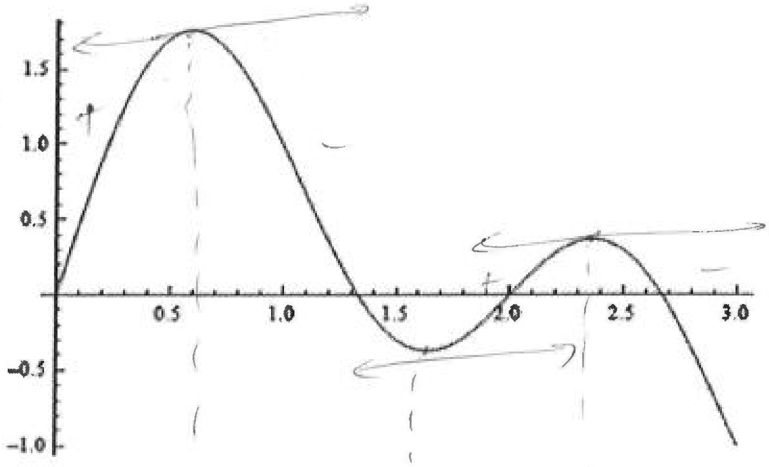
For each problem, *show your work*, and/or otherwise explain how you got your answer. Correct answers with insufficient justification may not receive full credit, and partial credit for incorrect answers can only be given based on work shown and/or written explanation.

When finding derivatives, all differentiation rules from sections 3.1-3.6 may be used *unless* the instructions for a problem specifically say otherwise. (Again: read all instructions carefully!)

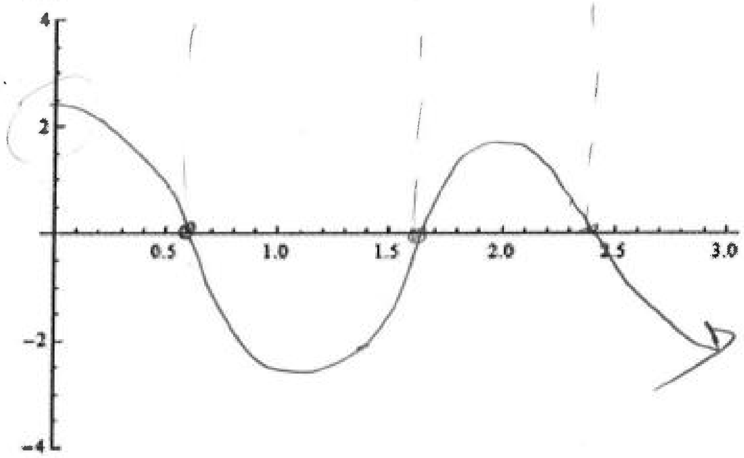
Please write all work and answers on this test, rather than using any separate sheets of paper. If you find that you need more space than what is provided, write any additional work on the back of the page.

1. Use the given graph of f to sketch a possible graph of f'. To receive full credit, you need to show that you understand how the graph of f' should behave relative to the graph of f. Write a brief explanation (one or two sentences should be enough) of how you came up with your sketch. In particular, explain how you decided where f' should be positive, negative, or zero in your sketch.

The graph of $y = f(x)$ is shown here:



Please sketch a graph of $y = f'(x)$ here:



(See Test 2A for explanations)

2. Suppose a rocket's altitude in feet after t seconds is given by the equation $h(t) = 160t + 12t^2 + \frac{1}{3}t^3$.

(a) Find $h'(t)$ and $h''(t)$.

$h'(t) = 160(1) + 12(2t) + \frac{1}{3}(3t^2)$

$h'(t) = 160 + 24t + t^2$ ft/sec.

$h''(t) = 24 + 2t$ ft/sec^2

(b) What do $h'(t)$ and $h''(t)$ represent? That is, what does each of these derivatives tell us about the rocket's flight? Write one or two sentences of explanation for each. For full credit, you must include the correct units of measurement.

(See Test 2A solutions)

3. Let $f(x) = \frac{-1}{5x}$. Use the *definition of the derivative* to show that $f'(x) = \frac{1}{5x^2}$.

Note:

- The instruction "use the definition of the derivative" means that, for this problem, you may *not* use any of the differentiation formulas from Chapter 3 of the text.
- Since you're being given the correct answer for $f'(x)$, it's especially important that you clearly show *all* necessary steps in your work.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{-1}{5(x+h)} - \frac{-1}{5x}}{h}$$

$$\frac{-1}{5(x+h)} + \frac{1}{5x}$$

$$\frac{-x}{5x(x+h)} + \frac{x+h}{5x(x+h)}$$

$$= \frac{h}{5x(x+h)}$$

$$= \lim_{h\to 0} \frac{\frac{h}{5x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{h}{5x(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{1}{5x(x+h)} \quad \leftarrow \text{Sub. } h=0$$

$$= \frac{1}{5x(x+0)} = \boxed{\frac{1}{5x^2}}$$

4. Find dy/dx for each of the following. Simplify your answers.

(a) $y = 6x^2 + 4\sqrt{x} + 12 + \frac{9}{x^2}$

$y = 6x^2 + 4x^{1/2} + 12 + 9x^{-2}$

$y' = 6(2x) + 4(\frac{1}{2}x^{-1/2}) + 0 + 9(-2x^{-3})$

$= 12x + 2x^{-1/2} + -18x^{-3}$

or, $12x + \frac{2}{\sqrt{x}} - \frac{18}{x^3}$

(b) $y = \frac{\sin(x^2)}{x^2}$ ← Quotient Rule

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$

$= \frac{x^2 \frac{d}{dx}(\sin(x^2)) - \sin(x^2)(2x)}{(x^2)^2}$

$\frac{d}{dx}(\sin(x^2))$ → $\cos(x^2) \cdot (2x)$

$= \frac{x^2 \cdot 2x \cdot \cos(x^2) - 2x\sin(x^2)}{x^4}$

$= \frac{2x(x^2\cos(x^2) - \sin(x^2))}{x^3}$

$= \frac{2(x^2\cos(x^2) - \sin(x^2))}{x^3}$

5. Show how to use the indicated method to find each of the following derivatives. Note that you're being given the correct answer for each of these, so it's especially important that you clearly show *all* necessary steps in your work.

(a) Use the quotient rule to show: if $y = \cot(x) = \dfrac{\cos(x)}{\sin(x)}$, then $\dfrac{dy}{dx} = -\csc^2(x)$.

$$\frac{dy}{dx} = \frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{\sin^2(x)}$$

$$= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$$

$= 1$

$$= \frac{-(\sin^2(x) + \cos^2(x))}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

(b) Use implicit differentiation to show: if $y = \sin^{-1}(x)$, then $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$.

$$y = \sin^{-1}(x)$$

$\frac{d}{dx}$ both sides

$$x = \sin(y)$$

$y' = \dfrac{dy}{dx}$

$$1 = \cos(y)\, y'$$

$$\frac{1}{\cos(y)} = y'$$

$$\cos^2(y) + \sin^2(y) = 1$$

$$\cos^2(y) + x^2 = 1$$

$$\cos^2(y) = 1 - x^2$$

$$\cos(y) = \sqrt{1-x^2}$$

$$\frac{1}{\sqrt{1-x^2}} = y'$$

6. For each of the following, find dy/dx using the specified method.

(a) $xy + 3x + 6 = y^3$ (use implicit differentiation)

$$\frac{d}{dx}(xy + 3x + 6) = \frac{d}{dx}(y^3)$$

$$xy' + y \cdot 1 + 3 = 3y^2 y'$$

$$-xy' \qquad\qquad -xy'$$

$$y + 3 = 3y^2 y' - xy'$$

$$y + 3 = y'(3y^2 - x)$$

$$\frac{y+3}{3y^2 - x} = y'$$

(b) $y = (\cos(x))^{\cot(x)}$ (use logarithmic differentiation)

$$\ln(y) = \ln\left((\cos(x))^{\cot(x)}\right)$$

$$\ln(y) = \cot(x) \cdot \ln(\cos(x))$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(\cot(x) \cdot \ln(\cos(x)))$$

$$\frac{y'}{y} = \cot(x) \cdot \frac{d}{dx}(\ln(\cos(x))) + \ln(\cos(x)) \cdot \frac{d}{dx}(\cot(x))$$

$$\frac{y'}{y} = \cot(x) \cdot \frac{-\sin(x)}{\cos(x)} + \ln(\cos(x)) \cdot (-\csc^2(x))$$

$$-1$$

$$y \cdot \frac{y'}{y} = \left(-1 + \ln(\cos(x)) \cdot (-\csc^2(x))\right) y. \rightarrow \left(-1 - \ln(\cos(x)) \cdot \csc^2(x)\right) (\cos(x))^{\cot(x)}$$