Brief Summary of Chapter 8 – Properties of Context-Free Languages

We will not cover Chapter 8 in any sort of detail; however, it is a good idea to be aware of some of the main results from that chapter. A brief summary is presented below, in parallel with some similar properties of regular languages (which we did study in some detail).

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| Context-Free Languages | Regular Languages |
| A language is context-free iff some nondeterministic pushdown automaton accepts it.  There is a “pumping lemma” which holds for any infinite context-free language; this lemma can be used to prove a given language is not context-free.  A few relatively simple, well-known examples of languages that are not context-free (assume for each): | A language is regular iff some nondeterministic finite automaton accepts it.  There is a “pumping lemma” which holds for any infinite regular language; this lemma can be used to prove that a given language is not regular.  A few relatively simple, well-known examples of (context-free) languages that aren’t regular (assume for each): |
|  |  |
| Closure Properties The family of context-free languages is closed under only the following operations (see Theorem 8.3):   * Union * Concatenation * Star Closure   Note that this family of languages is NOT closed under intersection or complementation. (This is Theorem 8.4 in the textbook.) | Closure Properties The family of regular languages is closed under all of the following operations:   * Union * Concatenation * Star Closure * Complementation * Intersection |

Additional notes on closure:

* The non-context-free language is the intersection of two context-free languages: and . Thus, the intersection of two context-free languages is not always context free.
* Since the family of context-free languages is closed under union but not under intersection, the family cannot be closed under complementation. This is due to the set identity . If the family of context-free languages were closed under complementation, this would show that must be context-free whenever and are. Since we know this is not the case (see above), it follows that the family of context-free languages cannot be closed under complementation.
* While the family of context-free languages is not closed under intersection, there is a related property which is satisfied by all context-free languages: **If is a context-free language and is a regular language, then is context-free**. (This is Theorem 8. 5 in the textbook.) This property is referred to as “closure under regular intersection.”

Section 8.2: Closure properties of context-free languages

Theorem 8.3:   
The family of context-free languages is closed under union, concatenation, and star-closure.

Outline of proof: Let and be context-free languages that do not share any variables – in particular, index each variable in the grammar for with the subscript . That is, if we had named a variable “A,” we’ll now call it in the grammar for , or in the grammar for . In particular, assume the grammars generating and have start variables and , respectively. So, if , , and if , then .

* Unions: The grammar that includes all productions from the grammars generating and , together with new start variable and productions , will generate :
  + If then .
  + If , then .
  + If is in neither nor , then cannot be derived from .
* Concatenation: The grammar that includes all productions from the grammars generating and , together with new start variable and the productions , will generate .  
    
  If for some , then .   
  If can’t be decomposed in this way, then can’t be derived from .
* Star Closure: The grammar that generates , together with the productions and , will generate .  
    
  If for some strings , then we can start any derivation with iterations of to give us . From here, we can generate any concatenation of strings from .  
    
  (Note: the production is required since must include .)