MATH 210, TEST #2
OCTOBER 23, 2015

Read all instructions carefully. Use the space provided on each page to write your work and answers. If you need more space than what is provided, use the back of the page. (You may ask for additional paper if necessary.)

For each question, clearly state your answer, and explain (through words, diagrams or both) how you arrived at your answer. Answers without sufficient justification, even if correct, will not receive full credit. (If you're not sure you're showing enough work and/or explaining sufficiently, you may ask about this during the test.)

1. Consider the relation $\propto$ on the set {1, 3, 5, 7, 9, 11}, which is defined as follows:

$$a \propto b \text{ iff } a + b \text{ is NOT a multiple of 4.}$$

For example, $1 \propto 5$ (1 is related to 5), since $1 + 5 = 6$, which is not a multiple of 4.
For example, $3 \not\propto 5$ (3 is not related to 5), since $3 + 5 = 8$, which is a multiple of 4.
One more example: $11 \propto 11$, since $11 + 11 = 22$, which is not a multiple of 4.

a) Draw a directed graph (or "digraph") for this relation.

b) Determine whether this relation is an equivalence relation. Explain* your answer.

c) If $\propto$ is an equivalence relation, write down its equivalence classes. If $\propto$ is not an equivalence relation, give a specific example showing why it isn't.

(*Note: Since we're working with a finite set, your explanation for part (b) doesn't necessarily need to be a formal proof. Just write enough to make it clear that you know what an equivalence relation is, and that you understand why this relation does or does not qualify.)

2. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, and let $B = \{-3, -2, -1, 0, 1, 2, 3\}$.

Define the function $f: A \to B$ as follows:

$$f(n) = \frac{n}{2}, \qquad \text{if } n \text{ is even}$$
$$f(n) = \frac{1-n}{2}, \qquad \text{if } n \text{ is odd}$$

a) Sketch an arrow diagram for this function.
b) Determine whether this function is one-to-one, onto, both, or neither. Explain your answers.
c) Draw an arrow diagram for $f^{-1}: B \to A$, and determine whether f^{-1} is a function. Explain your answer.

3. Define the relation $\prec$ on the set of natural numbers as follows:

$$x \prec y \text{ iff } y = kx \text{ for some } \textit{odd} \text{ integer, } k.$$

For example, $2 \prec 6$, since $6 = 2 \cdot 3$ and 3 is an odd integer.
On the other hand, $2 \not\prec 8$, since $8 = 2 \cdot 4$, and 4 is *not* an odd integer.

a. Prove that $\prec$ is a partial ordering on $\mathbb{N}$.

b. Draw a Hasse diagram for $\prec$ on the set of divisors of 30: {1, 2, 3, 5, 6, 10, 15, 30}.

4. Prove the theorem: for $n \in \mathbb{Z}$, n is divisible by 3 if and only if n^2 is divisible by 3.

5. Consider the proposition (for $x, y \in \mathbb{Z}$):

$$x \equiv y \ (mod\ 6) \rightarrow 3x \equiv 3y \ (mod\ 9)$$

a) Prove that the proposition is a theorem.

b) Show that the converse of the proposition is not a theorem.

6. Suppose g is a function with domain D and codomain C, and suppose the first line of a proof reads as follows:

"Assume (for a contradiction) that there exist $x, y \in D$ such that $x \neq y$ and $g(x) = g(y)$."

Which of the following would be the objective of a proof that starts out like this?

a) To prove that g is one-to-one
b) To prove that g is not one-to-one
c) To prove that g is onto
d) To prove that g is not onto

Explain your choice. You *must* give a written explanation to receive credit for this problem.