1

Unit 5: AC ~~fundamentals~~ Circuits

4.1. a) **Inductive Reactance :- (X_L)**

When an alternating quantity flows through an inductive coil, an emf is induced in it. "The inductive reactance of a coil is defined as the opposition offered by an inductive coil to the flow of alternating current through it due to its self inductance (L)".

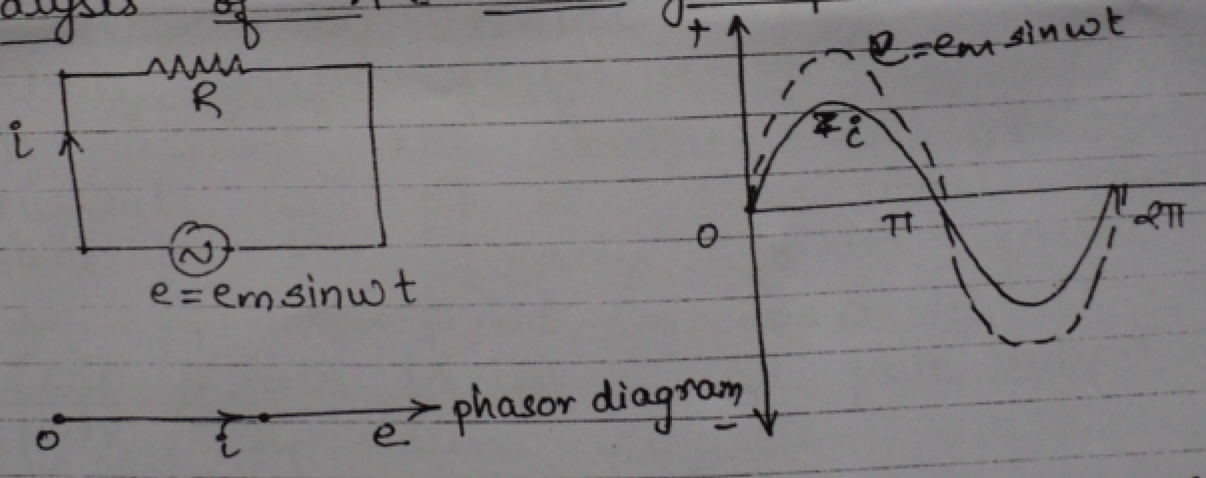
$$X_L = 2\pi f L \quad (\Omega)$$

Capacitive Reactance: (X_C)

Capacitive reactance of a capacitor is the opposition offered to the flow of alternating current through it, due to its capacitance

$$X_C = \frac{1}{2\pi f C} \quad (\Omega)$$

4.1.1 **Analysis of A.C. through pure resistive c...**



A pure resistance circuit has a pure resistance across which AC is applied.

$\therefore$ Applied AC voltage $= e = e_m \sin \omega t$.

There is a voltage drop across the resistor given by

$$e = iR \quad -(2)$$

$$\Rightarrow e_m \sin \omega t = iR$$

$$\Rightarrow i = \left(\frac{e_m}{R}\right) \sin \omega t \quad -(3)$$

5.1

The current value is maximum when $\sin\omega t = 1$

$$\Rightarrow \quad I_m = \frac{e_m}{R} \quad - (4)$$

$$\Rightarrow \quad \boxed{i = I_m \sin\omega t = I_m \sin\theta} \quad - (5)$$

Both current & voltage are in phase for a purely resistive cct

Instantaneous Power : $P = ei$

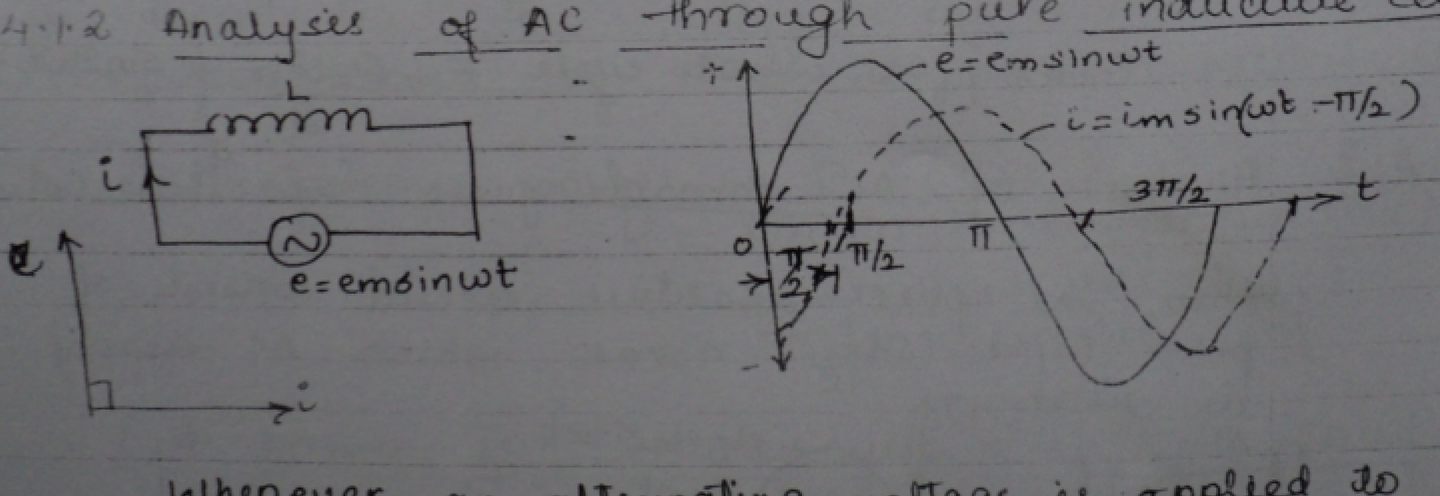
$$= \frac{e_m}{R}\sin\omega t \cdot i_m \sin\omega t$$

$$\Rightarrow P = e_m i_m \sin^2(\omega t) = e_m i_m \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$\Rightarrow P = \underbrace{\frac{e_m i_m}{2}}_{\substack{\text{constant}\\ \text{part}}} - \underbrace{\frac{e_m i_m \cos 2\omega t}{2}}_{\substack{\text{fluctuating}\\ \text{part}}}$$

$$\Rightarrow \quad P = \frac{e_m i_m}{2} = \frac{e_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} = e_{rms} \cdot i_{rms}$$

4.1.2 **Analysis of AC through pure inductive cct:-**



Whenever an alternating voltage is applied to a purely inductive coil, a back emf is produced due to self inductance. This back emf always opposes the change in current in the coil.

$$\therefore \quad e = L\frac{di}{dt}$$

$$e_m \sin\omega t = L\,\frac{di}{}$$

$$di = \frac{e_m}{L} \sin\omega t \, dt$$

$$i = \int \frac{e_m}{L} \sin\omega t \, dt$$

$$i = \frac{e_m(-\cos\omega t)}{\omega L} = \frac{e_m}{\omega L}(-\cos\omega t) \rightarrow \text{(rember}$$

$$\Rightarrow i = \frac{e_m}{\omega L} \sin(\omega t - \pi/2)$$

$$\boxed{i = \frac{e_m}{X_L} \sin(\omega t - \pi/2)}$$

The maximum value of i_m is obtained when $\sin(\omega t - \pi/2) = 1$

$$\Rightarrow i_m = \frac{e_m}{X_L}$$

$$\Rightarrow \boxed{i = i_m \sin(\omega t - \pi/2)}$$

$\Rightarrow$ Current lags the voltage by a phase difference of

Power consumed :- $P = e \cdot i$

$$= e_m \sin\omega t \cdot i_m \sin(\omega t - \pi/2)$$

$$= -e_m \sin\omega t \; i_m \cos\omega t$$

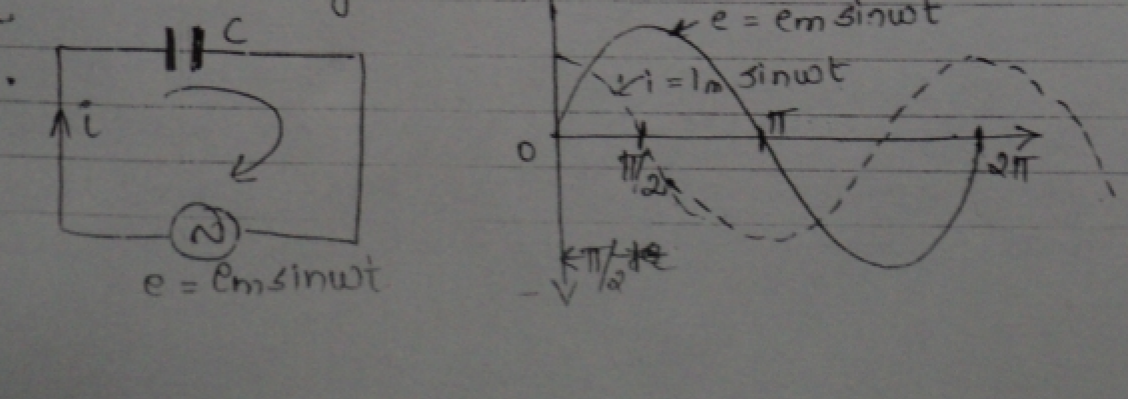
$$= \frac{-e_m i_m}{2} \sin 2\omega t$$

$$\Rightarrow P = -\frac{e_m i_m}{2} \sin 2\omega t.$$

$\therefore$ Power for whole cycle $= -\frac{e_m i_m}{2} \int_0^{2\pi} \sin 2\omega t$

4.1.2 Analysis of AC through pure Capacitive cct

When the pure capacitive circuit consists of pure capacitance across which AC supply is given.



4.1.b Definitions

1) Impedance - (Z) unit → ohms (Ω)
The total opposition offered either by resistance, capacitance or inductance or by any two of them or all of them in an AC cct is known as

2) Power factor - (p.f) (unit less)
a) Cosine of the angle between the applied voltage and current in the circuit ⇒ $\boxed{p.f = \cos\phi}$
b) It is also the ratio of resistance to impedance ⇒ $\boxed{p.f = R/Z}$
c) It is also the ratio of true power to apparent power ⇒ $p.f = \dfrac{\text{true power}}{\text{apparent power}} = \dfrac{VI\cos\phi}{VI}$.

3) Leading power factor :- eg. capacitive circuits.
The cct is said to have a leading power factor if the current leads the applied voltage by certain angle.

4) Lagging power factor :- eg. inductive circuit
The circuit is said to have lagging power factor if the current lags the applied voltage by certain angle

5) Unity power factor :- eg. pure resistive cct.
If circuit has both the voltage & currents in phase then they ~~have~~ are said to have unity power factor.
⇒ $pf = \cos\phi = 1 = \cos 0^\circ$.

6) Zero power factor :- eg. pure inductive or pure capacitive circuits

If the current leads or lags the applied voltage by 90° or $\pi/2^c$, then the circuit is said to have zero power factor
$pf = \cos\phi = \cos 90^\circ = 0$.