

MATH 105 Practice Exercises: Counting Problems

Note:

$P(n, k)$ denotes the number of ways to select a permutation of k elements from a set of n elements.

$C(n, k)$ denotes the number of ways to select a combination of k elements from a set of n elements.

Practice Exercises

1. Evaluate each of the following: $P(8,4)$, $P(9,5)$, $P(12,4)$
2. How many different 5-note melodies are possible, if each note can be any of the 12 notes from the standard 12-tone scale, and no note may be repeated?
3. How does the answer to #2 change if notes may be repeated?
4. How many ways are there to rearrange the letters of the word "MUSICAL"?
5. Evaluate each of the following: $C(8,4)$, $C(9,5)$, $C(12,4)$
6. How many 5-note chords are possible, if we're selecting from the 12-tone scale?
7. There are fourteen students in a class. In how many ways can we select four of these students under each of the following scenarios?
 - a) We are selecting a committee of four students
 - b) We are selecting "officers" – a president, vice president, secretary and treasurer (assuming that no student may hold multiple offices)
 - c) We are selecting a committee consisting of two males and two females – for this one, assume that there are nine female students and five male students in the class
8. Selecting notes from the 12-tone scale: how many different nine-note melodies include exactly four D's, exactly two F#'s, and no other repeated notes?
9. How many 7-note melodies are there with three C's, one D, one E, one F and one G?
(e.g., CCCDEFG, FGDECCC, CGDFCEC, FCCDECG, etc... there are a lot more!)
10. Find the number of different rearrangements of each of the following words:
MELODY RHYTHM AARDVARK TWITTER MISSISSIPPI

Card problems (optional – not covered in class)

NOTE: We have not discussed card problems in class this semester, so you will NOT be responsible for this material. However, if you happen to be familiar with card games (blackjack, poker, etc.), then these problems may be of interest to you

11. How many different ways are there to select a 5-card poker hand?
12. How many ways are there to select a “suits full house,” where three cards are all of one suit, and the other two cards share some other suit. (e.g., three clubs and two hearts would be a “suits full house.”)
13. How many ways are there to select a three-card hand which consists entirely of face cards (jack, queen, king)?
14. How many ways are there to select a four-card hand which consists of only numbered cards (2, 3, 4, ..., 10)?
15. How many ways are there to select “three of a kind” – that is, three cards all of one rank, and two other cards, each of different ranks? (e.g., Three kings, a jack, and a ten would count; three jacks and two tens would not count, since that’s a “full house,” as described in class)