Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings

As outcomes, Year 7 pupils should, for example:

Strategies for mental addition and subtraction

Count forwards and backwards from any number.
For example:
- Count on in 0.1s from 4.5.
- Count back from 4.05 in 0.01s.
- Count on from and back to zero in steps of ¾.

Identify positions of 0.1s and 0.01s on a number line.

Use a **spreadsheet** to replicate cells, e.g. to 'count' from 1 in steps of 1.

	A	B	C	D	E	F	G
1	1	= A1+1	= B1+1	= C1+1	= D1+1	= E1+1	= F1+1

	A	B	C	D	E	F	G
1	1	2	3	4	5	6	7

Modify the spreadsheet to count from 0.5 in steps of 0.1.

Add and subtract several small numbers.
For example:
- 4 + 8 + 12 + 6 + 13
- 5 – 4 + 8 – 10 – 7

Extend to adding and subtracting several small multiples of 10:
- 40 + 30 + 20
- 60 + 50 – 30

Continue to add and subtract any pair of two-digit whole numbers, such as 76 + 58, 91 – 47.
Extend to:
- adding and subtracting a two-digit whole number to or from a three-digit whole number;
- adding and subtracting decimals such as:

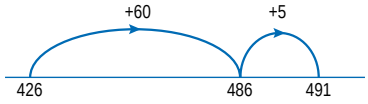
  8.6 ± 5.7 0.76 ± 0.58 0.82 ± 1.5

  by considering

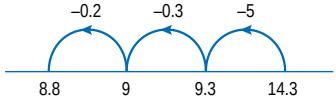
  86 ± 57 76 ± 58 82 ± 150

Use jottings such as an empty number line to support or explain methods for adding and subtracting mentally. Choose an appropriate method, such as one of the following:

Partition and deal with the most significant digits first.
For example:
- 426 + 65 = (426 + 60) + 5 = 486 + 5 = 491



- 14.3 – 5.5 = 14.3 – 5 – 0.3 – 0.2 = 9 – 0.2 = 8.8



As outcomes, Year 8 pupils should, for example:

Strategies for mental addition and subtraction

Consolidate and use addition and subtraction strategies from previous years. For example:

Add and subtract mentally pairs of integers.
Use strategies for addition and subtraction to add and subtract pairs of integers. For example:

- $^{-}3 + {}^{-}5 = \ldots$ $\quad$ $^{-}13 + {}^{-}25 = \ldots$
  $^{-}46 + {}^{-}59 = \ldots$ $\quad$ $^{-}100 + {}^{-}99 = \ldots$
  $^{-}9 - {}^{-}14 = \ldots$ $\quad$ $^{-}43 - {}^{-}21 = \ldots$
  $^{-}37 - {}^{-}25 = \ldots$ $\quad$ $^{-}7 - {}^{-}7 = \ldots$

- The result of subtracting one integer from another is $^{-}29$.
  What could the integers be?

- $\square + \square = {}^{-}46$

Add mentally several positive or negative numbers, including larger multiples of 10. For example:
- $5 + {}^{-}4 + 8 + {}^{-}10 + {}^{-}7$
- $250 + 120 - 190$

Calculate a mean using an assumed mean.
For example:
- Find the mean of 18.7, 18.4, 19.1, 18.3 and 19.5.
  Use 19.0 as the assumed mean.
  The differences are $^{-}0.3$, $^{-}0.6$, 0.1, $^{-}0.7$ and 0.5, giving a total difference of $^{-}1.0$.
  The actual mean is $19.0 - (1.0 \div 5) = 18.8$.

Link to integers (pages 48–9).

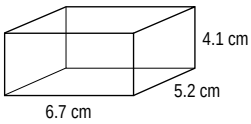
Add and subtract pairs of numbers of the same order (both with two significant figures). For example:
- 360 + 250
- 4800 – 1900
- 7.8 + 9.3
- 0.081 – 0.056

As outcomes, Year 9 pupils should, for example:

Strategies for mental addition and subtraction

Consolidate and use addition and subtraction strategies from previous years. For example:

- Find the length of wire in this framework.



$4(6.7) + 4(5.2) + 4(4.1) = 4 \times 16 = 64$ cm

Pupils should be taught to:

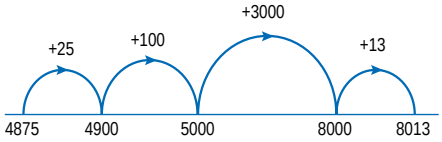
Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Mental addition and subtraction strategies (continued)

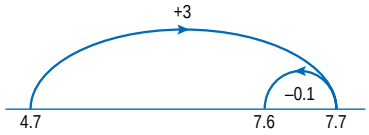
Find a difference by counting up from the smaller to the larger number. For example:

- 8013 – 4875 = 25 + 100 + 3000 + 13 = 3138

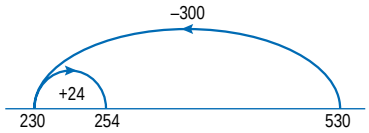


Use compensation, by adding or subtracting too much, and then compensating. For example:

- 4.7 + 2.9 = 4.7 + 3 – 0.1 = 7.7 – 0.1 = 7.6



- 530 – 276 = 530 – 300 + 24 = 230 + 24 = 254



Recognise special cases. For example:

Near doubles
- 8.5 + 8.2 = 16.7 (double 8.2 plus 0.3)
- 427 + 366 = 793 (double 400 plus 27 minus 34)

'Nearly' numbers
Add and subtract near 10s and near 100s, by adding or subtracting a multiple of 10 or 100 and adjusting. For example:

- 48 + 39
- 92 + 51
- 76 + 88
- 427 + 103
- 586 + 278
- 84 – 29
- 70 – 51
- 113 – 78
- 925 – 402
- 350 – 289

Use the relationship between addition and subtraction.
For example, recognise that knowing one of:

2.4 + 5.8 = 8.2 5.8 + 2.4 = 8.2
8.2 – 5.8 = 2.4 8.2 – 2.4 = 5.8

means that you also know the other three.

See Y456 examples (pages 40–7).

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Strategies for multiplication and division

Use factors. For example:

- 3.2×30 — $3.2 \times 10 = 32$; $32 \times 3 = 96$
- $156 \div 6$ — $156 \div 3 = 52$; $52 \div 2 = 26$

Use partitioning. For example:
For multiplication, partition either part of the product:

- 7.3×11 — $= (7.3 \times 10) + 7.3$; $= 73 + 7.3$; $= 80.3$

For division, partition the dividend (the number that is to be divided by another):

- $430 \div 13$ — $400 \div 13 = 30$ R 10; $30 \div 13 = 2$ R 4; $430 \div 13 = 32$ R 14; $= 33$ R 1

Recognise special cases where doubling or halving can be used. For example:

To multiply by 50, first multiply by 100 and then divide by 2.
For example:

- 1.38×50 — $1.38 \times 100 = 138$; $138 \div 2 = 69$

Double one number and halve the other. For example:

- 6×4.5 — $3 \times 9 = 27$
  12×7.5 — $6 \times 15 = 3 \times 30 = 90$

Use the relationship between multiplication and division.
For example, knowing one of these facts means you also know the other three:

$2.4 \times 3 = 7.2$ $\quad$ $3 \times 2.4 = 7.2$
$7.2 \div 3 = 2.4$ $\quad$ $7.2 \div 2.4 = 3$

See Y456 examples (pages 60–5).

As outcomes, Year 8 pupils should, for example:

Strategies for multiplication and division

Use factors. For example:

- 22×0.02 — $22 \times 0.01 = 0.22$; $0.22 \times 2 = 0.44$
- $420 \div 15$ — $420 \div 5 = 84$; $84 \div 3 = 28$
- $126 \div 18$ — $126 \div 6 = 21$; $21 \div 3 = 7$

Use partitioning. For example, for multiplication, partition either part of the product:

- 13×1.4 — $= (10 \times 1.4) + (3 \times 1.4)$; $= 14 + 4.2$; $= 18.2$
- 7.3×21 — $= (7.3 \times 20) + 7.3$; $= 146 + 7.3$; $= 153.3$

Use knowledge of place value to multiply and divide mentally any number by 0.1 or 0.01. For example:

- 3.6×0.1 — $3.6 \div 10$
- 99.2×0.01 — $99.2 \div 100$
- $^{-}1.8 \div 0.1$ — $^{-}1.8 \times 10$
- $0.35 \div 0.01$ — 0.35×100

$$99.2 \times 0.01 \quad 0.992$$
$$\times 10 \downarrow \quad \downarrow \times 100 \quad \uparrow \div 1000$$
$$992 \times 1 = 992$$

Recognise special cases where doubling or halving can be used. For example:

Extend doubling and halving methods to include decimals and negative numbers. For example:

- 3.4×4.5 — $1.7 \times 9 = 15.3$
- 8.12×2.5 — $4.06 \times 5 = 20.3$
- 22×3.01 — $11 \times 6.02 = 66.22$
- $^{-}17 \times 1.5$ — $^{-}8.5 \times 3 = {}^{-}25.5$
- $^{-}8.4 \times {}^{-}1.25$ — $^{-}4.2 \times {}^{-}2.5 = {}^{-}2.1 \times {}^{-}5 = 10.5$

Multiply by near 10s.
For example:

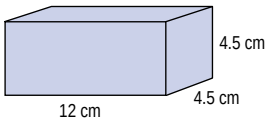
- $23 \times 11 = 230 \times 10 + 23 = 253$
- $75 \times 29 = 75 \times 30 - 75 = 2175$
- $8 \times {}^{-}19 = 8 \times ({}^{-}20 + 1) = {}^{-}160 + 8 = {}^{-}152$

As outcomes, Year 9 pupils should, for example:

Strategies for multiplication and division

Consolidate and use multiplication and division strategies from previous years. For example:

- Find the volume of this square-based cuboid.



$4.5 \times 4.5 \times 12 = \frac{9}{2} \times \frac{9}{2} \times 12$

$= 9 \times 9 \times 3$

$= 243\ \text{cm}^3$

or

$4.5 \times 4.5 \times 12 = (4.5 \times 12) \times 4.5$

$= 54 \times 4.5$

$= 216 + 27$

$= 243\ \text{cm}^3$

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know or derive quickly:

- simple decimal/fraction/percentage equivalents, such as:
  ¼ = 0.25 or 25% 0.23 is equivalent to 23%
  $\frac{7}{10}$ = 0.7 or 70% 57% is equivalent to 0.57 or $\frac{57}{100}$
- simple addition facts for fractions, such as:
  ¼ + ¼ = ½ ¼ + ½ = ¾
- some simple equivalent fractions for ¼ and ½, such as:
  ½ = $\frac{2}{4}$ = $\frac{3}{6}$ = $\frac{4}{8}$ = $\frac{5}{10}$ = $\frac{50}{100}$
  ¼ = $\frac{2}{8}$ = $\frac{3}{12}$ = $\frac{4}{16}$ = $\frac{5}{20}$ = $\frac{25}{100}$

Strategies for finding equivalent fractions, decimals and percentages

For example:

- Convert ⅛ into a decimal.
  (Know that ¼ = 0.25 so ⅛ is 0.25 ÷ 2 = 0.125.)
- Express ⅗ as a percentage.
  (Know that ⅗ = $\frac{6}{10}$ or $\frac{60}{100}$, so it is equivalent to 60%.)
- Express 23% as a fraction and as a decimal.
  (Know that 23% is equivalent to $\frac{23}{100}$ or 0.23.)
- Express 70% as a fraction in its lowest terms.
  (Know that 70% is equivalent to $\frac{70}{100}$, and cancel this to $\frac{7}{10}$.)

Use known facts such as ⅕ = 0.2 to convert fractions to decimals mentally. For example:

⅗ = 0.2 × 3 = 0.6

Find simple equivalent fractions.

For example:

- State three fractions equivalent to ⅗, such as:
  $\frac{6}{10}$, $\frac{30}{50}$, $\frac{24}{40}$
- Fill in the boxes:
  ¾ = $\frac{\square}{8}$ = $\frac{\square}{12}$ = $\frac{\square}{16}$ = $\frac{\square}{20}$
  $\frac{7}{\square}$ = $\frac{21}{30}$

Strategies for calculating fractions and percentages of whole numbers and quantities. For example:

- ⅛ of 20 = 2.5 (e.g. find one quarter, halve it)
- 75% of 24 = 18 (e.g. find 50% then 25% and add the results)
- 15% of 40 (e.g. find 10% then 5% and add the results)
- 40% of 400 kg (e.g. find 10% then multiply by 4)

- 60 pupils go to the gym club.
  25% of them are girls.
  How many are boys?

See Y456 examples (pages 24–5, 32–3).

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

As outcomes, Year 8 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know or derive quickly:
- decimal/fraction/percentage equivalents such as:
  - $\frac{1}{8}$ = 0.125 or 12½%
  - $\frac{3}{5}$ = 0.6 or 60%
  - $1\frac{3}{4}$ = 1.75 or 175%
  - $\frac{1}{3} \approx 0.33$ or 33⅓%
- the simplified form of fractions such as:
  - $\frac{3}{15} = \frac{1}{5}$
  - $\frac{14}{21} = \frac{2}{3}$

Know that $\frac{1}{3}$ is $0.\dot{3}$ and $\frac{2}{3}$ is $0.\dot{6}$.

Know that 0.03 is $\frac{3}{100}$ or 3%.

Strategies for finding equivalent fractions, decimals and percentages

For example:
- Express 136% as a decimal.
  (Know that 136% is equivalent to $\frac{136}{100}$ or 1.36.)
- Express 55% as a fraction in its lowest terms.
  (Know 55% is equivalent to $\frac{55}{100}$, cancel to $\frac{11}{20}$.)
- Express $\frac{13}{20}$ as a percentage.
  (Work out that $\frac{13}{20} = \frac{65}{100}$, so it is equivalent to 65%.)
- Convert $\frac{4}{25}$ into a decimal.
  (Work out that $\frac{4}{25} = \frac{16}{100}$, so it is equivalent to 0.16.)
- Convert 0.45 into a fraction.
  (Know that 0.45 = $\frac{45}{100}$, and simplify this to $\frac{9}{20}$.)
- Express 0.06 as a percentage.
  (Recognise this as $\frac{6}{100}$ or 6%.)

Use known facts such as $\frac{1}{8}$ = 0.125 to convert fractions to decimals mentally. For example:
$\frac{5}{8}$ = 0.125 × 5 = 0.625

Convert between improper fractions and mixed numbers. For example:
- Convert $7\frac{1}{3}$ into an improper fraction.
- Convert $\frac{36}{5}$ into a mixed number.

Strategies for calculating fractions and percentages

of whole numbers and quantities. For example:
- $\frac{3}{5}$ of 20 = 12 (e.g. find one fifth, multiply by 3)
- 1½ of 16 = 24 (e.g. find one half, add it to 16)
- 125% of 240 (e.g. find 25%, add it to 240)
- 35% of 40 (e.g. find 10% then 30% then 5%, add the last two results)

- There is a discount of 5% on a £45 coat in a sale. By how much is the coat's price reduced?
  (e.g. 1% = 45p so 5% = (45 × 5)p = £2.25
  or 10% = £4.50 so 5% = £2.25)

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

As outcomes, Year 9 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know that 0.005 is half of one per cent,
so that 37.5% = 37% + 0.5%
or 0.37 + 0.005 = 0.375

Strategies for finding equivalent fractions, decimals and percentages

For example:
- Express 0.625 as an equivalent percentage.
  (Recognise this as 62%, plus half of one per cent, or 62.5%.)
- Express 10.5 as an equivalent percentage.
  (Recognise this as 1000% plus 50%, or 1050%.)

Simplify fractions by cancelling highest common factors mentally. For example:
- Simplify:
  $\frac{85}{100}$ $\frac{630}{720}$

Strategies for calculating fractions and percentages

of numbers and quantities. For example:
- $\frac{2}{5}$ of 20.5 = 8.2
  (e.g. find one fifth, multiply by 2)
- $\frac{3}{8}$ of 6400 = 2400
  (e.g. find one eighth, multiply by 3)

- Find 20% of £3.50.
- Find 35% of £5.
- Increase 480 kg by 20%.
- Decrease 500 mm by 12%.
- 25% of a number is 12. What is the number?

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Word problems and puzzles (all four operations)

Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Arrange the digits 3, 5 and 2 to make the largest possible odd number.
- Write in figures the number two and a quarter million.
- A girl scored 67 in her first innings and 128 in her second innings. What was her total score?
- Pencils cost 37p each.
  How many pencils can you buy with £3.70?
- A 55 g bag of crisps has 20% fat. How much fat is that?
- A boy saved £215. He bought a Walkman for £69.
  How much money did he have left?
- A girl used 2 metres of wood to make 5 identical shelves.
  How long was each shelf?
- Estimate the value of 51×19.
- Find two numbers whose sum is 14 and whose product is 48.
- There are 12 green buttons and 4 white buttons in a tin.
  I choose one button at random from the tin.
  What is the probability it is a white button?

Written questions

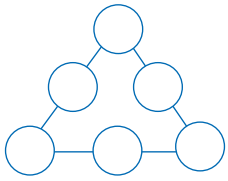
- Sandy and Michael dug a neighbour's garden.
  They were paid £32 to share for their hours of work.
  Sandy worked for 6 hours. Michael worked for 2 hours.
  How much should Sandy get paid?
- The mean of a, b and c is 6. a is 5 and b is 11. What is c?
- Tony, David and Estelle are playing a team game.
  They need to get a mean of 75 points to win.
  Tony scores 63 points, Estelle scores 77 points and David scores 77 points. Have they scored enough points to win?
- What is the value of $6n + 3$ when $n = 2.5$?

Solve problems or puzzles such as:

- Three consecutive integers add up to 87.
  What are they?

- Choose from 1, 2, 3, 4 and 5 to place in the boxes.
  In any question, you cannot use a number more than once.
  a. $\square - \square + \square = 5$
  b $\square + \square - \square = 4$
  c. $\square \times \square - \square = 3$
  d. $(\square + \square) \div \square = 2$
  e. $(\square + \square) \div (\square + \square) = 1$

- Use each of the numbers
  1, 2, 4, 6, 8, 12 once.
  Write one number in each circle.
  The product of the three numbers on each side of the triangle must be 48.



- Write any number up to 40.
  Multiply its last digit by 4 then add the other digit to this.
  Repeat the process until you get back to the original.
  What is the longest chain you can make?

As outcomes, Year 8 pupils should, for example:

Word problems and puzzles (all four operations)

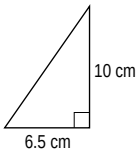
Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Write in figures the number that is one less than seven and a half million.
- Two angles fit together to make a straight line. One of them is 86°. What is the other?
- 1 ounce is about 28 grams. About how many grams are 3 ounces?
- Four oranges cost 37p. What do 12 oranges cost?
- How many metres are there in 2.5 kilometres?
- A person's heart beats 70 times in 1 minute. How many times does it beat in 1 hour?
- Carpet tiles are 50 cm by 50 cm. How many are needed to cover one square metre?
- Estimate the value of 502 ÷ 49.
- Solve 45 + x = 92.
- The probability that it will rain in August is 0.05. What is the probability it will not rain in August?

Written questions

- 14 × 39 = 546. What is 14 × 3.9?
- Four sunflowers have heights of 225 cm, 199 cm, 185 cm and 239 cm. What is their mean height?
- The sum of p and q is 12. The product of p and q is 27. Calculate the values of p and q.
- Find 25% of 10% of £400.
- Calculate the area of this triangle.



Solve problems or puzzles such as:

- Make 36 using any combination of +, –, ×, ÷, and brackets, and each of 1, 3, 3 and 5 once.

- The numbers 3 and 10 are written on the front of two cards. There is a different number on the back of each card. When the two cards are on the table, the sum of the two numbers showing is 12, 13, 14 or 15. What two numbers are on the back of the cards?

- Use each of the digits 1, 2, 3, 4, 5, 6, 7 once. Write them in the boxes to make this statement true:
  □□ + □□ + □□ + □ = 100

- Take a pair of consecutive integers. Square each of them. Find the difference of the two squares. Repeat with different pairs of consecutive integers. Repeat with a pair of numbers whose difference is 2, or 3, or 4 …

As outcomes, Year 9 pupils should, for example:

Word problems and puzzles (all four operations)

Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Two years ago Jim's height was 1.48 metres. Now Jim's height is 1.7 metres. How much has Jim grown?
- Two of the angles of a triangle are 47° and 85°. Calculate the third angle.
- You get $56 for £40. How many dollars do you get for £100?
- 75 miles per hour is about the same as 33 metres per second. About how many metres per second is 50 miles per hour?
- In a raffle, half of the tickets are bought by men. One third are bought by women. The rest are bought by children. What fraction of the tickets are bought by children?
- The ratio of men to women in a room is 3 to 5. There are 12 men. How many women are there?
- x = 2 and y = 3. Work out the value of x to the power y plus y to the power x.

Written questions

- The probability that a train will be late is 0.03. Of 50 trains, how many would you expect to be late?
- Find 1% of 2% of £1000.
- Some girls and boys have £32 between them. Each boy has £4 and each girl has £5. How many boys are there?

Solve problems or puzzles such as:

- You can use four 8s to make 10, e.g. (8 + 8)/8 + 8. Using any of +, –, ×, ÷ and brackets, and eight 8s, make the number 1000.

- Find two numbers:
  whose sum is 0.8 and whose product is 0.15;
  whose sum is $^{-}$11 and whose product is 28;
  whose difference is 4 and whose quotient is 3;
  whose difference is 2 and whose quotient is $^{-}$1.

- The product of two numbers is six times their sum. The sum of their squares is 325. What are the two numbers?

- Use each of the prime numbers 5, 7, 11, 13, 17, 19, 23 once. Write one in each circle so that the three primes in each line add up to the same prime number.