

NUMBERS AND THE NUMBER SYSTEM

Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals

As outcomes, Year 7 pupils should, for example:

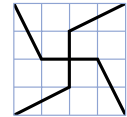
Use, read and write, spelling correctly:
*numerator, denominator, mixed number, proper fraction, improper fraction... decimal fraction, percentage...
 equivalent, cancel, simplify, convert...
 lowest terms, simplest form...*

Understand a fraction as part of a whole.

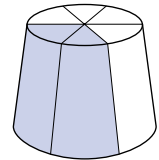
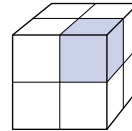
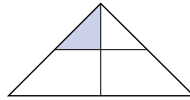
Use fraction notation to describe a proportion of a shape.

For example:

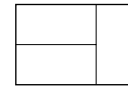
- Find different ways of dividing a 4 by 4 grid of squares into quarters using straight lines.



- Estimate the fraction of each shape that is shaded.



- Shade one half of this shape.



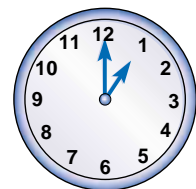
- Watch a **computer simulation** of a square being sectioned into fractional parts.
 Shade a fraction of a 6 by 6 grid of squares, e.g. one third.
 Convince a partner that exactly one third is shaded.

Relate fractions to division. Know that $4 \div 8$ is another way of writing $\frac{4}{8}$, which is the same as $\frac{1}{2}$.

Express a smaller number as a fraction of a larger one.

For example:

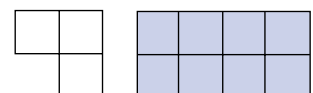
- What fraction of:
 - 1 metre is 35 centimetres?
 - 1 kilogram is 24 grams?
 - 1 hour is 33 minutes?
 - 1 yard is 1 foot?
- What fraction of a turn does the minute hand turn through between:
 - 7:15 p.m. and 7:35 p.m.?
 - 3:05 p.m. and 6:50 p.m.?



- What fraction of a turn takes you from facing north to facing south-west?

- What fraction of a turn is 90° , 36° , 120° , 450° ?

- What fraction of the big shape is the small one? ($\frac{3}{8}$)



Know the meaning of *numerator* and *denominator*.

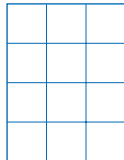
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to: *terminating decimal, recurring decimal... unit fraction...*

Use fraction notation to describe a proportion of a shape. For example:

- Draw a 3 by 4 rectangle.

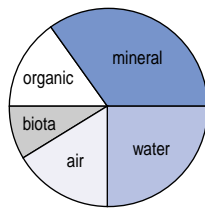
Divide it into four parts that are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$ of the whole rectangle. Parts must not overlap.



Now draw a 4 by 5 rectangle. Divide it into parts. Each part must be a unit fraction of the whole rectangle, i.e. with numerator 1.

Try a 5 by 6 rectangle. And a 3 by 7 rectangle?

- The pie chart shows the proportions of components in soil.

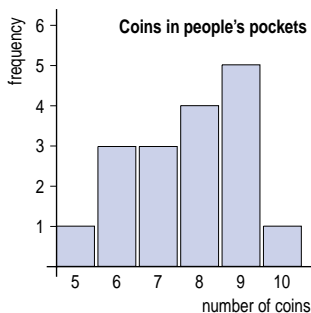


Estimate the fraction of the soil that is:
 a. water; b. air.

Relate fractions to division. Know that $43 \div 7$ is another way of writing $4\frac{3}{7}$, which is the same as $6\frac{1}{7}$.

Express a number as a fraction (in its lowest terms) of another. For example:

- What fraction of 180 is 120?
- What fraction of:
 1 foot is 3 inches?
 1 year is February?
- The bar chart shows the numbers of coins in people's pockets. What fraction of the total number of people had 7 coins in their pockets?



Link to enlargement and scale factor (pages 212–15).

As outcomes, Year 9 pupils should, for example:

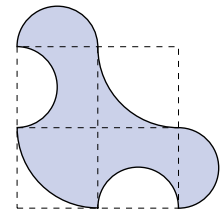
Use fraction notation to describe a proportion of a shape. For example:

- Estimate the fraction of each shape that is shaded.



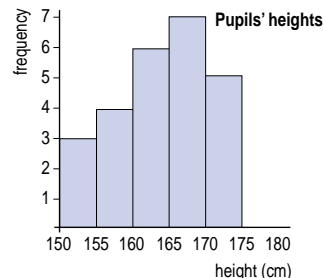
- The curves of this shape are semicircles or quarter circles.

Express the shaded shape as a fraction of the large dashed square.



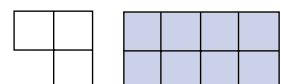
Express a number as a fraction (in its lowest terms) of another. For example:

- What fraction of 120 is 180? ($\frac{3}{2}$ or $1\frac{1}{2}$)
- This frequency diagram shows the heights of a class of girls, classified in intervals $150 \leq h < 155$, etc.



What fraction of the girls are between 150 cm and 160 cm tall?

- What fraction of the small shape is the large one? ($\frac{2}{3}$ or $2\frac{2}{3}$)



Link to enlargement and scale factor (pages 212–15).

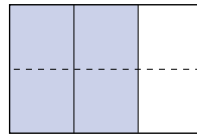
Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals (continued)

As outcomes, Year 7 pupils should, for example:

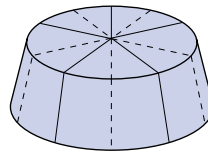
Simplify fractions by cancellation and recognise equivalent fractions.

Understand how equivalent fractions can be shown in diagrammatic form, with shapes sectioned into equal parts.



$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{5}{5} = 1$$



$$\frac{10}{10} = 1$$

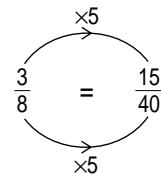
Find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

For example, recognise that because

$$3 \times 5 = 15$$

$$8 \times 5 = 40$$

it follows that $\frac{3}{8}$ is equivalent to $\frac{15}{40}$.



Know that if the numerator and the denominator have no common factors, the fraction is expressed in its lowest terms.

Answer questions such as:

- Cancel these fractions to their simplest form by looking for highest common factors:

$$\frac{9}{15}$$

$$\frac{12}{18}$$

$$\frac{42}{56}$$

- Find two other fractions equivalent to $\frac{4}{5}$.
- Show that $\frac{12}{18}$ is equivalent to $\frac{6}{9}$ or $\frac{4}{6}$ or $\frac{2}{3}$.
- Find the unknown numerator or denominator in:

$$\frac{1}{4} = \frac{\square}{48}$$

$$\frac{7}{12} = \frac{35}{\square}$$

$$\frac{36}{24} = \frac{\square}{16}$$

Link to finding the highest common factor (pages 54–5).

Continue to convert improper fractions to mixed numbers and vice versa: for example, change $\frac{34}{8}$ to $4\frac{1}{4}$, and $5\frac{7}{12}$ to $\frac{67}{12}$.

Answer questions such as:

- Convert $\frac{36}{5}$ to a mixed number.
- Which fraction is greater, $\frac{4}{7}$ or $\frac{29}{7}$?
- How many fifths are there in $7\frac{1}{5}$?
- The fraction $\frac{7}{14}$ has three digits, 7, 1 and 4. It is equal to $\frac{1}{2}$. Find all the three-digit fractions that are equal to $\frac{1}{2}$. Explain how you know you have found them all.
- Find all the three-digit fractions that are equal to $\frac{1}{3}$. And $\frac{1}{4}$...
- There is only one three-digit fraction that is equal to $1\frac{1}{2}$. What is it?
- Find all the three-digit fractions that are equal to $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$...

See Y456 examples (pages 22–3).

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Understand the equivalence of algebraic fractions.

For example:

$$\frac{ab}{ac} \equiv \frac{b}{c}$$

$$\frac{ab^2}{abc} = \frac{\overset{1}{a} \times \overset{1}{b} \times b}{\overset{1}{a} \times \overset{1}{b} \times c} = \frac{b}{c}$$

$$\frac{3ab}{6bc} = \frac{a}{2c}$$

Simplify algebraic fractions by finding common factors. For example:

- Simplify $\frac{3a + 2ab}{4a^2}$

Recognise when cancelling is inappropriate. For example, recognise that:

- $\frac{a + b}{b}$ is not equivalent to $a + 1$;
- $\frac{a + b}{b}$ is not equivalent to a ;
- $\frac{ab - 1}{b}$ is not equivalent to $a - 1$.

[Link to adding algebraic fractions \(pages 118–19\).](#)

NUMBERS AND THE NUMBER SYSTEM

Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals (continued)

As outcomes, Year 7 pupils should, for example:

Convert terminating decimals to fractions.

Recognise that each terminating decimal is a fraction: for example, $0.27 = \frac{27}{100}$.

Convert decimals (up to two decimal places) to fractions.

For example:

- Convert 0.4 to $\frac{4}{10}$ and then cancel to $\frac{2}{5}$.
- Convert 0.32 to $\frac{32}{100}$ and then cancel to $\frac{8}{25}$.
- Convert 3.25 to $3\frac{25}{100} = 3\frac{1}{4}$.

[Link to place value \(pages 36–9\).](#)

Convert fractions to decimals.

Convert a fraction to a decimal by using a known equivalent fraction. For example:

- $\frac{2}{8} = \frac{1}{4} = 0.25$
- $\frac{3}{5} = \frac{6}{10} = 0.6$
- $\frac{3}{20} = \frac{15}{100} = 0.15$

Convert a fraction to a decimal by using a known equivalent decimal. For example:

- Because $\frac{1}{5} = 0.2$
 $\frac{3}{5} = 0.2 \times 3 = 0.6$

See Y456 examples (pages 30–1).

Compare two or more simple fractions.

Deduce from a model or diagram that $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \dots$ and that, for example, $\frac{2}{3} < \frac{3}{4}$.

$\frac{1}{8}$									
$\frac{1}{7}$									
$\frac{1}{6}$									
$\frac{1}{5}$									
$\frac{1}{4}$									
$\frac{1}{3}$									
$\frac{1}{2}$									

Answer questions such as:

- Insert a $>$ or $<$ symbol between each pair of fractions:
 $\frac{1}{2} \square \frac{7}{10}$ $\frac{3}{8} \square \frac{1}{2}$ $\frac{1}{2} \square \frac{2}{3}$ $\frac{7}{15} \square \frac{1}{2}$
- Write these fractions in order, smallest first:
 $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{5}{6}$;
 $2\frac{1}{10}$, $1\frac{3}{10}$, $2\frac{1}{2}$, $1\frac{1}{5}$, $1\frac{3}{4}$.
- Which of $\frac{5}{6}$ or $\frac{6}{5}$ is nearer to 1?
 Explain your reasoning.

See Y456 examples (pages 22–3).

As outcomes, Year 8 pupils should, for example:

Convert decimals to fractions.

Continue to recognise that each **terminating decimal** is a fraction. For example, $0.237 = \frac{237}{1000}$.

Recognise that a **recurring decimal** is a fraction.

Convert decimals (up to three decimal places) to fractions. For example:

- Convert 0.625 to $\frac{625}{1000}$ and then cancel to $\frac{5}{8}$.

Link to percentages (pages 70–1).

Convert fractions to decimals.

Use division to convert a fraction to a decimal, without and with a calculator. For example:

- Use short division to work out that:
 $\frac{1}{5} = 0.2$ $\frac{3}{8} = 0.375$ $\frac{2}{8} = \dots$ $\frac{3}{7} = \dots$
- Use a **calculator** to work out that $\frac{7}{53} = \dots$

Investigate fractions such as $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{1}{11}$, ... converted to decimals. For example:

- Predict what answers you will get when you use a **calculator** to divide:
 3 by 3, 4 by 3, 5 by 3, 6 by 3, and so on.

Order fractions.

Compare and order fractions by converting them to fractions with a common denominator or by converting them to decimals. For example, find the larger of $\frac{7}{8}$ and $\frac{4}{5}$:

- using common denominators:
 $\frac{7}{8}$ is $\frac{35}{40}$, $\frac{4}{5}$ is $\frac{32}{40}$, so $\frac{7}{8}$ is larger.
- using decimals:
 $\frac{7}{8}$ is 0.875, $\frac{4}{5}$ is 0.8, so $\frac{7}{8}$ is larger.

Use equivalent fractions or decimals to position fractions on a number line. For example:

- Mark fractions such as $\frac{2}{5}$, $\frac{6}{20}$, $\frac{3}{15}$, $\frac{18}{12}$ on a number line graduated in tenths, then on a line graduated in hundredths.

Answer questions such as:

- Which is greater, 0.23 or $\frac{3}{16}$?
- Which fraction is exactly half way between $\frac{3}{5}$ and $\frac{5}{7}$?

As outcomes, Year 9 pupils should, for example:

Know that a recurring decimal is an exact fraction.

Know and use simple conversions for recurring decimals to fractions. For example:

- $0.333\ 333\dots = \frac{1}{3}$ ($= \frac{3}{9}$)
- $0.666\ 666\dots = \frac{2}{3}$
- $0.111\ 111\dots = \frac{1}{9}$
- $0.999\ 999\dots = \frac{9}{9} = 1$

Convert recurring decimals to fractions in simple cases, using an algebraic method. For example:

- $z = 0.333\ 333\dots$ (1)
 $10z = 3.333\ 333\dots$ (2)
 Subtracting (1) from (2) gives:
 $9z = 3$
 $z = \frac{1}{3}$

- **Comment on:**
 $z = 0.999\ 999\dots$
 $10z = 9.999\ 999\dots$
 $9z = 9$
 $z = 1$

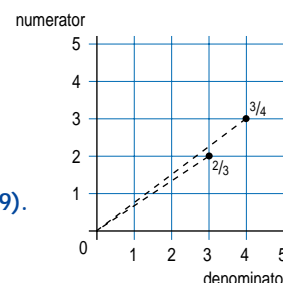
Order fractions.

Answer questions such as:

- The numbers $\frac{1}{2}$, a , b , $\frac{3}{4}$ are in increasing order of size. The differences between successive numbers are all the same. What is the value of b ?
- z is a decimal with one decimal place. Write a list of its possible values, if both these conditions are satisfied:
 $\frac{1}{3} < z < \frac{2}{3}$ $\frac{1}{6} < z < \frac{5}{6}$

Link to inequalities (pages 112–13).

Order fractions by graphing them. Compare gradients.



Link to gradients (page 167–9).

Investigate sequences such as:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, \frac{n}{n+1}$$

Investigate what happens as the sequence continues and n tends towards infinity. Convert the fractions to decimals and draw a graph of the decimal against the term number.