C2 Formulae

Remainder Theorem

To find the remainder when one algebraic function $f(x)$ is divided by $(x-a)$, we simply find $f(a)$ and that is the remainder. Therefore we know that if $f(a)=0$ for some value of a, then $x=a$ is a root of that function which means $(x-a)$ is a factor of that function.

Geometric Sequences

A geometric sequence is a sequence of numbers defined by two values:

- a - The first number in the sequence
- r - The common ratio. This means that each term is r times bigger than the previous term.

Using these facts we can show that the n-th term of a gerometric progression is:

$$x_n = ar^{n-1}$$

We can also show that the sum of the first n terms of the sequence is:

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$$

Here I have shown two possible representations, both of which are correct however the first is easier if $r>1$ and the second preferable if $r<1$. In the case of $r<1$ we can also find the infinite sum of the sequence:

$$S_\infty = \frac{a}{1-r}$$

Binomial Expansion

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ which can also be obtained from your calculator using the ${}_nC_r$ button. An example of this is:

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Trigonometry

C2 requires you to be able to manipulate various trigonometric equations and then solve them. To do this we turn to two VERY important identities. You must know these perfectly:

- $cos^2\theta + sin^2\theta \equiv 1$
- $tan\theta \equiv \frac{sin\theta}{cos\theta}$

Using these it is possible to rewrite many trigonometric equations in terms of just one trigonometric function, which makes solving the equations possible.
Once the equation is solved for one value of θ, we must remember that the oscillating nature of trigonometric functions means that there are often other possible answers. The calculator will always give you the answer that falls in the range $0° < \theta < 90°$ if the value was positive, since all trig functions are positive in that range. To find the other possible values we look at which other quadrants the function is positive in:

Range	Positive Function	How to find angle
$90° < \theta < 180°$	Only Sin	$180-\theta$
$180° < \theta < 270°$	Only Tan	$180+\theta$
$270° < \theta < 360°$	Only Cos	$360-\theta$

It is also important to remember that if the trigonometric function is squared in the final equation, that means there will be four solutions, one in each quadrant.

Logarithms

Logarithms are extremely useful when we have unknown values as a power. The important thing to know is that:

$$y = b^x \Rightarrow log_b y = x$$

There are also a few other log rules that you must remember:

- $log(ab) = log(a) + log(b)$
- $log(\frac{a}{b}) = log(a) - log(b)$
- $log(a^b) = b \times log(a)$
- $log_b(b^c) = b^{log_b(c)} = c$
- $log(1) = 0$

You may occasionally see the natural logarithm, which is written $ln(x) = log_e(x)$, where $e = 2.718281.....$ which is an irrational number.

Equation of a circle

The equation of a circle centred at (a,b) with radius r has the equation:

$$(x - a)^2 + (y - b)^2 = r^2$$

Area under a curve

The area under curve $f(x)$ between $x = a$ and $x = b$ is given by:

$$\int_a^b f(x)dx$$

Stationary Points on Graphs

A stationary point on a graph as any point with a zero gradient. These can be found by differentiating the function of the curve and setting it equal to zero. So to find stationary points of f(x):

$$\frac{d}{dx}f(x) = 0$$

To find out whether this is a maximum or minimum, we must differentiate again:

$$\frac{d^2}{dx^2}f(x)$$

If this is positive then the stationary point is a minimum. If it is negative then it the stationary point is a maximum.