C1 Formulae

Indices

These are extremely important and you must be comfortable manipulating indices as they occur often exams. You need to know these few rules:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$
- $(a^m)^n = a^{mn}$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Surds

When given a surd to simplify, we start by rationalising the denominator. For example:

$$\frac{a+\sqrt{b}}{c+\sqrt{d}} \times \frac{c-\sqrt{d}}{c-\sqrt{d}} = \frac{(a+\sqrt{b})(c-\sqrt{d})}{c^2-d}$$

There are a few rules which may help to simplify the surd:

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- $a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$

Co-ordinate Geometry

Given an equation of a line $y = f(x)$, we differentiate to find $\frac{dy}{dx} = f'(x)$ which gives us the gradient m of the tangent to the line at any given x value. The tangent of the normal $\hat{m}$ at this point can be found as $\hat{m} = \frac{-1}{m}$.
If you are given two points and want to find the gradient of the line between them:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

To find the equation of a straight line, you must know the gradient m of the line and one point through which the line passes $P(x_1, y_1)$. Then the equation of the line is give by:

$$y - y_1 = m(x - x_1)$$

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$\overrightarrow{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Graph Transformations

There are six types of transformations one can apply to the graph $y = f(x)$. These are summarised in the following table where a is just a constant:

$f(x) + a$	shifts the curve up by a units	$f(x + a)$	shifts the curve to the left by a units
$af(x)$	stretches the curve vertically	$f(ax)$	compresses the curve horizontally
$-f(x)$	reflects the curve in the x-axis	$f(-x)$	reflects the curves in the y-axis

The first column in the above table shows the three transformations that affect the y co-ordinates of a curve. These act exactly how you would expect them to. The second column shows the three transformations that affect the x co-ordinates which actually act in the opposite manner to how you would expect. Make sure you fully understand these!

Roots of an Equation

The roots of an equation $y = f(x)$ are the points at which $y = 0$. In the exam we are often given a quadratic equation and told that it has either *no real roots*, *equal real roots* or *distinct real roots.* Then asked to show that some equation or inequality holds. To do this we firstly rearrange the quadratic into our standard form $ax^2 + bx + c = 0$ and then substitiute the values of a,b and c into one of the following:

- $b^2 - 4ac < 0$ if the equation has no real roots
- $b^2 - 4ac = 0$ if the equation has equal real roots
- $b^2 - 4ac > 0$ if the equation has two distinct real roots

Sketching curves

There are two important points to consider when sketching a curve:

1. Shape - Work out what shape the curve will be, whether it is x^2, $-x^2$, x^3, $-x^3$, trigonmetric or even just linear.
2. Axes - Calculate the points at which the curves crosses the axes. This is done by simply setting $y = 0$ and solve for x and then setting $x = 0$ and solve for y

Once you have done these two things, you should be able to draw an adequate sketch of the given curve.

Calculus

This is made up of Differentiation and Integration. You will be expected to have a basic knowledge of both of these. The easiest place to start is to put your formula in a fairly standard form, by changing fractions to negative indices, roots to fractional indices and expanding brackets. Once you have your formula as a sum of powers of x then:

- Differentiation: $x^n \to nx^{n-1}$ (ie. multiply by power and then reduce power by 1)
- Integration: $x^n \to \dfrac{x^{n+1}}{n+1}$ (ie. increase power by 1 and then divide by new power)

You must make sure you are confident at performing both of these processes and remember which is which. Be very careful with your minus signs if you have negative powers.

Iterative Sequences

Sequences are often defined iteratively, which mean we are given some starting value and then a way to calculate each successive value, based on the last. So in the sequence $\{a_n\}$, we could be given:

$$a_1 = p$$

$$a_n = f(a_{n-1})$$

So given a_1 we simply substitute that into the function in the second equation to find out what a_2 is. Then we can substitute this value into the same function again to find the value for a_3 etc. If you ever find that you come across the same number twice then you find the sequence will start to loop over and over. For example, if $a_1 = p$, $a_2 = q$ and $a_3 = p$ then $a_4 = q$ and the sequence will just alternate $p, q, p, q, p, q...$

Arithmetic Sequences

An arithmetic Sequence is one that can be defined using two paramteters:

- a - This is the first term of the sequence
- d - This is the commom difference, so each term is d larger than the previous.

We can now define the n-th term of an aritmetic sequence as:

$$x_n = a + (n-1)d$$

And the sum of the first n terms of an aritmetic sequence as:

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a + l)$$

where l is the last term in the sequence.