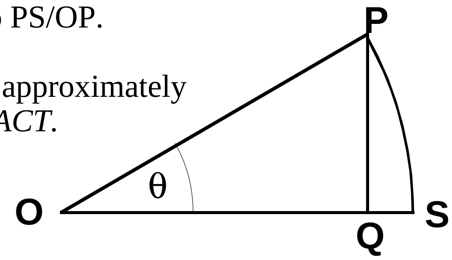
1. In the diagram at the right, arc PS is part of a circle with center at point O. Point O is called the circle's "center of curvature".
   a. Notice that ***if*** angle θ is small, ***then*** the length of line PQ is approximately equal to the length of arc PS. Also notice that the approximation becomes better as the angle becomes smaller. (Try calculating the difference when the angle is less than one degree!)
   b. Notice that sin θ = PQ/OP, and that it is approximately equal to PS/OP.
   c. By definition, the "**radian measure"** of angle θ is ___ ÷ ____.
   d. Conclusion: If θ is small, the *radian* measure of θ is approximately ________ to the *sine* measure of θ. *This is an IMPORTANT FACT.*



2. Let "R" represent the ratio of the *sine* measure of an angle to the *radian* measure of that angle.
   a. If the two measures are exactly equal, then **R =** ___.
   b. If they differ by only one percent, then **R =** _____ or _____.
   c. If you know the "R" value for any angle you can calculate the percentage disagreement between the two measures. Using a calculator, tabulate and plot a few points on a graph to show how R depends on A.
   d. Use the graph to decide roughly how small the angle must be for the approximation to be good within **5%, 1%,** and **0.1%.**

3. Does "R" have a limit as angle θ approaches zero? ______ -If so, determine its value.

4. Figure out similar approximation rules for tangents and cosines:
   "If θ is small and radian measure is used, then tan θ is approximately equal to _____ and cosθ is approximately equal to _____." (If you need a clue, look at the graphs!)

5. Given any small percentage uncertainty that some engineer might require, is it possible to find an angle which is not zero, but which is small enough so that the approximations above are valid? ___ -Is there some non-zero percentage that can never be achieved, no matter how small the angle is made? __

6. Use #3 to prove that for small displacements a pendulum bob is subject to a linear restoring force.

7. You can also use #3 to measure the earth's radius while watching a sunset over an ocean horizon. First squat down as close as you can to the water level and wait for the sun to disappear behind the horizon. When it does, start counting and stand up. You will see the sun again for a few seconds before it disappears again. (A diagram will help you see why.)
   a. Use the measured time interval to calculate how much the earth rotated while you were counting.
   b. If your initial altitude was zero, the diagram will show how the earth radius is related to that small angle and your small change in altitude. Remember to use the tricks discovered above to simplify the formula.
   c. You can also solve the problem with a larger initial altitude: To do so, you must use two simultaneous equations involving two angles.