Imagine that there has been a terrible accident in which an automobile failed to make a turn on a mountain road. Your job is to figure out how fast the car was going when it went over the edge of the cliff. To prepare for the job you decide to brush up on some physics. To model the problem you prepare a spring gun which can shoot a brass ball horizontally from a table top. The ball flies through the air just as the car did, and hits the floor some distance away. By taping a sheet of paper onto the floor you can easily record the impact point. Both the vertical distance from the point where the ball leaves the gun to the floor and the horizontal distance covered by the ball while it is in the air can be measured.

1. Using a diagram, define symbols to represent those distances. (Show how they are measured.)

2. It is clear that gravity causes the ball to descend to the floor after leaving the gun. (Without gravity the ball would have continued moving horizontally.) You also know how much acceleration is caused by gravity if the object is falling freely, and how the falling distance and falling time are related. But some people argue that the horizontal velocity given to the ball by the gun will cause gravity to have a different effect on that ball. To settle that argument you arrange for a second ball to be released from the same altitude at the moment when the first ball is shot forward.
   a. After making the necessary measurements, show how the falling time is calculated for the ball that is released. Also show how the uncertainty or range of that result is estimated. (See #11 on RS IV.)
   b. Do a simple experiment to find out if the same equation predicts flight time of the ball that is shot straight forward. In other words, find out if the horizontal motion of the ball causes it to have a different gravitational acceleration. *Explain* your observation and conclusion briefly and clearly.

3. Imagine shooting the ball horizontally from an outdoor table top on a clear day when the sun is directly overhead. The ball's shadow is always directly beneath it, on the level ground. To change the speed of the shadow you would have to change the ball's horizontal velocity. But no significant horizontal force acts on the ball while it is in the air. (The ball is dense enough so that air drag can be ignored.)
   a. Describe the motion of the ball's shadow under those conditions and illustrate with a graph.
   b. After making appropriate measurements, show how the ball's horizontal velocity is calculated.
   c. Show how the uncertainty or range of that result is estimated.

4. You know that some people will distrust your results. To demonstrate that you are correct, you decide to make a daring prediction and then test it by experiment. You will use the same spring gun to shoot the same ball straight up, after predicting how high it will go. Show how that prediction is made, and show how its uncertainty or range is estimated. (See #9 on RS VI.) Remember to define your symbols.

5. If the upshot experiment does not agree with your prediction as closely as you would like, then perhaps a mistaken assumption was made in #2. Use the actual height found by measurement to re-calculate the ball's speed. Also suggest possible reasons why the speed found in #2 was too big or too small.

6. Another way to convince the skeptics that you are qualified to be an expert witness is to predict what will happen when the ball is shot into a massive pendulum. (It's called a "ballistic pendulum", and is commonly used to measure bullet speeds.)
   a. The ball remains in the pendulum after impact. You already know the speed of the ball *before* impact. What familiar conservation principle enables you to calculate the speed of the ball and pendulum bob just *after* impact? State the principle clearly in words.
   b. Choose symbols to use in your equations: (Let ___ represent the velocity of the ball just before impact. Let ___ represent the velocity of the ball and pendulum just *after* impact. Let ___ represent the mass of the ball. Let ___ represent the mass of the pendulum, so the mass of the ball and pendulum together must be ___ + ___.) Use those symbols to write equation 6a: ___ = ______
   c. Solve that equation for the velocity just after impact, and check units: __ = _________
   d. Make whatever measurements are needed for that calculation and record them clearly. To avoid confusion, do *not* abbreviate four-letter words. Then calculate the speed and its range or uncertainty.

7. The pendulum swings forward and upward after impact.
   a. What kind of energy does it lose on the way up? ________ -What kind does it gain? ______ _______
   b. Does the *total* energy of the ball and pendulum change during the upswing? ___
   c. What symbol in 6b represents the velocity of the bob at the beginning of this swinging motion? __
   * d. Write the conservation principle mentioned in 7b as an equation in simplest form, using that symbol and others defined in #6b. If additional symbols are needed, *define* them. *Don't be vague.*
   * e. Use 7d to predict how far up the pendulum will swing, and show clearly how that prediction is made.

A spring gun shot a brass ball horizontally from a table top. The ball hit the floor some distance away, as shown in the diagram on the back of this paper. The vertical distance to the floor from the point where the ball left the gun (called "Y") was 0.85 ± 0.002 meter, and the horizontal distance ("X") covered by the ball while it was in the air was 2.45 ± 0.012 m.

1. We know how much acceleration is caused by gravity if the object is falling freely, and how that acceleration and falling time can be use to calculate the falling distance. But some people argue that the horizontal velocity given to the ball by the gun will cause gravity to have a different effect on that ball. To settle that argument, we arranged for a second ball to be released from the same altitude at the moment when the first ball was shot forward. Those two balls hit the floor at the same time. We concluded that the vertical acceleration caused by gravity does not depend on horizontal velocity.

2. Since the ball which fell straight down had a triangular speed-time graph, we know that Y = _____. (That's the formula for the area of such a triangle.) Solving for the unknown falling time, we get:

   $$\mathbf{t = (Y/a)^{1/2} = [(0.85\ m)/(9.8\ m/s^2)]^{1/2} = 0.43\ sec.}$$

   The relative uncertainty of "Y" was (±0.002 meter)/(0.85 m) = 0.0022 = 0.22%.
   In calculating the falling time we had to take the square root of Y, which doubles that percentage.
   Therefore the falling time was **t = 0.43 sec ± 0.44%**.

3. No significant horizontal force acted on the ball while it was in the air. (The ball is so dense that air drag can be ignored.) Therefore the horizontal component of the ball's velocity must have been constant, and equal to the velocity that it had when it left the gun. We calculated that velocity by dividing the horizontal distance ("X") by the travel time. When dividing those values we must subtract their percent uncertainties. The uncertainty of X is (± 0.012 meter)/(2.45 m) = ± 0.005 = ± 0.5%.

   Therefore, **V = (2.45 m ± 0.5%)÷(0.43 sec ± 0.44%) = 5.69 m/s ± 0.94%**

4. To test that result we decided to shoot the same ball straight up with the same spring gun, after predicting how high it would go. All of the ball's initial kinetic energy becomes gravitational potential energy on the way up, so $mV^2/2 = mgH$. ("H" represent its gain in altitude, and "m" represents its mass.) Solving for the unknown height, (exactly as in 9c on page 62) we find that

   $$\mathbf{H = V^2/g = (5.69\ m/s \pm 0.94\%)^2 \div (9.8\ m/s^2) = 3.08\ m \pm 1.9\%.}$$

5. When we tested this prediction we found that the ball actually went up about 1.6 meter, indicating that something might be slightly wrong with the results above. Perhaps the ball was not originally shot in a perfectly horizontal direction, or perhaps repeated operation of the gun caused its internal friction to be increased. Whatever the reason, it is clear that "V" should be re-calculated. Solving the energy equation in #3, we get $V = (2gH)^{1/2}$. Inserting the measured value for "H", we get

   $$\mathbf{V = [2(9.8\ m/s^2)(1.6\ m \pm 2\%)]^{1/2} = 24.8\ m/s \pm 1\%}$$

6. Next we predicted what would happen if we shot the ball into a massive "ballistic pendulum". Since the ball remains embedded in the pendulum, the momentum of the ball before impact must be equal to the momentum of the ball and pendulum together just after impact. Let "V" represent the velocity of the ball just before impact. Let "v" represent the velocity of the ball and pendulum just *after* impact. Let "m" represent the mass of the ball. Let "M" represent the mass of the pendulum, so the mass of the ball and pendulum together must be M + m.
   a. Using those symbols, the momentum conservation equation says **mV = (M + m)v.**
   b. Solving that equation and simplifying the result, we get **v = V/(m/M + 1)**
   c. With a balance we found that **M = 223 gram,** and **m = 69 gram**, so **M/m = 3.75**.
      The masses had much smaller percentage uncertainties than the lengths measured above.
      Inserting those values into eq. 6b, we get: **v = (5.73 m/s ± 1%) ÷ (3.75 + 1) = 1.02 m/s ± 1%.**

7. The pendulum swings forward after impact, gaining gravitational potential energy and losing kinetic energy, so $(mv)^2/2 = mgh$. Solving for the pendulum's gain in altitude, we get $\mathbf{h = mV^2/2g}$.
   Inserting values, we get $\mathbf{h = (1.02\ m/s \pm 1\%)^2 \div 19.6\ m/s^2 = 7.2\ cm \pm 2\%.}$
   Testing this prediction, we found that h = 7.0 ± .03 cm, indicating that our theory is ________.

Note: *All numerical values and some of the statements on this page are pure fiction.*