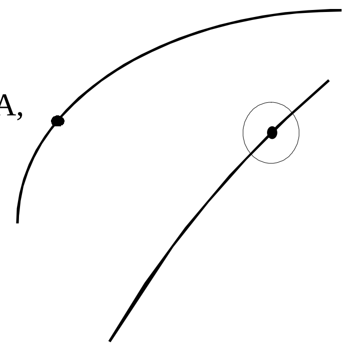
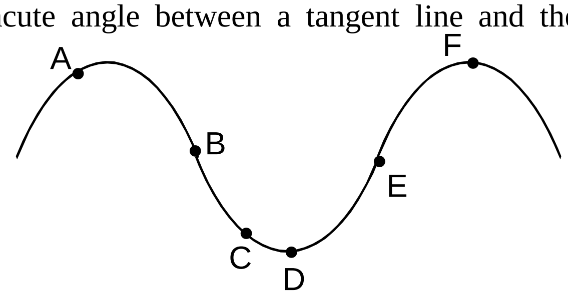


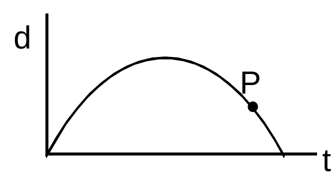
1. Here is a curve with a dot on it. Label the dot with the letter "A".
   Using a ruler, draw some straight lines through point A.

2. If we used a microscope to examine the part of the curve that is near point A, we would see something like the *second* picture at the right.
   In that picture I have circled a small region around point A.
   a. Look at the part that is inside the circle. When you make a magnified picture of a short part of a curve, (like the part that is circled in this diagram) it looks like a ________ line. (curved, straight, zig-zag)
   b. Using a ruler, *extend* the line inside the little circle in both directions.
   c. *That extended straight line is called a "**tangent line".*** We say that it is "tangent *to* the curve *at* point A". Notice that a line *cannot* be "tangent ***to*** a point".
   A line can only be tangent ***to*** a ________, ***at*** a ________.
   d. Most of the straight lines that you drew on the *first* diagram above were *not* tangent to the curve at point A. How many straight lines through point A ***can*** be tangent to the curve? ___
   e. Please illustrate 2d by labeling a tangent line in the top diagram.

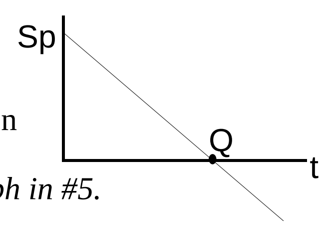
3. If you looked at a magnified picture of the region near point A in the top diagram you would see a bunch of intersecting lines forming angles with their vertexes at point A. You could measure those angles with a protractor. If a line is *tangent* to the curve at point A then the line and the magnified portion of the curve must *coincide*. That means the acute angle between a tangent line and the magnified portion of the curve must be ___ degrees.



4. Use a ruler to draw tangent lines at each of the labeled points on the curve at the right.
   a. Which tangent line has the greatest slope? __
   b. Which one has a small positive slope? ___
   c. Which one has a small negative slope? ___
   d. Which ones have slope equal to zero? ___
   e. Which one has a slope that is almost equal to the one in 4a but with the opposite sign? ___



5. We all know what is meant by the "slope" of a straight line. But sometimes we need to know the slope *at a point* on a *curved* line. For example, the displacement-time graph for a ball tossed straight up would look like the one sketched at the right, as on page 28. If the ball had a speedometer, we could look at it to see what speed the ball had at any point along its journey, such as at point "P".
   a. At the time corresponding to point "P" on this graph the ball had a _____tive speed.
   b. We can see that easily because a line tangent to the curve at point P has a _____tive slope.
   c. Please draw that line through point P, using a ruler.



6. To find the *acceleration* that an object had at any given time you would find the slope of a line tangent to the object's ______-time graph.
   For example, the graph at the right describes the *same motion as #5,* as on page 28. The numerical value of its slope at Q is ____ ____.
   Check the sign and units carefully. Lable point Q on the time axis of the graph in #5.

7. A circus clown is shot diagonally upward from a cannon.
   a. Draw a dotted line describing the curved path of the clown through the air from the cannon to the net. Then add a crude picture of the cannon. Draw a long straight arrow through its end to describe the emerging clown's velocity.
   b. If drawn correctly, the arrow must be ________ to the dotted curve at the point of release.
   (perpendicular, tangent, low, indifferent) -Does your diagram clearly illustrate that fact? ____
   c. Using 7b, draw the path of a coin released by a person riding a fast escalator diagonally upward as seen in the *ground* frame of reference. Label the sections of the path *before* and *after* release.

1. On pages 15 and 15R we learned how to transform *speed-time* data into *displacement-time* data**.** We have used that process several times since then. Mathematicians call it "____________tion".
   a. The first step is to find or choose the initial displacement value. (Sometimes it's given.)
   b. To find the next displacement in the data table we multiply the duration of the first time interval by the average ________ for that interval and then _______ that result to the previous displacement.
   c. Then we repeat step b for the rest of the time intervals.

2. On page 15 we also proved that the process in #1 is equivalent to calculating the _______ of a graph. (slope, area, color, mass) That fact is still recorded in #___ on RS II.

3. It is also possible to work the process *in reverse*, using a displacement-time data table to make a speed-time data table. This operation is called **"differentiation".** Here are the steps:
   a. Get "changes in displacement" by __________ing every adjacent pair of displacements in the list.
   b. Each change in displacement is ___________ed by the corresponding time interval to obtain the __________ speed for that part of the trip, as in #6 on page 8, #2 on page 9, and as on page 23.

4. The "differentiation" process described in #3 is equivalent to finding the __________s of all the dot-to-dot segments on the original graph as in #3 on RS II. Does this contradict #2? ___

5. When you **differentiate** a displacement-time graph, the resulting **"derivative"** is a graph of _________ vs time. If you differentiate a graph of *velocity* vs time, the resulting derivative is a graph of ____________ vs time. Does this contradict 1 above or #2 on RS I? ___ (At least check units!)

6. Sketch a *displacement vs time* graph describing the motion of a swinging pendulum bob. Make the first cycle look like the second. (You can use a sonic ranger to test this prediction.) Then sketch the bob's *velocity vs time* graph by following the instructions below. Find out if the computer agrees.

How to sketch the derivative of a given graph: (Save these instructions on RS II.)
Step I: Draw and label a pair of axes directly below the original graph.
Step II: Divide the original graph into short segments, short enough to be almost linear.
Step III: For each segment, decide whether the slope is positive or negative. Also decide whether the slope is great or small compared to the slopes of neighboring segments.
Step IV: Use those decisions to plot data points directly below, on the new graph.

7. When you *integrate* a function, the result that you get is called an **"integral"**.
   For example, #1 tells us that the **integral** of a velocity-time graph is a ___________ vs time graph.
   a. Suppose I integrate a set of data and then differentiate the result:
      What is the overall effect of that *pair* of operations? ____________________
   b. We must conclude that those two operations are _________. (alike, reciprocals, opposites)

8. Suppose a car accelerates "flat out" from 0 to 75 mph. (Save 8a and 8c on the back of RS II.)
   a. Sketch the car's speed-time graph. (See p. 27b or any issue of ***Road & Track*** Magazine.)
   b. On RS II you sketched a graph with similar *shape*. What was its title? _______ vs. ________
   c. Use #1 or the instructions below to transform the graph in 8a into a *displacement vs time* graph.

How to sketch the integral of a given graph: (Please copy this procedure onto RS II.)
Step I: Make the given graph into a series of dots.
Step II: Choose a starting point for the new graph, as in 1a. (Use given information if possible.)
Step III: For each dot on the given graph, make a short line segment on the new graph. Start the first segment at the point chosen in step II. Join the rest end-to-end. If a dot on the original graph has a large positive value, then give the corresponding segment a large positive slope. If the dot has a small negative value, then give the corresponding segment a small negative slope.

9. Use differentiation (as in #6) to transform the speed-time graph in 8a into a graph of *acceleration vs time*. (First use the hints below to decide whether the acceleration graph is straight or curved.)
   a. Suppose the car continues to accelerate flat out for several minutes: Will it suddenly stop gaining speed? ___ -Will it gradually stop gaining speed as it approaches a top speed? __ -Does 8a agree? __
   b. If the car slows down, then the acceleration becomes negative so that the acceleration graph must go below the time axis. Is that what happens in 8 and 9a? ____
   c. Is the acceleration graph in 9a linear? ___ Is it concave downward? ___ -Is it concave upward? ___
   d. Have you sketched that graph on this paper?___ Does it agree with 9a? __ with 9b?___ -with 9c?___