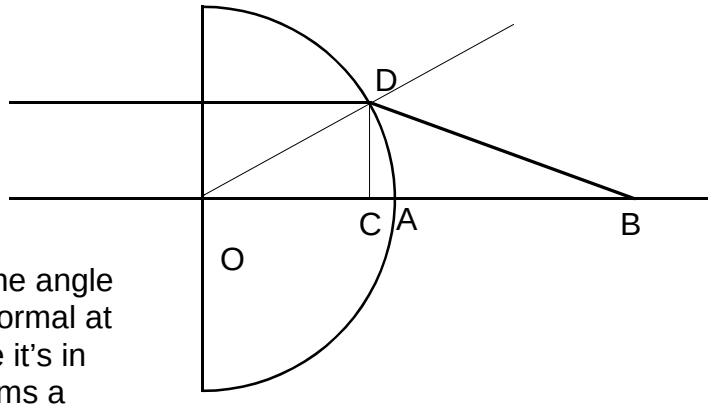
Problem of the Month **April 2002**



A light ray enters the flat side of a semi-circular lens normally, at a point R/2 = 5 cm. from the axis. The refractive index is 1.5. Predict distance AB.

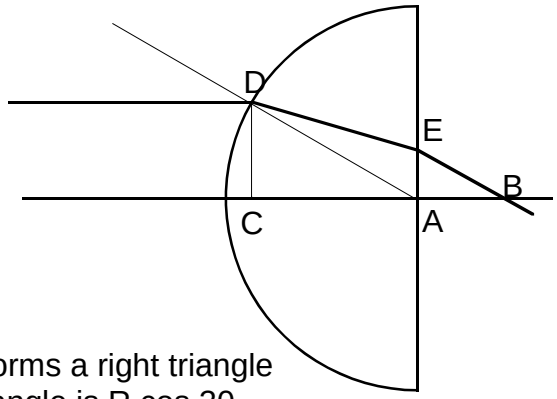
1: Since OCD is a 30-60-90 triangle, the angle "g" between the incident ray and the normal at point D is 30 deg. (I call it "g" because it's in the *glass*.) The emerging ray "DB" forms a larger angle "a" with that same normal. According to Mr. Snel, sin(a) = n sin(g), so a = 48.59 deg.

2: Extending the horizontal line to the right from point D splits that angle into two parts. The upper part is 30 deg, so the lower part (equivalent to angle DBA) is 48.59 - 30 deg = 18.59 deg. We can use that angle with distance DC to calculate distance CB:

CB = 5 cm/tan(18.59 deg) = 14.87 cm.

3: Now we must find distance CA, so we can subtract it from CB:
CA = OA - OC = R - R cos (g) = R (1 - cos g) = (10 cm)[1 - cos(30 deg)] = 1.34 cm

4: AB = CB - CA = 14.87 - 1.34 cm = **13.53 cm.**



Extra Credit: What if we turn the lens around so that the ray is first incident on the curved surface, again 5 cm from the axis?

1: The angle between the incident ray and the normal at D is 30 degrees again, but this time it's in air, so I'll call it "a". Angle "g" (angle EDA) is found by Snel's equation: sin(g) = sin(a)/n, so g = 19.47 deg.

2: Extending the incident ray horizontally to the right forms a right triangle in the glass with hypotenuse DE. The width of this triangle is R cos 30 = 8.66 cm. The small angle in this triangle is 30 deg - 19.47 deg = 10.53 deg, so its height is (8.66 cm)(tan 10.53 deg) = 1.609 cm. Therefore EA = 5.0 - 1.609 = 3.39 cm.

3: A new horizontal normal line can be drawn at E. The glass angle between ray DE and the new normal is g' = 10.53 deg, as found in step 2. The emerging ray EB forms a new air angle a' with that new normal. sin(a') = n sin(g'), so a' = 15.91 deg.

4: By alternate interior angles that new air angle a' is congruent to angle ABE, so

AB = EA/tan(a') = **11.89 cm.**.

Additional extra credit: One orientation of a hemispherical lens like the one above is better for starting fires than the other, because it will focus much more light energy on the tinder. Which one is it, why, and how much better is it?