1. A "harmonic series" is a set of waves or vibrations like this:
   $\mathbf{Y_1 = A_1 \sin(x)}, \quad \mathbf{Y_2 = A_2 \sin(2x)}, \ \ldots \quad \mathbf{Y_n = A_n \sin(nx)}, \ldots$
   a. How does the wavelength of the second wave compare with the wavelength of the first? _______
   b. What symbols represent the amplitudes of these waves? _____
   c. If these equations are supposed to represent vibrations, we should replace the "x" in each equation with "____", where the variable "__" represents time, and the constant "__" represents *angular* frequency, in radians per second. (Explained in chapter VIII, reviewed in chapter XI.)
   * d. Using just one time scale, sketch graphs of displacement (Y_n) vs time for the first three members of this harmonic series. Label them clearly.
   e. How does the period of the second harmonic compare with the period of the first, or "fundamental" harmonic? _____

2. Now imagine superimposing the motions produced by many different harmonic vibrations. This means that the displacement of our vibrating object at any time will be the *sum* of the "Y" values given by the equations in #1: $D = Y_1 + Y_2 + Y_3 + ...$
   a. Will the net displacement vs time graph resemble a sine curve? ___
   b. Will the displacement vs time graph be periodic? ____ If so, what period will it have? ____
   c. Let $A_1 = 1$, let $A_2 = 1/2$, and let $A_3 = 1/3$. Make a data table and a graph of $Y_1 + Y_2 + Y_3$ vs time. Plot enough points to show the shape of the graph over more than one full cycle. You are welcome to use more than the first three terms if you feel ambitious. This is easiest with a graphing calculator or a computer, although it can also be done by hand.
   * d. At $t = 0$ and also at the end of one full cycle, the slope of this sum must approach ____ as the number of terms approaches infinity. *Please explain*.
   * e. What is the limiting value of the slope at the *midpoint* of the cycle as “n” approaches infinity? ______ *Please explain*

3. Here are some other special harmonic series which you might like to explore if you are interested:
   a. Try the same series as in #2 but with the even (or odd) harmonics deleted.
   b. Try this series of amplitudes: $1, 0, 3^{-2}, 0, 7^{-2}, \ldots$
   c. Try some examples using cosines instead of sines.

4. The point being made in #2 and #3 is that almost any periodic wave shape can be produced by adding up a harmonic series. Joseph Fourier proved that theorem in 1807. He also showed how the appropriate amplitudes can be determined from the shape of any given periodic wave.

5. Notice that the summation of a harmonic series has the same period as its fundamental. Will that be true if we add terms with frequencies that are not members of the harmonic series?

6. Suppose we add waves with periods of 1/2 second and 1/3 second: What is the period of their sum?

7. A clarinet is approximately a cylinder with hard reflection at one end and soft reflection at the other.
   a. What mode numbers are possible in the clarinet's standing wave patterns?
   b. For that reason, certain harmonics are conspicuously absent from its tone. Which ones do you expect to be missing? (If you need help, review page 140.)

8. Members of the brass instrument family, such as trumpets and bugles, behave like a tube with *soft* reflection at each end. Should we expect any harmonics to be absent from their tones?

1. When two notes with certain frequency ratios are played or sung together the resulting sounds are considered by most people to be pleasing. Pythagoras and his students are said to have spent a lot of time studying these special frequency ratios. (Musicians call them "**intervals**".) If you are interested in music you should learn to recognize the sounds of these special intervals, since they are part of the basic vocabulary of music.
   a. The second harmonic (or two-humped standing wave pattern) on a guitar string or bugle sounds one octave higher than the first. That tells us that raising the pitch of a tone by one "octave" means exactly doubling its frequency. It's called an "octave" because it is the interval between the first and the eighth note of a diatonic ("do-re-mi..") scale.
   b. Raising the pitch of a tone by a "perfect fifth" means multiplying its frequency by 3/2. This is the interval between the second and third harmonics on a string or bugle. It is called a "fifth" because it is the interval between the first and fifth notes in a diatonic scale.
   c. Raising the pitch by a "perfect fourth" means multiplying the frequency by 4/3.
   d. Raising a pitch by a "major third" means multiplying the frequency by 5/4.
   e. A "minor third" means a factor of 6/5.

2. Here are some easy questions about intervals:
   a. Exactly how do you alter the frequency of a tone to make its pitch an octave lower?
      -You divide the frequency by ___ or multiply the frequency by ____.
   b. How do you lower the pitch by a fifth, a fourth, a major third, and a minor third?
      -You _________ the frequency by ____, ____, or _____.
   c. What happens to the frequency of a tone when we raise its pitch by *two* consecutive perfect fifths?
      -Its frequency is multiplied *twice* by a factor of ____, so the new frequency is ____ times the original frequency.

3. A "chord" is three or more tones played together. A "triad" is a three-note chord consisting of the first, third, and fifth notes of a scale. The first note is called the "root" of the triad.
   a. In a "major" triad the middle note is a "major third" above the root, so it has a frequency of ___ times the root's frequency.
   b. In a "minor" triad the interval from the root to the middle note is a "minor third", so the middle note's frequency is ___ times the root frequency.
   c. In both of those simple triads the upper note is a "fifth" above the lower note, so its frequency is ___ times the first frequency.
   d. The interval between the middle and upper tones in a major triad is a ______ third, because its frequency ratio is ____.
   e. The interval between the middle and upper notes in the minor triad is a _______ ______, because its frequency ratio is _____.

4. Let "F_1" represent the frequency of the root in a certain triad.
   The frequencies of the other two tones will be called "F_2" and "F_3", respectively.
   a. If it's a major triad, then F_2 = (___)F_1 and F_3 = (___)F_1, as in 3a.
   b. If it's a *minor* triad then the lower interval is a _______ ______ and the upper one is a _______ ________, so F_2 = (___)F_1 as in 3b.
   c. The waveform of a triad can be determined by adding up the waveforms of its three components: It will be a periodic function, but its period will *not* be the reciprocal of F_1. How can the period of the combined waveform be determined from F_1?
   d. Is the answer to 4c the same for both major and minor triads?
   e. Is it possible to raise or lower any of the notes of a triad by one octave without altering that period? *Please explain with specific examples.*

1. You need three chords to play the blues:
   a. You start with a major triad called a "tonic" triad, in honor of the musician's favorite beverage. Let's choose symbols to represent the frequencies of those three notes:

$$\mathbf{f_1}\ , \qquad \mathbf{f_3 = (5/4)f_1}\ , \qquad \text{and} \qquad \mathbf{f_5 = (3/2)f_1}$$

   b. Then you play another major triad, a perfect fourth higher:
   (Please fill in the blanks with fractions in "top-heavy" form.)

$$\mathbf{f_4} = (4/3)\mathbf{f_1}\ , \quad \mathbf{f_6} = (5/4)\mathbf{f_4} = \_\_\_\mathbf{f_1}\ , \qquad \mathbf{f_8} = \_\_\_\mathbf{f_4} = \_\_\_\mathbf{f_1}$$

   c. Before ending the second line of the blues, you always go back to the tonic triad. Then you start the third line with a triad a perfect *fifth* higher than the original:

$$\mathbf{f_5} = \_\_\_\_\mathbf{f_1}\ , \qquad \mathbf{f_7} = \_\_\_\mathbf{f_5} = \_\_\_\mathbf{f_1}\ , \qquad \mathbf{f_9} = \_\_\_\mathbf{f_5} = \_\_\_\mathbf{f_1}$$

   d. If you have trouble with high notes, you can bring that last note down an octave:

$$\mathbf{f_2 = (1/2)f_9 = \_\_\_f_1}$$

2. Now suppose you have constructed a musical instrument that can play notes with each of the frequencies listed above. If you tried to teach someone to play the instrument, you would need names for each note. Historical development leaves us with the following note names:

$\mathbf{f_1}$ = "C"	$\mathbf{f_3}$ = "E"	$\mathbf{f_5}$ = "G"	$\mathbf{f_7}$ = "b
$\mathbf{f_2}$ = "D"	$\mathbf{f_4}$ = "F"	$\mathbf{f_6}$ = "a"	$\mathbf{f_9}$ = "d"

The letter names correspond to the names of the white keys on a piano or synthesizer. This "diatonic" scale can be extended as far as you like in either direction by doubling or halving frequencies. For example, f_8 is twice the frequency of f_1, and is called “c”. If you try the activities below, you will find discover some other interesting things:
   a. Mark the locations of the frequencies on an ordinary number line.
   b. Use a calculator to determine the logarithms of the frequencies.
   Mark those numbers on another number line.
   c. Examine the triad formed by "D", "F", and "a". Is it a major triad, or is it minor? ___
   Are its frequency ratios exactly right? ____ If not, explain what is wrong.
   d. Critically examine the "E" triad and the "A" triad also.
   e. Why is it not necessary to examine the "F" and "G" triads?

3. If you did 3b, then you can see why there are black keys between some but not all of the adjacent pairs of white keys on a piano. The intervals represented by the larger spaces on your number line are called "whole steps". (Those are the spaces that are big enough for a "black" note to be squeezed in.) The intervals represented by the smaller spaces on the number line are called "semitones", or "half-steps". If you do the exercises suggested below, you will discover some interesting facts about whole steps and semitones:
   a. Calculate the frequency ratio for each adjacent pair of notes in the diatonic scale.
   Express them as rational fractions, and classify them as whole steps or semitones.
   b. Are all whole steps the same? ____ What about the semitones? ____
   c. Are two semitones exactly equivalent to a whole step?
   d. How many semitones can be squeezed into one octave if we fill in all of the gaps?

4. An “equal-tempered scale” can be constructed, in which all of the semitones *do* have equal frequency ratios, and there are exactly 12 semitones in each octave, as in 3d.
   a. How can that special frequency ratio be calculated?
   c. By how much do the notes in an equal-tempered scale differ from the notes in a diatonic scale?