

1. Simple Harmonic Motion: Recall that the displacement vs. time graph for any object in simple harmonic motion can be written as  $y = A \sin(\omega t)$ , where "A" represents the amplitude of the motion, "t" represents the time, and " $\omega$ " represents "angular frequency" in radians per second. (Angular frequency is just a proportionality constant which is equal to the regular frequency multiplied by \_\_, because there are \_\_ radians in a complete cycle.)
  - a. Use differentiation (#14 on RS XII) to create equations describing the velocity-time graph and the acceleration-time graph in terms of the same variables:  $v = \underline{\hspace{2cm}}$ ;  $a = \underline{\hspace{2cm}}$
  - b. Notice how the acceleration must be related to the displacement. With this and Newton's second law you can make a conclusion about how the restoring force must be related to the displacement:  
 $F = \underline{\hspace{2cm}}$
  - c. With that relationship and the definition of "spring constant" you can show how the frequency must be related to the spring constant ("k") and the mass of the object:  $f = \underline{\hspace{2cm}}$  *Do the units balance?* \_\_
  - d. Does this formula agree with what you wrote on RS VIII?  $\underline{\hspace{2cm}}$
  - e. Is the derivation above simpler than the ones used previously, or is it more complicated? \_\_
2. The Capacitive Reactance Formula: Consider a capacitor connected to a sine generator.
  - a. Write an equation describing the capacitor voltage vs. time graph, using  $\|V_c\|$  to represent the *amplitude* of the capacitor voltage:  $V_c = \|V_c\| (\underline{\hspace{2cm}})$
  - b. Use the capacitor law to determine the equation of the charge vs. time graph:  $Q = \underline{\hspace{2cm}}$
  - c. Use the definition of current to write the equation of the current vs. time graph:  $I = \underline{\hspace{2cm}}$
  - d. Express  $\|I\|$  in terms of  $\|V\|$  and other constants:  $\|I\| = \underline{\hspace{2cm}}$
  - e. Use the definition of capacitive reactance to discover a formula for calculating it from capacitance and angular frequency:  $X_c = \underline{\hspace{2cm}}$
  - f. Does this agree with earlier theories and with experimental data? \_\_
3. Impedance of a Series R-C Combination: (*This is new!*)  
The "Impedance" of *any* AC circuit component is defined as a ratio of the amplitude of the voltage across the component to the amplitude of the current through it:  $Z = \|V\| \div \|I\|$   
Suppose that the component in question is the series combination of a resistor and a capacitor, and that the current is  $I = \|I\| \cos \omega t$ , where  $\|I\|$  represents the current amplitude, and "omega" represents the angular frequency, as in #1.
  - a. Write an equation describing the resistor's voltage vs. time graph in terms of  $\|I\|$ , R,  $\omega$ , and t:  
 $V_r = \underline{\hspace{2cm}}$
  - b. Use #2d to write an equation describing the capacitor's voltage vs. time graph in terms of  $\|I\|$ ,  $X_c$ ,  $\omega$ , and t:  $V_c = \underline{\hspace{2cm}}$
  - c. The expression for  $V_r$  can be given a neat geometric interpretation: Imagine a vector with magnitude  $R \cdot \|I\|$  which is rotating at " $\omega$ " radians per second clockwise, with its tail tacked to the origin of a coordinate system. Notice that the x-component of that vector is equal at all times to  $V_r$ . Draw a diagram to illustrate this. Label it clearly.
  - d. A similar interpretation can be given to the expression for  $V_c$ . At all times, the angle between these rotating vectors must be \_\_ degrees. Illustrate that with a new diagram. Label each part clearly.
  - e. The sum of  $V_r$  and  $V_c$  must therefore be equal at all times to the \_\_-component of the \_\_ of the two vectors described above.
  - f. Notice that the diagram for that vector sum forms a \_\_ triangle which also rotates clockwise as time goes on. The x-component of the sum is equal at all times to the voltage across the \_\_\_\_\_.
  - g. Now you can use simple geometry to express the *amplitude of the combination voltage* in terms of *current amplitude, R, and  $X_c$* : Combination Voltage Amplitude =  $\|I\| (\underline{\hspace{2cm}})$
  - h. Notice that the blank which you just filled in fits the definition of "impedance". Therefore the formula for the impedance of a series R-C combination must be:  $Z = \underline{\hspace{2cm}}$
  - i. Will the voltage vs. time graph for the combination be in phase with the current vs. time graph? \_\_  
If not, how can their phase difference be predicted from R and  $X_c$ ? Phase diff. =  $\underline{\hspace{2cm}}$   
*The "Phase Diagram" that you made for 3f can help.*
- \* 4. The reciprocal of impedance is called "admittance". Using #3 as a model, show how the admittance of a *parallel* R-C combination can be predicted.

1. Turn the calculator on and hit the **Y=** key. (For TI-85 or 86 hit **GRAPH**, then **F1**.) Delete or switch off any equations already stored there. Then put in the equation of a sine curve:  $y = \sin x$
2. To avoid having stray data points on your graph, hit the **STATPLOT** key and select choice 4.
3. To choose appropriate scales for the axes of your graph, hit **ZOOM**, and select 7 (TRIG).
4. When you enter that selection (or whenever you hit the **GRAPH** key) your graph should appear. Sketch it and show it to your teacher.
5. Now let's zoom in to examine the region near the origin on that graph:
  - a. Hit the **ZOOM** key and select **ZOOM IN** by hitting 2. (On a TI-85 hit **Z IN** or **F2**.)
  - b. Hit **ENTER** to see an enlarged picture of the central region on the screen.  
-Is the curvature still visible? \_\_\_\_
  - c. Repeat 5a-b several times to see a greatly magnified picture of the region near the origin.
  - d. In the region *very* close to the origin a sine curve *strongly* resembles a \_\_\_\_\_ line.
6. Hit the **TRACE** key. (On a TI-85 hit **GRAPH**, then **F4**.) Move the cursor along the line. Notice that the coordinates of points on the line are displayed on the screen.
  - a. Record the coordinates of one of those points (not the origin) keeping many digits:  

$$X = \underline{\hspace{2cm}}, \quad Y = \underline{\hspace{2cm}}$$
  - b. Use those coordinates to calculate the slope of your line without rounding off:  
 Slope = \_\_\_\_\_ when x is in \_\_\_\_\_s. (degrees, radians, grads)
7. Hit the **Y=** key. (On a TI-85 hit **GRAPH**, then **F1**.) Below the original equation enter a new equation in "y = mx" form to describe the straight line that you saw on the screen in #6 & 7. (Use the slope recorded in 6b.) Leave the original equation ( $y = \sin x$ ) above the new one. Then hit **ZOOM** and select 7 again. (On a TI-85 use **F2**, **ENTER**.) Sketch what you see after entering that choice, and show it to your teacher.
8. Turn on the **TRACE**. Move the cursor along the straight line until its x-value is 180 degrees or the equivalent. Record the *special y-value* that you find there:  $Y = \underline{\hspace{2cm}}$
9. Repeat # 1-8 using  $y = 2\sin x$  or  $y = \sin(2x)$  or  $y = \sin(x/2)$ .  
Share the results with your classmates and make some observations:
  - a. Doubling the amplitude of a sine curve causes the slope near the origin to be multiplied by \_\_\_\_.
  - b. Doubling the amplitude causes the "special y value" in #8 to \_\_\_\_\_.
  - c. Doubling the "argument" of a sine function (i.e. changing the equation from " $y = \sin x$ " to " $y = \sin 2x$ ") causes the "period" (width of one cycle) to be \_\_\_\_\_ed by 2 and causes the steepest slope to be \_\_\_\_\_ed by 2. The "special" Y value is \_\_\_\_\_ed.
  - d. Cutting the argument in half causes the period to \_\_\_\_\_ and the steepest slope to \_\_\_\_\_.
10. Let "A" represent the amplitude of a sine curve, and let "P" represent its period:  
For example the period of the original curve that you saw in #4 was  $P = \underline{\hspace{1cm}} \underline{\hspace{1cm}}$ .  
In 9a the amplitude was  $A = \underline{\hspace{1cm}}$  units.
  - a. The special Y value found in #8 depends only on the \_\_\_\_\_ of the sine curve.
  - b. In **every** case we find that the special Y-value is roughly \_\_\_\_\_ times the value of that \_\_\_\_\_.
  - c. In **every** case we find that the steepest slope is approximately \_\_\_\_\_ times the value of A/P.